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A note on recent cyclic fixed point results in dislocated quasi-b-metric spaces

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Abstract

The purpose of this paper is to establish some fixed point results for cyclic contractions in the setting of dislocated quasi-b-metric spaces. We verify that some previous cyclic contraction results in dislocated quasi-b-metric spaces are just equivalent to the non-cyclic ones in the same spaces. Moreover, by using two examples, we highlight the superiority of the results obtained.

MSC: 54H25; 47H10

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1 Introduction and preliminaries

French mathematician Poinćare was first to use the concept of fixed point in 'Poinćare's final theorem' during the period of 1895 to 1900, from restricting the existence of periodic solution for three body problem to the existence of fixed point under some conditions of planar continuous transformations. In 1910, Brouwer proved that there exists at least one fixed point for the polyhedron continuous map in finite dimensional space, and this opened the situation of fixed point theory research. Particularly in 1922, Polish mathematician Banach innovated Banach contraction mapping principle by using Picard iteration method. Due to its beautiful assertion and successful way of solving the implicit function existence theorem, the existence of a solution for a differential equation with initial value condition, fixed point theory caught the eyes of scholars and it sparkles people's inspirations towards in-depth and extensive research. Especially in recent decades, with the development of the computer, many people have coped with numerous applications by utilizing a variety of iteration methods to approach the fixed point and hence they made a breakthrough and brought this subject gradually to perfection. Nowadays fixed point theory plays a crucial role in nonlinear functional analysis. Just because of this, in this paper, we start our fixed point investigation based on some previous work.

To start this article, we first of all recall some basic knowledge.

In 1922, Banach [1] introduced the Banach contraction mapping principle as follows:

A self-map T on a metric space (X,d) has a unique fixed point if there exists $k \in [0,1)$ such that

$$d(Tx,Ty) \le kd(x,y), \quad \forall x,y \in X.$$

After that, based on this finding, a large number of fixed point results have appeared in recent years. Generally speaking, there usually are two generalizations on them. One is from mappings. The other is from spaces.

Concretely, for one thing, from mappings, for example, the concept of a Kannan contraction mapping was introduced in 1969 by Kannan [2] as follows:

A self-map T on a metric space (X,d) is called a Kannan contraction mapping if there exists $k \in [0, \frac{1}{2})$ such that

$$d(Tx, Ty) \leq k\bigl[d(x, Tx) + d(y, Ty)\bigr], \quad \forall x, y \in X.$$

In this case, T has a unique fixed point.

Recently, the cyclic contraction mapping has become popular for research activities (see [3–11]). Let A and B be nonempty subsets of a metric space (X,d) and let $T : A \cup B \to A \cup B$ be a mapping. T is called a cyclic map if and only if $T(A) \subseteq B$ and $T(B) \subseteq A$. In 2003, Kirk *et al.* [4] introduced a cyclic contraction mapping as follows:

A cyclic map $T : A \cup B \to A \cup B$ is said to be a cyclic contraction mapping if there exists $k \in [0,1)$ such that

$$d(Tx, Ty) \leq kd(x, y), \quad \forall x \in A, \forall y \in B.$$

Then T has a unique fixed point in $A \cap B$.

In 2010, Karapınar and Erhan [12] introduced a Kannan type cyclic contraction mapping as follows:

Let A and B be nonempty subsets of a metric space (X,d). A mapping $T : A \cup B \to A \cup B$ is called a Kannan type cyclic contraction mapping if there exists $k \in [0, \frac{1}{2})$ such that

$$d(Tx, Ty) \leq k\bigl[d(x, fx) + d(y, fy)\bigr], \quad \forall x \in A, \forall y \in B.$$

If (X,d) is still complete, then T has a unique fixed point in $A \cap B$ [4, 12].

