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$F(\psi,\varphi)$-Contractions for α-admissible mappings on M-metric spaces

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Abstract

In this paper, we introduce certain α-admissible mappings which are $F(\psi,\varphi)$-contractions on M-metric spaces, and we establish some fixed point results. Our results generalize and extend some well-known results on this topic in the literature.

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1 Introduction and preliminaries

Geraghty in [10] introduced an interesting class of auxiliary functions to refine the Banach contraction mapping principle. Let $\mathcal{F}$ be the function $\beta:[0,\infty)\to[0,1)$ which satisfies the condition

$$\lim_{n\to\infty}\beta(t_n)=1 \quad \text{implies} \quad \lim_{n\to\infty}t_n=0.$$

By using $\mathcal{F}$, Geraghty [10] proved the following theorem.

Theorem 1.1 ([10]) *Let (X,d) be a complete metric space and $T:X\to X$ be an operator. Suppose that there exists $\beta\in\mathcal{F}$ satisfying the condition*

$$\beta(t_n)\to 1 \quad \textit{implies} \quad t_n\to 0.$$

If T satisfies the following inequality

$$d(Tx,Ty)\le\beta\bigl(d(x,y)\bigr)d(x,y) \quad \textit{for any } x,y\in X, \tag{1}$$

then T has a unique fixed point.

We now present definitions, lemmas, remarks, and examples that we will use.

Definition 1.2 ([4]) Let $f:X\to X$ and $\alpha:X\times X\to[0,+\infty)$. We say that f is an α-admissible mapping if $\alpha(x,y)\ge 1$ implies $\alpha(fx,fy)\ge 1$ for all $x,y\in X$.

Definition 1.3 ([12]) Let Ψ denote all functions $\psi : [0,\infty) \to [0,\infty)$ satisfying:

(i) ψ is strictly increasing and continuous,

(ii) $\psi(t) = 0$ if and only if $t = 0$.

Definition 1.4 ([5]) An ultra altering distance function is a continuous, nondecreasing mapping $\varphi : [0,\infty) \to [0,\infty)$ such that $\varphi(t) > 0$ for $t > 0$.

Remark 1.5 We let Φ denote the class of the ultra altering distance functions.

Definition 1.6 ([5]) A mapping $F : [0,\infty)^2 \to \mathbb{R}$ is called a C-*class* function if it is continuous and satisfies the following axioms:

1. $F(s,t) \leq s$;
2. $F(s,t) = s$ implies that either $s = 0$ or $t = 0$ for all $s,t \in [0,\infty)$.

We denote the C-class functions by $\mathcal{C}$.

Example 1.7 ([5]) The following functions are elements of $\mathcal{C}$:

1. $F(s,t) = s - t$.
2. $F(s,t) = ms$, $0 < m < 1$.
3. $F(s,t) = \frac{s}{(1+t)^r}$; $r \in (0,\infty)$.
4. $F(s,t) = s\beta(s)$, $\beta : [0,\infty) \to (0,1)$ and is continuous.
5. $F(s,t) = s - (\frac{2+t}{1+t})t$.
6. $F(s,t) = \sqrt[n]{\ln(1 + s^n)}$.

Definition 1.8 ([18], [22, Definition 1.1]) A partial metric on a nonempty set X is a function $p : X \times X \to \mathbb{R}^+$ such that, for all $x,y,z \in X$:

(p1) $p(x,x) = p(y,y) = p(x,y) \iff x = y$,

(p2) $p(x,x) \leq p(x,y)$,

(p3) $p(x,y) = p(y,x)$,

(p4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$.

A partial metric space is a pair (X,p) such that X is a nonempty set and p is a partial metric on X.

For more details and examples see [14–16].

Definition 1.9 ([7]) Let X be a nonempty set. A function $\mu : X \times X \to \mathbb{R}^+$ is called an m-metric if the following conditions are satisfied:

(m1) $\mu(x,x) = \mu(y,y) = \mu(x,y) \iff x = y$,

(m2) $m_{xy} \leq \mu(x,y)$,

(m3) $\mu(x,y) = \mu(y,x)$,

(m4) $(\mu(x,y) - m_{xy}) \leq (\mu(x,z) - m_{xz}) + (\mu(z,y) - m_{zy})$,

where

$$m_{xy} := \min\{\mu(x,x), \mu(y,y)\}.$$

Then the pair (X,μ) is called an M-metric space. The following notation is useful in the sequel:

$$M_{xy} := \max\{\mu(x,x), \mu(y,y)\}.$$

Remark 1.10 ([7]) For every $x, y \in X$,

1. $0 \leq M_{xy} + m_{xy} = \mu(x,x) + \mu(y,y)$;
2. $0 \leq M_{xy} - m_{xy} = |\mu(x,x) - \mu(y,y)|$;
3. $M_{xy} - m_{xy} \leq (M_{xz} - m_{xz}) + (M_{zy} - m_{zy})$.

2 Topology on *M*-metric space

It is clear that each M-metric m on X generates a T_0 topology τ_m on X. The set

$$\{B_\mu(x,\varepsilon) : x \in X, \varepsilon > 0\},$$

where

$$B_\mu(x,\varepsilon) = \{y \in X : \mu(x,y) < m_{x,y} + \varepsilon\},$$

for all $x \in X$ and $\varepsilon > 0$, forms the base of τ_m.

