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Fixed point results for Geraghty quasi-contraction type mappings in dislocated quasi-metric spaces

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Abstract

In this paper, fixed point results for a newly introduced Geraghty quasi-contraction type mappings are proved in more general metric spaces called T -orbitally complete dislocated quasi-metric spaces. Geraghty quasi-contraction type mappings generalize, among others, Ciric's quasi-contraction mappings and other Geraghty quasi-contractive type mappings in the literature. Fixed point results are obtained without imposing a continuity condition on the mapping, thereby further generalizing some other related work in the literature. An example is given to show the validity of results obtained.

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1 Introduction and preliminaries

Geraghty [1] generalized the Banach [2] contraction mapping in metric spaces by using an auxiliary function instead of a constant.

Let F be the family of all functions $\beta : [0, \infty) \rightarrow [0, 1)$ which satisfy the condition

$$\lim_{n \rightarrow \infty} \beta(t_n) = 1 \quad \text{implies} \quad \lim_{n \rightarrow \infty} t_n = 0.$$

Using such a function, Geraghty [1] proved the following theorem.

Theorem 1.1 ([1]) *Let (X, d) be a complete metric space and let T be a self-mapping on X . Suppose that there exists $\beta \in F$ such that, for all $x, y \in X$,*

$$d(Tx, Ty) \leq \beta(d(x, y))d(x, y), \tag{1}$$

then T has a unique fixed point $x^ \in X$ and $\{T^n x\}$ converges to x^* for all $x \in X$.*

Since then, some authors have generalized and extended the Banach contraction principle in diverse ways (see [3–7]).

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Many extensions of Banach contraction mapping have also been investigated using different forms of contractive conditions. Asadi [8] proved some fixed point results satisfying certain contraction principles on a convex metric space. Özyurt [9] established fixed point results for extended large contraction via (c)-comparison function in a complete metric space while Öztürk [10] introduced F -contraction and proved fixed point results for F -contractive iterates in a metric space. Some interesting results using other contractive conditions include [11–15].

In 2000, Hitzler [16] introduced a space known as dislocated metric space in which the self distance of points is not necessarily zero and showed that the popular Banach contraction mapping is also valid in the space. Dislocated metric space has applications in semantic analysis of logical programming, electronic engineering and in topology [16]. Zeyada et al. [17] further generalized the concept of dislocated metric space and introduced the idea of dislocated quasi-metric space. In this new notion, the symmetric property is also omitted. Some other papers have been published containing fixed point results for self-mappings with different contraction conditions in metric spaces and their generalizations including dislocated metric spaces and dislocated quasi-metric spaces (see [17–26]).

Definition 1.2 ([17]) Let X be a non-empty set and let $d : X \times X \rightarrow \mathbb{R}^+$ be a function such that the following are satisfied:

- (i) $d(x, y) = d(y, x) = 0$ implies that $x = y$;
- (ii) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called dislocated quasi-metric on X and the pair (X, d) is called a dislocated quasi-metric space.

As an improvement of α -admissible maps introduced by Samet et al. [24] and Karapınar et al. [26], Popescu [5] introduced the following concepts, which were used to prove the existence and uniqueness of fixed point results in a complete metric space.

Definition 1.3 ([5]) Let $T : X \rightarrow X$ be a self-mapping and $\alpha : X \times X \rightarrow \mathbb{R}^+$ be a function. Then T is said to be α -orbital admissible if $\alpha(x, Tx) \geq 1$ implies $\alpha(Tx, T^2x) \geq 1$.

Definition 1.4 ([5]) Let $T : X \rightarrow X$ be a self-mapping and $\alpha : X \times X \rightarrow \mathbb{R}^+$ be a function. Then T is said to be triangular α -orbital admissible if T is α -orbital admissible and $\alpha(x, y) \geq 1, \alpha(y, Ty) \geq 1$ imply $\alpha(x, Ty) \geq 1$.

Lemma 1.5 ([5]) Let $T : X \rightarrow X$ be a triangular α -orbital admissible mapping. Assume that there exists $x_1 \in X$ such that $\alpha(x_1, Tx_1) \geq 1$. Define a sequence $\{x_n\}$ by $x_{n+1} = Tx_n$. Then, we have $\alpha(x_n, x_m) \geq 1$ for all $m, n \in \mathbb{N}$ with $n < m$.

The following definition by Ćirić [25] on quasi-contraction mappings in metric spaces are also true for dislocated quasi-metric spaces.