For another thing, from spaces, there are too many generalizations of metric spaces. For instance, we have the b-metric space, quasi-metric space, quasi-b-metric space, dislocated metric space (or metric-like space), dislocated b-metric space (or b-metric-like space), dislocated quasi-metric space (or quasi-metric-like space), and the dislocated quasi-b-metric space (or quasi-b-metric-like space) (see [3, 5, 9–11, 13–18]). Their definitions are as follows:

Let X be a nonempty set and $s \geq 1$ a real number. Let $d : X \times X \to [0, \infty)$ be a mapping and $x, y, z \in X$:

(d1) $d(x,y) = 0 \Leftrightarrow x = y$;
(d2) $d(x,y) = 0 \Rightarrow x = y$;
(d3) $d(x,y) = 0 = d(y,x) \Rightarrow x = y$;
(d4) $d(x,y) = d(y,x)$;
(d5) $d(x,z) \leq d(x,y) + d(y,z)$;
(d6) $d(x,z) \leq s[d(x,y) + d(y,z)]$, $s \geq 1$.

Then

(1) (X,d) is called a metric space if (d1), (d4), and (d5) hold;
(2) (X,d) is called a b-metric space if (d1), (d4), and (d6) hold;

(3) (X,d) is called a quasi-metric space if (d1) and (d5) hold;
(4) (X,d) is called a quasi-b-metric space if (d1) and (d6) hold;
(5) (X,d) is called a dislocated metric space if (d2), (d4), and (d5) hold;
(6) (X,d) is called a dislocated b-metric space if (d2), (d4), and (d6) hold;
(7) (X,d) is called a dislocated quasi-metric space if (d3) and (d5) hold;
(8) (X,d) is called a dislocated quasi-b-metric space if (d3) and (d6) hold.

Despite the fact that the given examples were previously known, we thought it is useful for easy reference to give a full review.

Example 1.1

(a) Let $X = \mathbb{R}$ and $d : X \times X \to [0,\infty)$ be defined as

$$d(x,y) = \begin{cases} x - y, & x \geq y, \\ 1, & \text{otherwise}. \end{cases}$$

Then (X,d) is a quasi-metric space, but it is not a metric space.

(b) Let $X = [0,\infty)$ and $d : X \times X \to [0,\infty)$ be defined as

$$d(x,y) = \begin{cases} 0, & x = y, \\ (x+y)^2, & \text{otherwise}. \end{cases}$$

Then (X,d) is a b-metric space, but it is not a metric space.

(c) Let $X = C([0,1],\mathbb{R})$ with the usual partial ordering, and let $d : X \times X \to \mathbb{R}^+$ be defined as

$$d(f,g) = \begin{cases} \int_0^1 (g(t) - f(t))^3\, dt, & f \leq g, \\ \int_0^1 (f(t) - g(t))^3\, dt, & f \geq g. \end{cases}$$

Then (X,d) is a quasi-b-metric space, but it is not a quasi-metric space and b-metric space.

(d) Let $X = \mathbb{R}^+$ and $d : X \times X \to \mathbb{R}^+$ be defined as $d(x,y) = \max\{x,y\}$. Then (X,d) is a dislocated metric space, but it is not a metric space.

(e) Let $X = [0,1]$ and $d : X \times X \to \mathbb{R}^+$ be defined as $d(x,y) = |x-y| + x$. Then (X,d) is a dislocated quasi-metric space, but it is not a dislocated metric space, and it is not a quasi-metric space.

(f) Let $X = [0,\infty)$ and $d : X \times X \to [0,\infty)$ be defined as $d(x,y) = (x+y)^2$. Then (X,d) is a dislocated b-metric space, but it is not a b-metric space.