Definition 2.1 ([7]) Let (X,μ) be an M-metric space. Then:

1. A sequence $\{x_n\}$ in an M-metric space (X,m) converges to a point $x \in X$ if
$$\lim_{n\to\infty} \left(\mu(x_n,x) - m_{x_n,x}\right) = 0. \tag{2}$$
2. A sequence $\{x_n\}$ in an M-metric space (X,m) is called an m-Cauchy sequence if
$$\lim_{n,m\to\infty} \left(\mu(x_n,x_m) - m_{x_n,x_m}\right) \quad \text{and} \quad \lim_{n,m\to\infty} (M_{x_n,x_m} - m_{x_n,x_m}) \tag{3}$$
exist (and are finite).
3. An M-metric space (X,m) is said to be complete if every m-Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_m, to a point $x \in X$ such that
$$\left(\lim_{n\to\infty} \left(\mu(x_n,x) - m_{x_n,x}\right) = 0 \text{ and } \lim_{n\to\infty} (M_{x_n,x} - m_{x_n,x}) = 0\right).$$

Lemma 2.2 ([7]) *Assume that $x_n \to x$ and $y_n \to y$ as $n \to \infty$ in an M-metric space (X,m). Then*

$$\lim_{n\to\infty} \left(\mu(x_n,y_n) - m_{x_n,y_n}\right) = \mu(x,y) - m_{xy}.$$

Lemma 2.3 ([7]) *Assume that $x_n \to x$ as $n \to \infty$ in an M-metric space (X,m). Then*

$$\lim_{n\to\infty} \left(\mu(x_n,y) - m_{x_n,y}\right) = \mu(x,y) - m_{x,y}$$

for all $y \in X$.

Lemma 2.4 ([7]) *Assume that $x_n \to x$ and $x_n \to y$ as $n \to \infty$ in an M-metric space (X,m). Then $\mu(x,y) = m_{xy}$. Further if $\mu(x,x) = \mu(y,y)$, then $x = y$.*

3 Methods

Many authors studied the class of $\alpha - \psi$ contractive type mappings and obtained fixed point results for this new class of mappings in metric spaces. Their results contain several well-known fixed point theorems including the Banach contraction principle.

The goal of this article is to introduce the class of $F(\psi, \varphi)$-contractions and investigate the existence and uniqueness of fixed points for α-admissible mappings on M-metric spaces.

4 Discussion and main results

We start this section with the following main theorem.

Theorem 4.1 *Let (X, μ) be a complete M-metric space and $T : X \to X$ be an α-admissible mapping. Suppose that the following condition is satisfied*:

$$\big(\psi\big(\mu(Tx, Ty)\big) + l\big)^{\alpha(x,Tx)\alpha(y,Ty)} \le F\big(\psi\big(\mu(x, y)\big), \varphi\big(\mu(x, y)\big)\big) + l \tag{4}$$

for all $x, y \in X$ and $l \ge 1$, where $\psi \in \Psi$, $\varphi \in \Phi$, and $F \in \mathcal{C}$. Suppose that either

(a) *T is continuous,*

or

(b) *if $\{x_n\}$ is a sequence in X such that $x_n \to x$, $\alpha(x_n, x_{n+1}) \ge 1$ for all n, then $\alpha(x, Tx) \ge 1$.*

If there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \ge 1$, then T has a fixed point.

Proof Let $x_0 \in X$ be such that $\alpha(x_0, Tx_0) \ge 1$. Define a sequence $\{x_n\}$ in X by $x_n = T^n x_0 = Tx_{n-1}$ for all $n \in \mathbb{N}$. Since T is an α-admissible mapping and $\alpha(x_0, Tx_0) \ge 1$, we deduce that $\alpha(x_1, x_2) = \alpha(Tx_0, T^2 x_0) \ge 1$. Continuing this process, we get $\alpha(x_n, Tx_n) \ge 1$ for all $n \in \mathbb{N} \cup \{0\}$. From inequality (4) we have

$$\begin{aligned}\psi\big(\mu(Tx_{n-1}, Tx_n)\big) + l &\le \big(\psi\big(\mu(Tx_{n-1}, Tx_n) + l\big)\big)^{\alpha(x_{n-1},Tx_{n-1})\alpha(x_n,Tx_n)} \\ &\le F\big(\psi\big(\mu(x_{n-1}, x_n)\big), \varphi\big(\mu(x_{n-1}, x_n)\big)\big) + l.\end{aligned}$$

Then we have

$$\psi\big(\mu(x_n, x_{n+1})\big) \le F\big(\psi\big(\mu(x_{n-1}, x_n)\big), \varphi\big(\mu(x_{n-1}, x_n)\big)\big) \le \psi\big(\mu(x_{n-1}, x_n)\big). \tag{5}$$

We want to prove that $\mu(x_n, x_{n+1}) \to 0$, as $n \to \infty$. If $\mu(x_{n_0}, x_{n_0+1}) = 0$, for some $n_0 \in \mathbb{N}$, then by (5)

$$0 \le \mu(x_{n_0+1}, x_{n_0+2}) \le F\big(\psi\big(\mu(x_{n_0}, x_{n_0+1})\big), \varphi\big(\mu(x_{n_0}, x_{n_0+1})\big)\big) \le \psi\big(\mu(x_{n_0}, x_{n_0+1})\big),$$

hence from the properties of functions F, ψ, and φ we have $\mu(x_{n_0+1}, x_{n_0+2}) = 0$ which means

$$\mu(x_n, x_{n+1}) = 0 \quad \text{for all } n \ge n_0, \quad \text{and thus} \quad \mu(x_n, x_{n+1}) \to 0 \quad \text{as } n \to \infty.$$

Now let

$$\mu(x_n, x_{n+1}) > 0 \quad \text{for all } n \in \mathbb{N}.$$

Inequality (5) implies that $\mu(x_n, x_{n+1}) \leq \mu(x_{n-1}, x_n)$. It follows that the sequence $\{\mu(x_n, x_{n+1})\}$ is decreasing. Thus, there exists $m \in \mathbb{R}_+$ such that

$$\lim_{n\to\infty} \mu(x_n, x_{n+1}) = m.$$

We want to prove that $m = 0$. Let $m > 0$. From (5) we have

$$\begin{aligned}\limsup_{n\to\infty} \psi\bigl(\mu(x_n, x_{n+1})\bigr) &\leq \limsup_{n\to\infty} F\bigl(\psi\bigl(\mu(x_{n-1}, x_n)\bigr), \varphi\bigl(\mu(x_{n-1}, x_n)\bigr)\bigr)\\ &\leq \limsup_{n\to\infty} \psi\bigl(\mu(x_{n-1}, x_n)\bigr).\end{aligned}$$