Definition 1.6 ([25]) Let $T : X \rightarrow X$ be a self-mapping on a metric space. For each $x \in X$ and for any positive whole number n ,

$$O_T(x, n) = \{x, Tx, \dots, T^n x\} \quad \text{and} \quad O_T(x, \infty) = \{x, Tx, \dots, T^n x, \dots\}.$$

The set $O_T(x, \infty)$ is called the orbit of T at x and the metric space X is called T -orbitally complete if every Cauchy sequence in $O_T(x, \infty)$ is convergent in X .

It is clear that every complete dislocated quasi-metric space is T -orbitally complete. But the converse does not hold in general.

The purpose of this paper is to prove some fixed point results in dislocated quasi-metric space using new concepts of Geraghty quasi-contraction type self-mappings that the authors just introduced and proved fixed point results in the context of metric spaces [27]. The result is obtained by dropping the restriction of continuity and proving the existence and uniqueness of fixed point in an orbitally complete (which is a relaxation of completeness) dislocated quasi-metric space. This result generalizes many existing related work in the literature [1–5, 16, 17, 22–27].

2 Main results

Let Φ denote the class of the functions $\phi : [0, \infty) \rightarrow [0, \infty)$ which satisfy the following conditions:

- (i) ϕ is non-decreasing;
- (ii) ϕ is continuous;
- (iii) $\phi(t) = 0 \iff t = 0$.

First, we state the following new mapping introduced by the authors in [27].

Definition 2.1 Let (X, d) be a metric space and $\alpha : X \times X \rightarrow \mathbb{R}^+$ be a function. A self-mapping $T : X \rightarrow X$ is called an α - ϕ -Geraghty quasi-contraction type mapping if there exists $\beta \in F$ such that, for all $x, y \in X$,

$$\alpha(x, y)\phi(d(Tx, Ty)) \leq \beta(\phi(M_T(x, y)))\phi(M_T(x, y)), \tag{2}$$

where $M_T(x, y) = \max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}$.

Remark 2.2

- (i) From inequality (2) above, if $\alpha(x, y) = 1$, $\phi(t) = t$ and $M_T(x, y) = d(x, y)$, then we have the Geraghty [1] contraction mapping defined on a metric space. In addition, if $\beta(t) = q$; where $q \in [0, 1)$, we have the Banach contraction mapping [2]. Inequality (2) also generalizes, among others, those of Popescu [5], Karapınar [4, 28] and Cho et al. [3].
- (ii) Definition 2.1 is also true for a dislocated quasi-metric space since every metric space is a dislocated quasi-metric space but the converse is not necessarily true. An example, which is inspired by Rahman and Sarwar [22], is provided to buttress this fact.

Example 2.3 Let $X = \mathbb{R}$ and $d(x, y) = |x|$ for all $x, y \in X$. Let $\beta(t) = \frac{1}{t}$ for all $t > 0$. Then $\beta \in F$. Let $\phi(t) = 2t$ and a mapping $T : X \rightarrow X$ be defined by

$$T(x) = \begin{cases} \frac{1}{2+x}, & \text{if } x \in [0, 1], \\ 1, & \text{if } x > 1. \end{cases}$$

Define a function $\alpha : X \times X \rightarrow [0, \infty)$ by

$$\alpha(x, y) = \begin{cases} 1, & \text{if } 0 \leq x, y \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Then T is an α - ϕ -Geraghty quasi-contraction type mapping defined on a dislocated quasi-metric space but not on a metric space.

Theorem 2.4 *Let (X, d) be a T -orbitally complete dislocated quasi-metric space such that $T : X \rightarrow X$ is a self-mapping. Suppose $\alpha : X \times X \rightarrow \mathbb{R}^+$ is a function satisfying the following conditions:*

- (i) T is an α - ϕ -Geraghty quasi-contraction type mapping.
- (ii) T is triangular α -orbital admissible mapping.
- (iii) There exists $x_1 \in X$ such that $\alpha(x_1, Tx_1) \geq 1$.