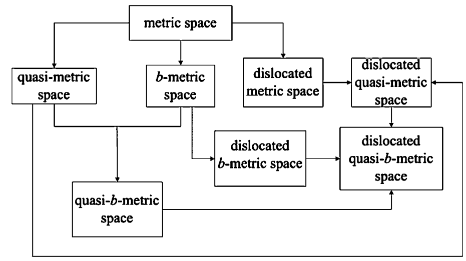
(g) Let $X = \mathbb{R}$ and $d : X \times X \to [0,\infty)$ be defined as $d(x,y) = |x-y|^2 + \frac{|x|}{n} + \frac{|y|}{m}$, where $n,m \in \mathbb{N} \setminus \{1\}$, $n \neq m$. Then (X,d) is a dislocated quasi-b-metric space, but it is not a quasi-b-metric space, dislocated b-metric space and dislocated quasi-metric space.

So, we have the process diagram (see Figure 1), where arrows stand for generalization relationships.

Also, scholars are interested in dislocated quasi-b-metric spaces since they are more general spaces. Based on this fact, we consider fixed point results in such spaces.

For the sake of reader, we recall the following concepts and results.

Figure 1 Process diagram.



Definition 1.2 ([5]) Let (X,d) be a dislocated quasi-b-metric space and $\{x_n\}_{n\in\mathbb{N}}$ be a sequence in X. Then we say that:

(i) $\{x_n\}_{n\in\mathbb{N}}$ converges to $x\in X$ if

$$\lim_{n\to\infty} d(x_n,x)=0=\lim_{n\to\infty} d(x,x_n).$$

In this case x is called a *dqb*-limit of $\{x_n\}$, and we write it as $x_n\to x$ $(n\to\infty)$.

(ii) $\{x_n\}_{n\in\mathbb{N}}$ is a *dqb*-Cauchy sequence if

$$\lim_{n,m\to\infty} d(x_n,x_m)=0=\lim_{n,m\to\infty} d(x_m,x_n).$$

(iii) (X,d) is *dqb*-complete if every *dqb*-Cauchy sequence is convergent in X.

Definition 1.3 ([5], Definition 2.8) Let A and B be nonempty subsets of a dislocated quasi-b-metric space (X,d). A cyclic map $T:A\cup B\to A\cup B$ is said to be a dislocated quasi-b-metric-cyclic-Banach contraction if there exists $k\in[0,\frac{1}{s})$, $s\geq 1$, such that

$$d(Tx,Ty)\leq kd(x,y) \tag{1.1}$$

for all $x\in A$, $y\in B$.

Definition 1.4 ([5], Definition 2.11) Let A and B be nonempty subsets of a dislocated quasi-b-metric space (X,d). A cyclic map $T:A\cup B\to A\cup B$ is said to be a dislocated quasi-b-metric-cyclic-Kannan contraction if there exists $k\in[0,\frac{1}{2s})$, $s\geq 1$, such that

$$d(Tx,Ty)\leq k\bigl[d(x,fx)+d(y,fy)\bigr] \tag{1.2}$$

for all $x\in A$, $y\in B$.

In [5], the authors proved the following main results.

Theorem 1.5 ([5], Theorem 2.9) *Let A and B be nonempty closed subsets of a dqb-complete dislocated quasi-b-metric space (X,d). Let T be a cyclic mapping that satisfies the condition of a dislocated quasi-b-metric-cyclic-Banach contraction. Then T has a unique fixed point in $A\cap B$.*

Theorem 1.6 ([5], Theorem 2.12) *Let A and B be nonempty closed subsets of a dqb-complete dislocated quasi-b-metric space (X,d). Let T be a cyclic mapping that satisfies the* *condition of a dislocated quasi-b-metric-cyclic-Kannan contraction. Then T has a unique fixed point in $A \cap B$.*

2 Main results

In this section, we consider and generalize some previous results. We also prove that the results from Theorem 1.5 and Theorem 1.6 are just equivalent to the respective ordinary fixed point results in the same framework.

First, we recall the following lemma ([8], Remark 2.13).

Lemma 2.1 *If some ordinary fixed point theorem in the framework of metric (resp. b-metric) spaces has a true cyclic-type extension, then these two theorems are equivalent.*

Now we announce the following result.