Hence we get

$$\psi(m) \leq F\bigl(\psi(m), \varphi(m)\bigr) \leq \psi(m),$$

so

$$F\bigl(\psi(m), \varphi(m)\bigr) = \psi(m).$$

Using the properties of functions F, ψ, and φ, we obtain that $\psi(m) = 0$ or $\varphi(m) = 0$, so then $m = 0$, which is a contradiction. Therefore

$$\mu(x_n, x_{n+1}) \to 0 \quad \text{as } n \to \infty. \tag{6}$$

Now we prove that $\{x_n\}$ is an M-Cauchy sequence in (X, μ). We have

$$\begin{aligned}&\lim_{n\to\infty} \mu(x_n, x_{n+1}) = 0,\\ &0 \leq m_{x_n,x_{n+1}} \leq \mu(x_n, x_{n+1}) \quad \Rightarrow \quad \lim_{n\to\infty} m_{x_n,x_{n+1}} = 0,\end{aligned}$$

and

$$m_{x_n,x_{n+1}} = \min\bigl\{\mu(x_n, x_n), \mu(x_{n+1}, x_{n+1})\bigr\} \quad \Rightarrow \quad \lim_{n\to\infty} \mu(x_n, x_n) = 0.$$

On the other hand,

$$m_{x_n,x_m} = \min\bigl\{\mu(x_n, x_n), \mu(x_m, x_m)\bigr\} \quad \Rightarrow \quad \lim_{n,m\to\infty} m_{x_n,x_m} = 0,$$

so

$$\lim_{n,m\to\infty} (M_{x_n,x_m} - m_{x_n,x_m}) = 0.$$

We show

$$\lim_{n,m\to\infty} \bigl(\mu(x_n, x_m) - m_{x_n,x_m}\bigr) = 0.$$

Let

$$M^*(x,y) := \mu(x,y) - m_{x,y}, \quad \forall x,y \in X.$$

If $\lim_{n,m\to\infty} M^*(x_n,x_m) \neq 0$, there exist $\varepsilon > 0$ and $\{l_k\} \subset \mathbb{N}$ such that

$$M^*(x_{l_k},x_{n_k}) \geq \varepsilon.$$

Suppose that k is the smallest integer which satisfies the above equation such that

$$M^*(x_{l_k-1},x_{n_k}) < \varepsilon.$$

Now by (m4) we have

$$\varepsilon \leq M^*(x_{l_k},x_{n_k}) \leq M^*(x_{l_k},x_{l_k-1}) + M^*(x_{l_k-1},x_{n_k}) < M^*(x_{l_k},x_{l_k-1}) + \varepsilon.$$

Thus

$$\lim_{k\to\infty} M^*(x_{l_k},x_{n_k}) = \varepsilon,$$

which means

$$\lim_{k\to\infty} \bigl(\mu(x_{l_k},x_{n_k}) - m_{x_{l_k},x_{n_k}}\bigr) = \varepsilon.$$

On the other hand,

$$\lim_{k\to\infty} m_{x_{l_k},x_{n_k}} = 0,$$

so we have

$$\lim_{k\to\infty} \mu(x_{l_k},x_{n_k}) = \varepsilon. \tag{7}$$

Again by (m4) we have

$$M^*(x_{l_k},x_{n_k}) \leq M^*(x_{l_k},x_{l_k+1}) + M^*(x_{l_k+1},x_{n_k+1}) + M^*(x_{n_k+1},x_{n_k})$$

and

$$M^*(x_{l_k+1},x_{n_k+1}) \leq M^*(x_{l_k},x_{l_k+1}) + M^*(x_{l_k},x_{n_k}) + M^*(x_{n_k+1},x_{n_k}),$$

and taking the limit as $k \to +\infty$, together with (6) and (7), we have

$$\lim_{k\to\infty} \mu(x_{l_k+1},x_{n_k+1}) = \varepsilon. \tag{8}$$

Now by (4), (7), and (8) we have

$$\begin{aligned}\psi\bigl(\mu(x_{m_k+1},x_{n_k+1})\bigr) + l &\leq \bigl(\psi\bigl(\mu(x_{m_k+1},x_{n_k+1})\bigr) + l\bigr)^{\alpha(x_{m_k},Tx_{m_k})\alpha(x_{n_k},Tx_{n_k})} \\ &= \bigl(\psi\bigl(\mu(Tx_{m_k},Tx_{n_k}) + l\bigr)\bigr)^{\alpha(x_{m_k},Tx_{m_k})\alpha(x_{n_k},Tx_{n_k})}\end{aligned}$$

$$\begin{aligned} &\le F\bigl(\psi\bigl(\mu(x_{m_k},x_{n_k})\bigr),\varphi\bigl(\mu(x_{m_k},x_{n_k})\bigr)\bigr)+l \\ &\le \psi\bigl(\mu(x_{m_k},x_{n_k})\bigr)+l. \end{aligned}$$

Therefore we get

$$\begin{aligned} \psi\bigl(\mu(x_{m_k+1},x_{n_k+1})\bigr) &\le F\bigl(\psi\bigl(\mu(x_{m_k},x_{n_k})\bigr),\varphi\bigl(\mu(x_{m_k},x_{n_k})\bigr)\bigr) \\ &\le \psi\bigl(\mu(x_{m_k},x_{n_k})\bigr). \end{aligned}$$