Then T has a fixed point $x^ \in X$ and $\{T^n x_1\}$ converges to x^* .*

Proof Let $x_1 \in X$ such that $\alpha(x_1, Tx_1) \geq 1$. Define a sequence $\{x_i\}$ by $x_{i+1} = T^i x_1$, for $1 \leq i \leq n - 1$. If $x_i = x_{i+1}$ for some $1 \leq i \leq n - 1$, then obviously T has a fixed point. Consequently, throughout the proof, we suppose that $x_i \neq x_{i+1}$ for all $i \geq 1$. By Lemma 1.5, used recursively, we have

$$\alpha(x_i, x_{i+1}) \geq 1 \quad \forall i \geq 1. \tag{3}$$

By (2), for $1 \leq j \leq n$ we get

$$\begin{aligned} \phi(d(T^i x, T^j x)) &= \phi(d(TT^{i-1} x, TT^{j-1} x)) \\ &\leq \alpha(T^{i-1} x, T^{j-1} x) \phi(d(TT^{i-1} x, TT^{j-1} x)) \\ &\leq \beta(\phi(M_T(T^{i-1} x, T^{j-1} x))) \phi(M_T(T^{i-1} x, T^{j-1} x)), \end{aligned} \tag{4}$$

where

$$\begin{aligned} \phi(M_T(T^{i-1} x, T^{j-1} x)) &= \phi(\max\{d(T^{i-1} x, T^{j-1} x), d(T^{i-1} x, T^i x), d(T^{j-1} x, T^j x), \\ &\quad d(T^{i-1} x, T^j x), d(T^{j-1} x, T^i x)\}) \\ &\leq \phi(\delta[O_T(x, n)]), \end{aligned}$$

where $\delta[O_T(x, n)] = \max\{d(T^i x, T^j x) : 0 \leq i \leq j \leq n\}$.

Note that, since the functions belonging to F are strictly smaller than one, inequality (2) implies $\phi(d(T^i x, T^j x)) \leq \phi(M_T(T^{i-1} x, T^{j-1} x))$ for all $0 \leq i \leq j \leq n$.

The case $\phi(M_T(T^{i-1} x, T^{j-1} x)) = \phi(d(T^i x, T^j x))$ is impossible. For

$$\begin{aligned} \phi(d(T^i x, T^j x)) &\leq \beta(\phi(M_T(T^{i-1} x, T^{j-1} x))) \phi(M_T(T^{i-1} x, T^{j-1} x)) \\ &\leq \beta(\phi(d(T^i x, T^j x))) \phi(d(T^i x, T^j x)) \\ &< \phi(d(T^i x, T^j x)), \end{aligned}$$

is a contradiction. Thus, we conclude that $\phi(d(T^i x, T^j x)) < \phi(M_T(T^{i-1} x, T^{j-1} x))$ for all $0 \leq i \leq j \leq n$. Thus, the sequence $\{d(T^i x, T^j x)\}$ is positive and decreasing. Consequently, there exists $r \geq 0$ such that

$$\lim_{i,j \rightarrow \infty} d(T^i x, T^j x) = r.$$

We claim that $r = 0$. Suppose, on the contrary, that $r > 0$. Then, from (4) we have

$$\frac{\phi(d(T^i x, T^j x))}{\phi(M_T(T^{i-1} x, T^{j-1} x))} \leq \beta(\phi(M_T(T^{i-1} x, T^{j-1} x))) < 1,$$

$$\lim_{i,j \rightarrow \infty} \beta(\phi(M_T(T^{i-1} x, T^{j-1} x))) = 1.$$

Since $\beta \in F$, by definition, it implies that

$$\lim_{i,j \rightarrow \infty} \phi(M_T(T^{i-1} x, T^{j-1} x)) = 0 \tag{5}$$

and so

$$r = \lim_{i,j \rightarrow \infty} (d(T^i x, T^j x)) = 0,$$

which is a contradiction.

Next, we show that the sequence $\{d(T^i x, T^j x)\}$ is Cauchy. On the contrary, suppose

$$\epsilon = \lim_{m,n \rightarrow \infty} d(T^{n-1} x, T^{m-1} x) > 0, \quad n \geq m. \tag{6}$$

Using the triangle inequality,

$$d(T^{n-1} x, T^{m-1} x) \leq d(T^{n-1} x, T^n x) + d(T^n x, T^m x) + d(T^m x, T^{m-1} x)$$

implying

$$d(T^{n-1} x, T^{m-1} x) - d(T^{n-1} x, T^n x) - d(T^m x, T^{m-1} x) \leq d(T^n x, T^m x).$$