Theorem 2.2 *Theorem* 1.5 (*that is*, [5], *Theorem* 2.9) *is equivalent with the following claim.*

Claim 1 *Let (X,d) be a dqb-complete dislocated quasi-b-metric space with $s \geq 1$, and let $T: X \to X$ be a mapping. Assume that there exists $k \in [0, \frac{1}{s})$ such that for all $x, y \in X$,*

$$d(Tx, Ty) \leq kd(x, y). \tag{2.1}$$

Then T has a unique fixed point in X.

Theorem 2.3 *Theorem* 1.6 (*that is*, [5], *Theorem* 2.12) *is equivalent with the following claim.*

Claim 2 *Let (X,d) be a dqb-complete dislocated quasi-b-metric space with $s \geq 1$, and let $T: X \to X$ be a mapping. Assume that there exists $k \in [0, \frac{1}{2s})$ such that for all $x, y \in X$,*

$$d(Tx, Ty) \leq k\bigl[d(x, fx) + d(y, fy)\bigr]. \tag{2.2}$$

Then T has a unique fixed point in X.

In order to prove the above two theorems, we use the following crucial lemma.

Lemma 2.4 *Let $\{A_i\}_{i=1}^{p}$ be nonempty closed subsets of a dqb-complete dislocated quasi-b-metric space (X,d), and suppose that $T: \bigcup_{i=1}^{p} A_i \to \bigcup_{i=1}^{p} A_i$ satisfies the following conditions:*

(1) *$T(A_i) \subseteq A_{i+1}$ for $1 \leq i \leq p$ where $A_{p+1} = A_1$;*
(2) *there exists $k \in [0, \frac{1}{s})$ such that for all $x \in \bigcup_{i=1}^{p} A_i$,*

$$d\bigl(T^2x, Tx\bigr) \leq kd(Tx, x), \qquad d\bigl(Tx, T^2x\bigr) \leq kd(x, Tx). \tag{2.3}$$

Then $\bigcap_{i=1}^{p} A_i \neq \emptyset$.

Proof If $k = 0$, then for all $x \in \bigcup_{i=1}^{p} A_i$, by (2.3) and (d3), we have $T(Tx) = Tx$, *i.e.*, Tx is a fixed point of T. Thus by (1), it is not hard to verify that $Tx \in \bigcap_{i=1}^{p} A_i$. That is, $\bigcap_{i=1}^{p} A_i \neq \emptyset$.

Now, let $k \in (0, \frac{1}{s})$ and $x \in \bigcup_{i=1}^{p} A_i$. Then by (2.3), on the one hand, we arrive at

$$\begin{aligned} d\bigl(T^{n+1}x, T^n x\bigr) &= d\bigl(T^2 T^{n-1}x, TT^{n-1}x\bigr) \\ &\leq kd\bigl(T^n x, T^{n-1}x\bigr) = kd\bigl(T^2 T^{n-2}x, TT^{n-2}x\bigr) \\ &\leq k^2 d\bigl(T^{n-1}x, T^{n-2}x\bigr) \leq \cdots \leq k^n d(Tx, x). \end{aligned} \tag{2.4}$$

On the other hand, we have

$$\begin{aligned} d\bigl(T^n x, T^{n+1}x\bigr) &= d\bigl(TT^{n-1}x, T^2 T^{n-1}x\bigr) \\ &\leq kd\bigl(T^{n-1}x, T^n x\bigr) = kd\bigl(TT^{n-2}x, T^2 T^{n-2}x\bigr) \\ &\leq k^2 d\bigl(T^{n-2}x, T^{n-1}x\bigr) \leq \cdots \leq k^n d(x, Tx). \end{aligned} \tag{2.5}$$

Hence by (2.4) and (2.5), we easily speculate that

$$\lim_{n,m\to\infty} d\bigl(T^n x, T^m x\bigr) = 0, \qquad \lim_{n,m\to\infty} d\bigl(T^m x, T^n x\bigr) = 0.$$