Letting $k\to\infty$ in the above inequality, we get

$$\psi(\varepsilon)\le F\bigl(\psi(\varepsilon),\varphi(\varepsilon)\bigr)\le\psi(\varepsilon),$$

so

$$F\bigl(\psi(\varepsilon),\varphi(\varepsilon)\bigr)=\psi(\varepsilon).$$

Using the properties of F, ψ, and φ, we obtain $\psi(\varepsilon)=0$ or $\varphi(\varepsilon)=0$, and then $\varepsilon=0$, which is a contradiction. Therefore $\{x_n\}$ is an M-Cauchy sequence. Now, by the completeness of X, $x_n\to x$ for some $x\in X$ in the τ_m topology, i.e.,

$$\lim_{n\to\infty}\bigl(\mu(x_n,x)-m_{x_n,x}\bigr)=0$$

and

$$\lim_{n\to\infty}(M_{(x_n,x)}-m_{x_n,x})=0.$$

However, $\lim_{n\to\infty} m_{x_n,x}=0$, hence $\lim_{n\to\infty}\mu(x_n,x)=0$, and by Remark 1.10

$$\mu(x,x)=0.$$

Now suppose (a) holds. Then T is continuous and we have

$$\lim_{n\to\infty}\bigl(\mu(Tx_n,Tx)-m_{Tx_n,Tx}\bigr)=0,$$

i.e.,

$$\lim_{n\to\infty}\bigl(\mu(x_{n+1},Tx)-m_{x_{n+1},Tx}\bigr)=0,$$

and similar to the above, we have $\lim_{n\to\infty} m_{x_{n+1},Tx}=0$. Hence $\lim_{n\to\infty}\mu(x_{n+1},Tx)=0$ and by Remark 1.10, $\mu(Tx,Tx)=0$. On the other hand, $x_n\to x$ as $n\to\infty$ so by Lemma 2.3, we get

$$\bigl(\mu(x_n,Tx)-m_{x_n,Tx}\bigr)\to\bigl(\mu(x,Tx)-m_{x,Tx}\bigr)=\mu(x,Tx)\quad\text{as } n\to\infty,$$

but we have

$$\bigl(\mu(x_n,Tx)-m_{x_n,Tx}\bigr)\to 0\quad\text{as } n\to\infty.$$

Thus

$$\mu(x, Tx) = 0,$$

therefore $\mu(x, Tx) = \mu(Tx, Tx) = \mu(x, x) = 0$ and by (m1) we get

$$Tx = x.$$

Next suppose (b) holds. Then $\alpha(x, Tx) \geq 1$. Now by (4) we have

$$\begin{aligned}\psi\big(\mu(Tx_n, Tx)\big) + l &\leq \big(\psi\big(\mu(Tx_n, Tx)\big) + l\big)^{\alpha(x_n, Tx_n)\alpha(x,Tx)} \\ &\leq F\big(\psi\big(\mu(x_n, x)\big), \varphi\big(\mu(x_n, x)\big)\big) + l,\end{aligned}$$

that is, $\psi(\mu(Tx_n, Tx)) \leq F(\psi(\mu(x_n, x)), \varphi(\mu(x_n, x))) \leq \psi(\mu(x_n, x))$, and so we get

$$\mu(Tx_n, Tx) \to 0 \quad \text{as } n \to \infty.$$

On the other hand,

$$0 \leq m_{Tx_n, Tx} \leq \mu(Tx_n, Tx) \to 0 \quad \text{as } n \to \infty.$$

Thus $Tx_n \to Tx$ in the τ_m topology.

The proof of $Tx = x$ follows as in (a). □

Theorem 4.2 *Let (X, μ) be a complete M-metric space and $T : X \to X$ be an α-admissible mapping. Suppose that the following condition is satisfied*:

$$\big(\alpha(x, Tx)\alpha(y, Ty) + 1\big)^{\psi(\mu(Tx,Ty))} \leq 2^{F(\psi(\mu(x,y)),\varphi(\mu(x,y)))} \tag{9}$$

for all $x, y \in X$, where $\psi \in \Psi$, $\varphi \in \Phi$, and $F \in \mathcal{C}$. Suppose that either

(a) *T is continuous,*

or

(b) *if $\{x_n\}$ is a sequence in X such that $x_n \to x$, $\alpha(x_n, x_{n+1}) \geq 1$ for all n, then $\alpha(x, Tx) \geq 1$.*

If there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$, then T has a fixed point.

Proof Let $x_0 \in X$ be such that $\alpha(x_0, Tx_0) \geq 1$. Define a sequence $\{x_n\}$ in X by $x_n = T^n x_0 = Tx_{n-1}$ for all $n \in \mathbb{N}$. Since T is an α-admissible mapping and $\alpha(x_0, Tx_0) \geq 1$, we deduce that $\alpha(x_1, x_2) = \alpha(Tx_0, T^2x_0) \geq 1$. Continuing this process, we get $\alpha(x_n, Tx_n) \geq 1$ for all $n \in \mathbb{N} \cup \{0\}$. From inequality (9) we have

$$\begin{aligned}2^{\psi(\mu(Tx_{n-1},Tx_n))} &\leq \big(\alpha(x_{n-1}, Tx_{n-1})\alpha(x_n, Tx_n) + 1\big)^{\psi(\mu(Tx_{n-1},Tx_n))} \\ &\leq 2^{F(\psi(\mu(x_{n-1},x_n)),\varphi(\mu(x_{n-1},x_n)))}.\end{aligned}$$

Then we have

$$\psi\big(\mu(x_n, x_{n+1})\big) \leq F\big(\psi\big(\mu(x_{n-1}, x_n)\big), \varphi\big(\mu(x_{n-1}, x_n)\big)\big) \leq \psi\big(\mu(x_{n-1}, x_n)\big). \tag{10}$$