Now, using ϕ , we have

$$\begin{aligned} \phi(d(T^{n-1} x, T^{m-1} x) - d(T^{n-1} x, T^n x) - d(T^m x, T^{m-1} x)) &\leq \phi(d(T^n x, T^m x)) \\ &\leq \alpha(x_n, x_m) \phi(d(T^{n-1} x, T^{m-1} x)) \\ &\leq \beta(\phi(M_T(T^{n-1} x, T^{m-1} x))) \\ &\quad \times \phi(M_T(T^{n-1} x, T^{m-1} x)). \end{aligned}$$

Taking the limits and using (6) we get

$$\phi(\epsilon) \leq \lim_{m,n \rightarrow \infty} \beta(\phi(M_T(T^{n-1} x, T^{m-1} x))) \phi(\epsilon),$$

$$1 \leq \lim_{m,n \rightarrow \infty} \beta(\phi(M_T(T^{n-1}x, T^{m-1}x))).$$

Therefore $\lim_{m,n \rightarrow \infty} \beta(\phi(M_T(T^{n-1}x, T^{m-1}x))) = 1$ and so $\lim_{m,n \rightarrow \infty} \phi(M_T(T^{n-1}x, T^{m-1}x)) = 0$. Thus $\lim_{m,n \rightarrow \infty} d(T^{n-1}x, T^{m-1}x) = 0$, which contradicts our assumption.

Thus, the sequence $\{d(T^i x, T^j x)\}$ is Cauchy. Since X is T -orbitally complete, there exists $x^* \in X$ such that $\lim_{n \rightarrow \infty} T^n x = x^*$. To show that $Tx^* = x^*$, suppose that

$$d(x^*, Tx^*) = \lim_{i \rightarrow \infty} d(T^i x, Tx^*) > 0.$$

Now,

$$d(x^*, Tx^*) \leq d(x^*, T^i x) + d(T^i x, Tx^*)$$

implying

$$d(x^*, Tx^*) - d(x^*, T^i x) \leq d(T^i x, Tx^*).$$

Applying ϕ , we have

$$\begin{aligned} \phi(d(x^*, Tx^*) - d(x^*, T^i x)) &\leq \phi(d(T^i x, Tx^*)) \\ &\leq \alpha(T^{i-1}x, x^*)\phi(d(TT^{i-1}x, Tx^*)) \\ &\leq \beta(\phi(M_T(T^{i-1}x, x^*)))\phi(M_T(T^{i-1}x, x^*)), \end{aligned}$$

where $M_T(T^{i-1}x, x^*) = \max\{d(T^{i-1}x, x^*), d(x^*, Tx^*), d(T^{i-1}x, T^i x), d(x^*, T^i x), d(T^{i-1}x, Tx^*)\}$.

Taking limits as i tends to infinity gives

$$\lim_{i \rightarrow \infty} \phi(d(x^*, Tx^*)) \leq \lim_{i \rightarrow \infty} \beta(\phi(M_T(T^{i-1}x, x^*))) \lim_{i \rightarrow \infty} \phi(d(x^*, Tx^*)).$$

Also

$$1 \leq \lim_{i \rightarrow \infty} \beta(\phi(M_T(T^{i-1}x, x^*))).$$

So, by the definition of $\beta \in F$, we get

$$\begin{aligned} \lim_{i \rightarrow \infty} \beta(\phi(M_T(T^{i-1}x, x^*))) = 1 &\rightarrow \lim_{i \rightarrow \infty} \phi(M_T(T^{i-1}x, x^*)) = 0 \\ &\rightarrow \phi(d(x^*, Tx^*)) = 0 \\ &\rightarrow d(x^*, Tx^*) = 0, \end{aligned}$$

a contradiction. Thus, we obtain $d(x^*, Tx^*) = 0$. Similarly, $d(Tx^*, x^*) = 0$. That is, $x^* = Tx^*$ and the fixed point of T is x^* . □

To ensure the uniqueness of a fixed point, consider the following hypothesis:

(J): For all $x \neq y \in \text{Fix}(T)$ there exists $w \in X$ such that $\alpha(x, w) \geq 1$, $\alpha(y, w) \geq 1$ and $\alpha(w, Tw) \geq 1$.

$\text{Fix}(T)$ denotes the set of fixed points of T .