In other words, we demonstrate that $\{T^n x\}$ is a *dqb*-Cauchy sequence in dislocated quasi-b-metric space $(\bigcup_{i=1}^{p} A_i, d)$. Note that $(\bigcup_{i=1}^{p} A_i, d)$ is *dqb*-complete, and one establishes that $\{T^n x\}$ converges to some $z \in \bigcup_{i=1}^{p} A_i$. However, in view of (1), $T^n x$ has infinite terms lying in each A_i, $i \in \{1, 2, \ldots, p\}$. As A_i is closed for all $i \in \{1, 2, \ldots, p\}$, we claim that $z \in \bigcap_{i=1}^{p} A_i$. Therefore, $\bigcap_{i=1}^{p} A_i \neq \emptyset$. □

Proof of Theorem 2.2 Putting $A_i = X$ for $i \in \{1, 2, \ldots, p\}$ in Theorem 1.5, we obtain Claim 1. Conversely, let Claim 1 hold. We shall prove that Theorem 1.5 also holds. Indeed, if $x \in \bigcup_{i=1}^{m} A_i$, then by virtue of Lemma 2.4, one establishes that $\{T^n x\}$ is a *dqb*-Cauchy sequence, further, $\bigcap_{i=1}^{p} A_i \neq \emptyset$. Now that $(\bigcap_{i=1}^{p} A_i, d)$ is a *dqb*-complete dislocated quasi-b-metric space, and we restrict T to $(\bigcap_{i=1}^{p} A_i, d)$ and hence the condition (2.1) holds for all $x, y \in \bigcap_{i=1}^{p} A_i$, then Claim 1 implies that T has a unique fixed point in $\bigcap_{i=1}^{p} A_i$. Accordingly, Theorem 1.5 is satisfied. □

Proof of Theorem 2.3 We could use the same method as in the proof of Theorem 2.2 and hence the proof is omitted. □

Further, we announce the result for the existence of fixed point under cyclical consideration in the framework following dislocated quasi-b-metric spaces.

Lemma 2.5 *Let* $(X = \bigcup_{i=1}^{p} A_i, d)$ *be a dqb-complete dislocated quasi-b-metric space. If* $T : X \to X$ *satisfies* (1) *of Lemma* 2.4, *and for all* $x \in X = \bigcup_{i=1}^{p} A_i$, *the corresponding Picard sequence* $\{T^n x\}$ *is a dqb-Cauchy sequence, then* $\bigcap_{i=1}^{p} A_i \neq \emptyset$.

Proof Since (X, d) is *dqb*-complete, then *dqb*-Cauchy sequence $\{T^n x\}$ converges to some $z \in X$. We shall prove that $z \in \bigcap_{i=1}^{p} A_i$. Actually, observing that $T(A_i) \subseteq A_{i+1}$ for all $i \in \{1, 2, \ldots, p\}$ and $T(A_{p+1}) \subseteq A_1$, we conclude that $\{T^n x\}$ has infinite terms in A_i for all $i \in \{1, 2, \ldots, p\}$. As A_i is closed for all $i \in \{1, 2, \ldots, p\}$, we claim that $z \in \bigcap_{i=1}^{p} A_i$. Consequently, $\bigcap_{i=1}^{p} A_i \neq \emptyset$. □

The following two examples support our Theorem 2.2 and Theorem 2.3.

Example 2.6 ([5], Example 2.10) Let $X = [-1, 1]$ and $Tx = -\frac{x}{5}$. Suppose that $A = [-1, 0]$ and $B = [0, 1]$. Define the function $d : X \times X \to [0, \infty)$ by

$$d(x,y) = |x-y|^2 + \frac{|x|}{10} + \frac{|y|}{11}.$$

Then (X, d) is a dislocated quasi-b-metric space with the coefficient $s = 2$. As a consequence of $d(1, 1) \neq 0$, so (X, d) is not a quasi-b-metric space. Also, it is not a dislocated b-metric space based on the fact that $d(1, 0) \neq d(0, 1)$. Clearly, $X = A \cup B$ and $T(A) \subseteq B$, $T(B) \subseteq A$. We shall prove (2.1), where $k \in [\frac{1}{5}, \frac{1}{2}) \subseteq [0, \frac{1}{s})$.