Now similar to the proof in Theorem 4.1, we get

$$\mu(x_n, x_{n+1}) \to 0 \quad \text{as } n \to \infty. \tag{11}$$

Now we prove that $\{x_n\}$ is an M-Cauchy sequence in (X, μ). We have

$$\lim_{n\to\infty} \mu(x_n, x_{n+1}) = 0,$$

$$0 \le m_{x_n,x_{n+1}} \le \mu(x_n, x_{n+1}) \quad \Rightarrow \quad \lim_{n\to\infty} m_{x_n,x_{n+1}} = 0,$$

and

$$m_{x_n,x_{n+1}} = \min\{\mu(x_n, x_n), \mu(x_{n+1}, x_{n+1})\} \quad \Rightarrow \quad \lim_{n\to\infty} \mu(x_n, x_n) = 0.$$

On the other hand,

$$m_{x_n,x_m} = \min\{\mu(x_n, x_n), \mu(x_m, x_m)\} \quad \Rightarrow \quad \lim_{n,m\to\infty} m_{x_n,x_m} = 0,$$

so

$$\lim_{n,m\to\infty} (M_{x_n,x_m} - m_{x_n,x_m}) = 0.$$

We show

$$\lim_{n,m\to\infty} \big(\mu(x_n, x_m) - m_{x_n,x_m}\big) = 0.$$

Let

$$M^*(x, y) := \mu(x, y) - m_{x,y}, \quad \forall x, y \in X.$$

If $\lim_{n,m\to\infty} M^*(x_n, x_m) \neq 0$, there exist $\varepsilon > 0$ and $\{l_k\} \subset \mathbb{N}$ such that

$$M^*(x_{l_k}, x_{n_k}) \ge \varepsilon.$$

Suppose that k is the smallest integer which satisfies the above equation such that

$$M^*(x_{l_k-1}, x_{n_k}) < \varepsilon.$$

Again as in the proof in Theorem 4.1, we obtain that

$$\lim_{k\to\infty} \mu(x_{m_k}, x_{n_k}) = \varepsilon \tag{12}$$

and

$$\lim_{k\to\infty} \mu(x_{l_k+1}, x_{n_k+1}) = \varepsilon. \tag{13}$$

Now by (9), (12), and (13) we have

$$\begin{aligned}2^{\psi(\mu(x_{m_k+1},x_{n_k+1}))} &\le \bigl(\alpha(x_{m_k}, Tx_{m_k})\alpha(x_{n_k}, Tx_{n_k}) + 1\bigr)^{\psi(\mu(x_{m_k+1},x_{n_k+1}))}\\ &\le 2^{F(\psi(\mu(x_{m_k},x_{n_k})),\varphi(\mu(x_{m_k},x_{n_k})))}.\end{aligned}$$

Therefore we get

$$\psi\bigl(\mu(x_{m_k+1},x_{n_k+1})\bigr) \le F\bigl(\psi\bigl(\mu(x_{m_k},x_{n_k})\bigr),\varphi\bigl(\mu(x_{m_k},x_{n_k})\bigr)\bigr) \le \psi\bigl(\mu(x_{m_k},x_{n_k})\bigr).$$

Letting $k \to \infty$ in the above inequality, we get

$$\psi(\varepsilon) \le F\bigl(\psi(\varepsilon),\varphi(\varepsilon)\bigr) \le \psi(\varepsilon),$$

so

$$F\bigl(\psi(\varepsilon),\varphi(\varepsilon)\bigr) = \psi(\varepsilon).$$

Using the properties of functions F, ψ, and φ, we obtain that $\psi(\varepsilon) = 0$, or $\varphi(\varepsilon) = 0$, and then $\varepsilon = 0$, which is a contradiction. Therefore $\{x_n\}$ is an M-Cauchy sequence.

Now, by the completeness of X, $x_n \to x$ for some $x \in X$ in the τ_m topology, i.e.,

$$\lim_{n\to\infty}\bigl(\mu(x_n,x) - m_{x_n,x}\bigr) = 0$$

and

$$\lim_{n\to\infty}(M_{(x_n,x)} - m_{x_n,x}) = 0.$$

However, $\lim_{n\to\infty} m_{x_n,x} = 0$, hence $\lim_{n\to\infty} \mu(x_n,x) = 0$ and by Remark 1.10

$$\mu(x,x) = 0.$$

Now suppose (a) holds. Then, as in the proof in Theorem 4.1, we have $Tx = x$. Next suppose (b) holds. Then $\alpha(x, Tx) \ge 1$. From (9) we have

$$\begin{aligned}2^{\psi(\mu(Tx_n,Tx))} &\le \bigl(\alpha(x_n, Tx_n)\alpha(x, Tx) + 1\bigr)^{\psi(\mu(Tx_n,Tx))}\\ &\le 2^{F(\psi(\mu(x_n,x)),\varphi(\mu(x_n,x)))},\end{aligned}$$

that is, $\psi(\mu(Tx_n, Tx)) \le F(\psi(\mu(x_n,x)),\varphi(\mu(x_n,x))) \le \psi(\mu(x_n,x))$, and so we get

$$\mu(Tx_n, Tx) \to 0 \quad \text{as } n \to \infty.$$

On the other hand,

$$0 \le m_{Tx_n,Tx} \le \mu(Tx_n, Tx) \to 0 \quad \text{as } n \to \infty.$$

Thus $Tx_n \to Tx$ in the τ_m topology.

The proof of $Tx = x$ follows as in (a). □

Theorem 4.3 *Let (X,μ) be a complete M-metric space and $T:X\to X$ be an α-admissible mapping. Suppose that the following condition is satisfied*:

$$\alpha(x,Tx)\alpha(y,Ty)\psi\big(\mu(Tx,Ty)\big)\le F\big(\psi\big(\mu(x,y)\big),\varphi\big(\mu(x,y)\big)\big) \tag{14}$$

for all $x,y\in X$, where $\psi\in\Psi$, $\varphi\in\Phi$, and $F\in\mathcal{C}$. Suppose that either

(a) *T is continuous,*

or

(b) *if $\{x_n\}$ is a sequence in X such that $x_n\to x$, $\alpha(x_n,x_{n+1})\ge 1$ for all n, then $\alpha(x,Tx)\ge 1$.*

If there exists $x_0\in X$ such that $\alpha(x_0,Tx_0)\ge 1$, then T has a fixed point.