Theorem 2.5 *Adding condition (J) to the conditions of Theorem 2.4, we find that x^* is a unique fixed point of T .*

Proof From the proof of Theorem 2.4, x^* is a fixed point of T . Assume that x_1^* and x_2^* are distinct fixed points of T . By (J), there exists $w \in X$ such that $\alpha(x_1^*, w) \geq 1$, $\alpha(x_2^*, w) \geq 1$ and $\alpha(w, Tw) \geq 1$. By condition (ii) in Theorem 2.4, $\alpha(x_1^*, T^n w) \geq 1$ and $\alpha(x_2^*, T^n w) \geq 1$ for all $n \geq 1$. So

$$\begin{aligned} \phi(d(x_1^*, T^{n+1}w)) &\leq \alpha(x_1^*, T^n w)\phi(d(Tx_1^*, T^{n+1}w)) \\ &\leq \beta(\phi(M_T(x_1^*, T^n w)))\phi(M_T(x_1^*, T^n w)) \end{aligned}$$

for all $n \geq 1$, where

$$\begin{aligned} M_T(x_1^*, T^n w) &= \max\{d(x_1^*, T^n w), d(T^n w, T^{n+1}w), d(x_1^*, Tx_1^*), d(Tx_1^*, T^n w), \\ &\quad d(x_1^*, T^{n+1}w)\} \\ &= \max\{d(x_1^*, T^n w), d(T^n w, T^{n+1}w), d(x_1^*, T^{n+1}w), d(x_1^*, T^n w)\}. \end{aligned}$$

We deduce, by Theorem 2.4, that the sequence $\{T^n w\}$ converges to a fixed point z . As $n \rightarrow \infty$ in the above inequality, we obtain $\lim_{n \rightarrow \infty} M_T(x_1^*, T^n w) = d(x_1^*, z)$. If $x_1^* \neq z$, then

$$\frac{\phi(d(x_1^*, T^{n+1}w))}{\phi(M_T(x_1^*, T^n w))} \leq \beta(\phi(M_T(x_1^*, T^n w))) < 1$$

and as $n \rightarrow \infty$, $\lim_{n \rightarrow \infty} \beta(\phi(M_T(x_1^*, T^n w))) = 1$ implies that $\lim_{n \rightarrow \infty} \phi(M_T(x_1^*, T^n w)) = 0$. Thus $d(x_1^*, z) = 0$, a contradiction. Therefore, $x_1^* = z$. Similarly, $x_2^* = z$. Thus $x_1^* = x_2^*$, a contradiction and hence the fixed point of T is unique. □

In inequality (2) taking $\phi(t) = t$, an α -Geraghty quasi-contraction type mapping is obtained and Theorem 2.4 reduces to the following.

Corollary 2.6 *Let (X, d) be a T -orbitally complete dislocated quasi-metric space such that $T : X \rightarrow X$ is a self-mapping. Suppose $\alpha : X \times X \rightarrow \mathbb{R}^+$ is a function satisfying the following conditions:*

- (i) T is an α -Geraghty quasi-contraction type mapping.
- (ii) T is triangular α -orbital admissible mapping.
- (iii) There exists $x_1 \in X$ such that $\alpha(x_1, Tx_1) \geq 1$.

Then T has a fixed point $x^ \in X$ and $\{T^n x_1\}$ converges to x^* .*

Remark 2.7

- (i) Suppose the α - ϕ -Geraghty quasi-contraction type mapping is defined on a metric space, then Theorem 2.4 reduces to the result obtained by the authors in [27].
- (ii) Moreover, suppose continuity condition is imposed on the mapping T , if defined on a complete metric space, which is a stronger restriction than orbital completeness, $\phi(t) = t$ and $M_T(x, y) = \{d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Ty)+d(y, Tx)}{2}\}$ the result in Popescu [5] is obtained.

(iii) The results in Karapinar [4, 28] and Cho et al. [3] are also corollaries to our result. Therefore, Theorem 2.4 is an improvement and a generalization of other related work and hence an addition to the library of mappings in the literature.

The following examples validate Theorem 2.4.

Example 2.8 Let $X = [0, \infty)$ and $d(x, y) = x$ for all $x, y \in X$. Let $\beta(t) = \frac{1}{1+t}$ for all $t > 0$. Then $\beta \in F$. Let $\phi(t) = 2t$ and a mapping $T : X \rightarrow X$ be defined by

$$T(x) = \begin{cases} \frac{1}{3}x, & \text{if } x \in [0, 1], \\ 2x, & \text{if } x > 1. \end{cases}$$

Define a function $\alpha : X \times X \rightarrow [0, \infty)$ by

$$\alpha(x, y) = \begin{cases} 1, & \text{if } 0 \leq x, y \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

One can easily see that X is a dislocated quasi-metric space but not a metric space. Also, the self-mapping T is not continuous at $x = 1$.