Indeed, for all $x, y \in X$, we have

$$\begin{aligned} d(Tx, Ty) &= d\left(-\frac{x}{5}, -\frac{y}{5}\right) \\ &= \left|-\frac{x}{5} + \frac{y}{5}\right|^2 + \frac{|x|}{50} + \frac{|y|}{55} \\ &= \frac{1}{5}\left(\frac{1}{5}|x-y|^2 + \frac{|x|}{10} + \frac{|y|}{11}\right) \\ &\leq \frac{1}{5}\left(|x-y|^2 + \frac{|x|}{10} + \frac{|y|}{11}\right) \\ &\leq kd(x,y), \end{aligned}$$

where $k \in [\frac{1}{5}, \frac{1}{2}) \subseteq [0, \frac{1}{s})$. Hence, all the conditions of Claim 1 are satisfied, that is, T has a unique fixed point in X. Now, using Theorem 2.2, we see that T has a unique fixed point in $A \cap B = \{0\}$, *i.e.*, 0 is the unique fixed point of T.

Example 2.7 ([5], Example 2.13) Let $X = [-1, 1]$ and $Tx = -\frac{x}{7}$. Suppose that $A = [-1, 0]$ and $B = [0, 1]$. Define the function $d : X \times X \to [0, \infty)$ by

$$d(x,y) = |x-y|^2 + 3|x| + 2|y|.$$

Then (X, d) is a dislocated quasi-b-metric space with the coefficient $s = 2$. Owing to $d(1, 1) \neq 0$, hence (X, d) is not a quasi-b-metric space. Also, it is not a dislocated b-metric space because of $d(1, 0) \neq d(0, 1)$. Further, similar to the preceding example, we can show (2.2), where $k \in [\frac{2}{23}, \frac{1}{2}) \subseteq [0, \frac{1}{s})$. Thereupon all the conditions of Claim 2 are satisfied, it ensures us that T has a unique fixed point in X. Now, by Theorem 1.6, T has a unique fixed point in $A \cap B = \{0\}$. Finally by Theorem 2.3, 0 is a unique fixed point of T.

The following lemmas are useful in proving of all main results in the framework of dislocated quasi-b-metric spaces. The proofs are almost the same as in [19] for the case of b-metric spaces and hence we omit them.

Lemma 2.8 *Let (X, d) be a dislocated quasi-b-metric space with $s \geq 1$, and suppose that $\{x_n\}$ and $\{y_n\}$ are dqb-convergent to x, y, respectively. Then we have*

$$\frac{1}{s^2}d(x,y) \leq \liminf_{n\to\infty} d(x_n, y_n) \leq \limsup_{n\to\infty} d(x_n, y_n) \leq s^2 d(x,y)$$

and

$$\frac{1}{s^2}d(y,x) \leq \liminf_{n\to\infty} d(y_n,x_n) \leq \limsup_{n\to\infty} d(y_n,x_n) \leq s^2 d(y,x).$$

In particular, if $x = y$, then $\lim_{n\to\infty} d(x_n,y_n) = 0$ and $\lim_{n\to\infty} d(y_n,x_n) = 0$. Moreover, for each $z \in X$, it follows that

$$\frac{1}{s}d(x,z) \leq \liminf_{n\to\infty} d(x_n,z) \leq \limsup_{n\to\infty} d(x_n,z) \leq sd(x,z)$$

and

$$\frac{1}{s}d(z,x) \leq \liminf_{n\to\infty} d(z,x_n) \leq \limsup_{n\to\infty} d(z,x_n) \leq sd(z,x).$$