Proof Let $x_0\in X$ be such that $\alpha(x_0,Tx_0)\ge 1$. Define a sequence $\{x_n\}$ in X by $x_n=T^nx_0=Tx_{n-1}$ for all $n\in\mathbb{N}$. Since T is an α-admissible mapping and $\alpha(x_0,Tx_0)\ge 1$, we deduce that $\alpha(x_1,x_2)=\alpha(Tx_0,T^2x_0)\ge 1$. Continuing this process, we get $\alpha(x_n,Tx_n)\ge 1$ for all $n\in\mathbb{N}\cup\{0\}$. From inequality (14) we have

$$\begin{aligned}\psi\big(\mu(Tx_{n-1},Tx_n)\big)&\le\alpha(x_{n-1},Tx_{n-1})\alpha(x_n,Tx_n)\psi\big(\mu(Tx_{n-1},Tx_n)\big)\\&\le F\big(\psi\big(\mu(x_{n-1},x_n)\big),\varphi\big(\mu(x_{n-1},x_n)\big)\big).\end{aligned}$$

Then we have

$$\psi\big(\mu(x_n,x_{n+1})\big)\le F\big(\psi\big(\mu(x_{n-1},x_n)\big),\varphi\big(\mu(x_{n-1},x_n)\big)\big)\le\psi\big(\mu(x_{n-1},x_n)\big). \tag{15}$$

Now, similar to the proof in Theorem 4.1, we get

$$\mu(x_n,x_{n+1})\to 0\quad\text{as } n\to\infty. \tag{16}$$

Now we prove that $\{x_n\}$ is an M-Cauchy sequence in (X,μ). We have

$$\begin{aligned}&\lim_{n\to\infty}\mu(x_n,x_{n+1})=0,\\&0\le m_{x_n,x_{n+1}}\le\mu(x_n,x_{n+1})\quad\Rightarrow\quad\lim_{n\to\infty}m_{x_n,x_{n+1}}=0,\end{aligned}$$

and

$$m_{x_n,x_{n+1}}=\min\big\{\mu(x_n,x_n),\mu(x_{n+1},x_{n+1})\big\}\quad\Rightarrow\quad\lim_{n\to\infty}\mu(x_n,x_n)=0.$$

On the other hand,

$$m_{x_n,x_m}=\min\big\{\mu(x_n,x_n),\mu(x_m,x_m)\big\}\quad\Rightarrow\quad\lim_{n,m\to\infty}m_{x_n,x_m}=0,$$

so

$$\lim_{n,m\to\infty}(M_{x_n,x_m}-m_{x_n,x_m})=0.$$

We show

$$\lim_{n,m\to\infty}\big(\mu(x_n,x_m)-m_{x_n,x_m}\big)=0.$$

Let

$$M^*(x,y) := \mu(x,y) - m_{x,y}, \quad \forall x,y \in X.$$

If $\lim_{n,m\to\infty} M^*(x_n,x_m) \neq 0$, there exist $\varepsilon > 0$ and $\{l_k\} \subset \mathbb{N}$ such that

$$M^*(x_{l_k},x_{n_k}) \geq \varepsilon.$$

Suppose that k is the smallest integer which satisfies the above equation such that

$$M^*(x_{l_k-1},x_{n_k}) < \varepsilon.$$

Again as in the proof in Theorem 4.1, we obtain that

$$\lim_{k\to\infty} \mu(x_{m_k},x_{n_k}) = \varepsilon \tag{17}$$

and

$$\lim_{k\to\infty} \mu(x_{l_k+1},x_{n_k+1}) = \varepsilon. \tag{18}$$

Now by (14), (17), and (18) we have

$$\begin{aligned}\psi\big(\mu(x_{m_k+1},x_{n_k+1})\big) &\leq \alpha(x_{m_k},Tx_{m_k})\alpha(x_{n_k},Tx_{n_k})\psi\big(\mu(x_{m_k+1},x_{n_k+1})\big)\\ &\leq F\big(\psi\big(\mu(x_{m_k},x_{n_k})\big),\varphi\big(\mu(x_{m_k},x_{n_k})\big)\big).\end{aligned}$$

Therefore we get

$$\psi\big(\mu(x_{m_k+1},x_{n_k+1})\big) \leq F\big(\psi\big(\mu(x_{m_k},x_{n_k})\big),\varphi\big(\mu(x_{m_k},x_{n_k})\big)\big) \leq \psi\big(\mu(x_{m_k},x_{n_k})\big).$$

Letting $k \to \infty$ in the above inequality, we get

$$\psi(\varepsilon) \leq F\big(\psi(\varepsilon),\varphi(\varepsilon)\big) \leq \psi(\varepsilon),$$

so

$$F\big(\psi(\varepsilon),\varphi(\varepsilon)\big) = \psi(\varepsilon).$$

Using the properties of functions F, ψ, and φ, we obtain that $\psi(\varepsilon) = 0$, or $\varphi(\varepsilon) = 0$, then $\varepsilon = 0$, which is a contradiction. Therefore $\{x_n\}$ is an M-Cauchy sequence.