Condition (iii) of Theorem 2.4 is satisfied with $x_1 = 1$.

For condition (ii), let x, y be such that $\alpha(x, y) \geq 1$. Then, $x, y \in [0, 1]$, and $Tx, Ty \in [0, 1]$. Moreover, $\alpha(y, Ty) = \alpha(x, Tx) = 1$ and $\alpha(Tx, T^2x) = 1$. Thus, T is triangular α -orbital admissible. Therefore, (ii) is satisfied.

Finally, we prove that condition (i) is satisfied. If $0 \leq x, y \leq 1$, then $\alpha(x, y) = 1$ and

$$\begin{aligned} \beta(\phi(M_T(x, y)))\phi(M_T(x, y)) - \alpha(x, y)\phi(d(Tx, Ty)) &= \beta(\phi(M_T(x, y)))\phi(M_T(x, y)) \\ &\quad - \phi(d(Tx, Ty)) \\ &= \frac{2M_T(x, y)}{1 + 2M_T(x, y)} - 2Tx \\ &\geq 0. \end{aligned}$$

Therefore, $\alpha(x, y)\phi(d(Tx, Ty)) \leq \beta(\phi(M_T(x, y)))\phi(M_T(x, y))$ for $0 \leq x, y \leq 1$.

If $x \in [0, 1], y > 1$, or $x, y > 1$ then, obviously, $\alpha(x, y) = 0$ and we have

$$\alpha(x, y)\phi(d(Tx, Ty)) \leq \beta(\phi(M_T(x, y)))\phi(M_T(x, y)).$$

Therefore, all assumptions of Theorem 2.4 and Theorem 2.5 are satisfied, and hence T has a unique fixed point $x^* = 0$.

Example 2.9 Consider the set $X = \{0\} \cup \{\frac{1}{n} : n \in \mathbb{N}\} \cup \mathbb{N}$ and a dislocated quasi-metric $d(x, y) = |x - y| + x, \forall x, y \in X$. Let $\phi(t) = \frac{t}{2}$ and $\beta(t) = \frac{1}{t} \forall t > 0$, then $\beta \in F$. Define the mapping $T : X \rightarrow X$ by

$$T(x) = \begin{cases} \frac{1}{x}, & \text{if } x \geq 1, \\ x, & \text{otherwise.} \end{cases}$$

Also define the function $\alpha : X \times X \rightarrow \mathbb{R}^+$ by $\alpha(x, y) = 1 \forall x, y \in X$.

X is also a dislocated quasi-metric space but neither a metric space nor a dislocated metric space and the self-mapping T is not continuous.

Condition (iii) of Theorem 2.4 is also satisfied with $x_1 = 1$.

Obviously, condition (ii) is satisfied. Let x, y be such that $\alpha(x, y) \geq 1$. Then $x, y \in X$ and $Tx, Ty \in X$. Moreover, $\alpha(y, Ty) = \alpha(x, Tx) = 1$ and $\alpha(Tx, T^2x) = 1$. Thus, T is triangular α -orbital admissible and (ii) is satisfied.

Finally, we prove that condition (i) is satisfied. For $\forall x, y \in X, \alpha(x, y) = 1$ and

$$\begin{aligned} \beta(\phi(M_T(x, y)))\phi(M_T(x, y)) - \alpha(x, y)\phi(d(Tx, Ty)) &= \beta(\phi(M_T(x, y)))\phi(M_T(x, y)) \\ &\quad - \frac{d(Tx, Ty)}{2} \\ &= \beta\left(\frac{M_T(x, y)}{2}\right)\left(\frac{M_T(x, y)}{2}\right) \\ &\quad - \frac{|Tx - Ty| + Tx}{2} \\ &= \left(\frac{2}{M_T(x, y)}\right)\left(\frac{M_T(x, y)}{2}\right) \\ &\quad - \frac{|Tx - Ty| + Tx}{2} \\ &= \frac{2 - (|Tx - Ty| + Tx)}{2} \\ &\geq 0. \end{aligned}$$

Therefore, $\alpha(x, y)\phi(d(Tx, Ty)) \leq \beta(\phi(M_T(x, y)))\phi(M_T(x, y)), \forall x, y \in X$.

Therefore, all assumptions of Theorem 2.4, Theorem 2.5 are satisfied, and hence T has a unique fixed point at $x^* = 1$.

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Authors' contributions

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