Now, by the completeness of X, $x_n \to x$ for some $x \in X$ in the τ_m topology, i.e.,

$$\lim_{n\to\infty} \big(\mu(x_n,x) - m_{x_n,x}\big) = 0$$

and

$$\lim_{n\to\infty} (M_{(x_n,x)} - m_{x_n,x}) = 0.$$

However, $\lim_{n\to\infty} m_{x_n,x} = 0$, hence $\lim_{n\to\infty} \mu(x_n,x) = 0$ and by Remark 1.10

$$\mu(x,x) = 0.$$

Now suppose (a) holds. Then, as in the proof in Theorem 4.1, we have $Tx = x$. Next suppose (b) holds. Then $\alpha(x, Tx) \geq 1$. From (14) we have

$$\begin{aligned}\psi\bigl(\mu(Tx_n, Tx)\bigr) &\leq \alpha(x_n, Tx_n)\alpha(x, Tx)\psi\bigl(\mu(Tx_n, Tx)\bigr)\\ &\leq F\bigl(\psi\bigl(\mu(x_n,x)\bigr), \varphi\bigl(\mu(x_n,x)\bigr)\bigr),\end{aligned}$$

that is, $\psi(\mu(Tx_n, Tx)) \leq F(\psi(\mu(x_n,x)), \varphi(\mu(x_n,x))) \leq \psi(\mu(x_n,x))$, and so we get

$$\mu(Tx_n, Tx) \to 0 \quad \text{as } n \to \infty.$$

On the other hand,

$$0 \leq m_{Tx_n,Tx} \leq \mu(Tx_n, Tx) \to 0 \quad \text{as } n \to \infty.$$

Thus $Tx_n \to Tx$ in the τ_m topology.
The proof of $Tx = x$ follows as in (a). □

Theorem 4.4 *Assume that all of the hypotheses of Theorems 4.1 or 4.2 or 4.3 hold. In addition, suppose the following condition is satisfied*:
(c) *if $Tx = x$ then $\alpha(x, Tx) \geq 1$.*
Then the fixed point of T is unique.

Proof Suppose that $u, v \in X$ are two fixed points of T such that $u \neq v$. Then $\alpha(u, Tu) \geq 1$ and $\alpha(v, Tv) \geq 1$.

For Theorem 4.1, we have

$$\begin{aligned}\psi\bigl(d(Tu, Tv)\bigr) + l &\leq \bigl(\psi\bigl(d(Tu, Tv)\bigr) + l\bigr)^{\alpha(u,Tu)\alpha(v,Tv)}\\ &\leq F\bigl(\psi\bigl(d(u,v)\bigr), \varphi\bigl(d(u,v)\bigr)\bigr) + l, \qquad (19)\\ \psi\bigl(d(Tu, Tu)\bigr) + l &\leq \bigl(\psi\bigl(d(Tu, Tu)\bigr) + l\bigr)^{\alpha(u,Tu)\alpha(u,Tu)}\\ &\leq F\bigl(\psi\bigl(d(u,u)\bigr), \varphi\bigl(d(u,u)\bigr)\bigr) + l. \qquad (20)\end{aligned}$$

For Theorem 4.2, we have

$$\begin{aligned}2^{\psi(\mu(Tu,Tv))} &\leq \bigl(\alpha(u, Tu)\alpha(v, Tv) + 1\bigr)^{\psi(\mu(Tu,Tv))}\\ &\leq 2^{F(\psi(\mu(u,v)),\varphi(\mu(u,v)))}, \qquad (21)\\ 2^{\psi(\mu(Tu,Tu))} &\leq \bigl(\alpha(u, Tu)\alpha(u, Tu) + 1\bigr)^{\psi(\mu(Tu,Tu))}\\ &\leq 2^{F(\psi(\mu(u,u)),\varphi(\mu(u,u)))}. \qquad (22)\end{aligned}$$

For Theorem 4.3, we have

$$\begin{aligned}\psi\bigl(\mu(Tu,Tv)\bigr) &\le \bigl(\alpha(u,Tu)\alpha(v,Tv)+1\bigr)\psi\bigl(\mu(Tu,Tv)\bigr)\\ &\le F\bigl(\psi\bigl(\mu(u,v)\bigr),\varphi\bigl(\mu(u,v)\bigr)\bigr), \qquad (23)\\ \psi\bigl(\mu(Tu,Tu)\bigr) &\le \bigl(\alpha(u,Tu)\alpha(u,Tu)+1\bigr)\psi\bigl(\mu(Tu,Tu)\bigr)\\ &\le F\bigl(\psi\bigl(\mu(u,u)\bigr),\varphi\bigl(\mu(u,u)\bigr)\bigr). \qquad (24)\end{aligned}$$

Therefore equations (19), (20), (21), (22), (23), and (24) imply that

$$\begin{aligned}&F\bigl(\psi\bigl(\mu(u,v)\bigr),\varphi\bigl(\mu(u,v)\bigr)\bigr)=\psi\bigl(\mu(Tu,Tv)\bigr)=\psi\bigl(\mu(u,v)\bigr),\\ &F\bigl(\psi\bigl(\mu(u,u)\bigr),\varphi\bigl(\mu(u,u)\bigr)\bigr)=\psi\bigl(\mu(Tu,Tu)\bigr)=\psi\bigl(\mu(u,u)\bigr),\\ &F\bigl(\psi\bigl(\mu(v,v)\bigr),\varphi\bigl(\mu(v,v)\bigr)\bigr)=\psi\bigl(\mu(Tv,Tv)\bigr)=\psi\bigl(\mu(v,v)\bigr),\end{aligned}$$

and so from the properties of functions F, ψ, and φ, we have

$$\mu(u,v)=\mu(u,u)=\mu(v,v)=0.$$

Therefore by (m1)

$$u=v. \qquad \square$$

5 Consequences

From Theorems 4.1, 4.2, and 4.3 we obtain the following corollaries as an extension of several known results in the literature.

If we let $\varphi(t)=\psi(t)=t$, we get the following three corollaries.

Corollary 5.1 *Let (X,μ) be a complete M-metric space and $T:X\to X$ be an α-admissible mapping. Suppose that the following condition is satisfied:*

$$\bigl(\mu(Tx,Ty)+l)\bigr)^{\alpha(x,Tx)\alpha(y,Ty)}\le F\bigl(\mu(x,y)\bigr),\mu(x,y))+l \qquad (25)$$

for all $x,y\in X$ and $l\ge 1$, where $\psi\in\Psi$, $\varphi\in\Phi$, and $F\in\mathcal{C}$. Suppose that either

(a) *T is continuous,*

or

(b) *if $\{x_n\}$ is a sequence in X such that $x_n\to x$, $\alpha(x_n,x_{n+1})\ge 1$ for all n, then $\alpha(x,Tx)\ge 1$.*

If there exists $x_0\in X$ such that $\alpha(x_0,Tx_0)\ge 1$, then T has a fixed point.

Corollary 5.2 *Let (X,μ) be a complete M-metric space and $T:X\to X$ be an α-admissible mapping. Suppose that the following condition is satisfied:*

$$\bigl(\alpha(x,Tx)\alpha(y,Ty)+1\bigr)^{\mu(Tx,Ty)}\le 2^{F(\mu(x,y)),\mu(x,y))} \qquad (26)$$

for all $x,y\in X$, where $\psi\in\Psi$, $\varphi\in\Phi$, and $F\in\mathcal{C}$. Suppose that either

(a) *T is continuous,*

or

(b) *if $\{x_n\}$ is a sequence in X such that $x_n \to x$, $\alpha(x_n, x_{n+1}) \geq 1$ for all n, then $\alpha(x, Tx) \geq 1$.*
If there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$, then T has a fixed point.

Corollary 5.3 *Let (X, μ) be a complete M-metric space and $T : X \to X$ be an α-admissible mapping. Suppose that the following condition is satisfied*:

$$\alpha(x, Tx)\alpha(y, Ty)\mu(Tx, Ty) \leq F\big(\mu(x, y), \mu(x, y)\big) \tag{27}$$

for all $x, y \in X$, where $\psi \in \Psi$, $\varphi \in \Phi$, and $F \in \mathcal{C}$. Suppose that either
(a) *T is continuous,*
or
(b) *if $\{x_n\}$ is a sequence in X such that $x_n \to x$, $\alpha(x_n, x_{n+1}) \geq 1$ for all n, then $\alpha(x, Tx) \geq 1$.*
If there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$, then T has a fixed point.

Lemma 5.4 ([7]) *Every p-metric and metric is an M-metric.*

If we let $\beta \in \mathcal{F}$, $\varphi(t) = \psi(t) = t$ and $F(s, t) = \beta(s)s$, by Lemma 5.4 we get three results of Hussein *et al.* [13] (they are the immediate consequences of our results).

Corollary 5.5 ([13, Theorem 4]) *Let (X, d) be a complete metric space and $T : X \to X$ be an α-admissible mapping. Assume that there exists a function $\beta : \mathbb{R}^+ \to [0, 1]$ such that, for any bounded sequence $\{t_n\}$ of positive reals, $\beta(t_n) \to 1$ implies $t_n \to 0$ and*

$$\big(d(Tx, Ty) + l\big)^{\alpha(x,Tx)\alpha(y,Ty)} \leq \beta\big(d(x, y)\big)d(x, y) + l \tag{28}$$

for all $x, y \in X$ where $l \geq 1$. Suppose that either
(a) *T is continuous,*
or
(b) *if $\{x_n\}$ is a sequence in X such that $x_n \to x$, $\alpha(x_n, x_{n+1}) \geq 1$ for all n, then $\alpha(x, Tx) \geq 1$.*
If there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$, then T has a fixed point.

Corollary 5.6 ([13, Theorem 6]) *Let (X, d) be a complete metric space and $T : X \to X$ be an α-admissible mapping. Assume that there exists a function $\beta : \mathbb{R}^+ \to [0, 1]$ such that, for any bounded sequence $\{t_n\}$ of positive reals, $\beta(t_n) \to 1$ implies $t_n \to 0$ and*

$$\big(\alpha(x, Tx)\alpha(y, Ty) + 1\big)^{d(Tx,Ty)} \leq 2^{\beta(d(x,y))d(x,y)} \tag{29}$$

for all $x, y \in X$. Suppose that either
(a) *T is continuous,*
or
(b) *if $\{x_n\}$ is a sequence in X such that $x_n \to x$, $\alpha(x_n, x_{n+1}) \geq 1$ for all n, then $\alpha(x, Tx) \geq 1$.*
If there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$, then T has a fixed point.

Corollary 5.7 ([13, Theorem 8]) *Let (X, d) be a complete metric space and $T : X \to X$ be an α-admissible mapping. Assume that there exists a function $\beta : \mathbb{R}^+ \to [0, 1]$ such that, for any bounded sequence $\{t_n\}$ of positive reals, $\beta(t_n) \to 1$ implies $t_n \to 0$ and*

$$\big(\alpha(x, Tx)\alpha(y, Ty)\big)d(Tx, Ty) \leq \beta\big(d(x, y)\big)d(x, y) \tag{30}$$

for all $x, y \in X$. Suppose that either

(a) *T is continuous,*

or

(b) *if $\{x_n\}$ is a sequence in X such that $x_n \to x$, $\alpha(x_n, x_{n+1}) \geq 1$ for all n, then $\alpha(x, Tx) \geq 1$.*

If there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$, then T has a fixed point.

6 Conclusion

Recently, the authors in [17] introduced the class of α-ψ contractive type mappings and obtained a fixed point result for this new class of mappings in the set-up of metric spaces. Their result contains several well-known fixed point theorems including the Banach contraction principle. Matthews (1994) in [18] established fixed point theorems in partial metric spaces. The authors in [7] introduced M-metric spaces which extend p-metric spaces and the authors established some new fixed point theorems.

In this paper, we introduce the class of $F(\psi, \varphi)$-contractions and investigate the existence and uniqueness of fixed points for α-admissible mappings on M-metric spaces. We also show that the fixed point results in [13] and Geraghty's theorem [10] (Theorem 1.1) are immediate consequences of our results. For further results, we refer the reader to [1–4, 6, 8–12, 19–21, 23].

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