Lemma 2.9 *Let (X,d) be a dislocated quasi-b-metric space with $s \geq 1$ and let $\{x_n\}$ be a sequence in X such that*

$$\lim_{n\to\infty} d(x_n,x_{n+1}) = 0.$$

If $\{x_n\}$ is not a dqb-Cauchy sequence, then there exist $\varepsilon > 0$ and two sequences $\{m(k)\}$ and $\{n(k)\}$ of positive integers such that, for the sequences

$$d(x_{m(k)},x_{n(k)}), \qquad d(x_{m(k)},x_{n(k)+1}), \qquad d(x_{m(k)+1},x_{n(k)}), \qquad d(x_{m(k)+1},x_{n(k)+1})$$

and

$$d(x_{n(k)},x_{m(k)}), \qquad d(x_{n(k)+1},x_{m(k)}), \qquad d(x_{n(k)},x_{m(k)+1}), \qquad d(x_{n(k)+1},x_{m(k)+1}),$$

we have

$$\begin{aligned}
&\varepsilon \leq \liminf_{n\to\infty} d(x_{m(k)},x_{n(k)}) \leq \limsup_{n\to\infty} d(x_{m(k)},x_{n(k)}) \leq s\varepsilon, \\
&\frac{\varepsilon}{s} \leq \liminf_{n\to\infty} d(x_{m(k)},x_{n(k)+1}) \leq \limsup_{n\to\infty} d(x_{m(k)},x_{n(k)+1}) \leq s^2\varepsilon, \\
&\frac{\varepsilon}{s} \leq \liminf_{n\to\infty} d(x_{m(k)+1},x_{n(k)}) \leq \limsup_{n\to\infty} d(x_{m(k)+1},x_{n(k)}) \leq s^2\varepsilon, \\
&\frac{\varepsilon}{s^2} \leq \liminf_{n\to\infty} d(x_{m(k)+1},x_{n(k)+1}) \leq \limsup_{n\to\infty} d(x_{m(k)+1},x_{n(k)+1}) \leq s^3\varepsilon,
\end{aligned}$$

and

$$\begin{aligned}
&\varepsilon \leq \liminf_{n\to\infty} d(x_{n(k)},x_{m(k)}) \leq \limsup_{n\to\infty} d(x_{n(k)},x_{m(k)}) \leq s\varepsilon, \\
&\frac{\varepsilon}{s} \leq \liminf_{n\to\infty} d(x_{n(k)+1},x_{m(k)}) \leq \limsup_{n\to\infty} d(x_{n(k)+1},x_{m(k)}) \leq s^2\varepsilon, \\
&\frac{\varepsilon}{s} \leq \liminf_{n\to\infty} d(x_{n(k)},x_{m(k)+1}) \leq \limsup_{n\to\infty} d(x_{n(k)},x_{m(k)+1}) \leq s^2\varepsilon, \\
&\frac{\varepsilon}{s^2} \leq \liminf_{n\to\infty} d(x_{n(k)+1},x_{m(k)+1}) \leq \limsup_{n\to\infty} d(x_{n(k)+1},x_{m(k)+1}) \leq s^3\varepsilon.
\end{aligned}$$

Finally, we shall announce a generalization of the recent Geraghty type result from [15], Theorem 2.1, used in the setting of b-metric spaces. In order to use our ideas in the framework of dislocated quasi-b-metric spaces, we shall consider the class of functions $\mathcal{B}_s$, where $\beta \in \mathcal{B}_s$ satisfies $\beta : [0, +\infty) \to [0, \frac{1}{s})$ and has the property

$$\beta(t_n) \to \frac{1}{s} \quad \text{implies} \quad t_n \to 0.$$

An example of such a β is given by $\beta(t) = \frac{1}{s}e^{-t}$ for $t > 0$ and $\beta(0) \in [0, \frac{1}{s})$.

The following result is a generalization of [15], Theorem 2.1, since we do not assume that the dislocated quasi-b-metric d is continuous. With regard to its proof, we omit it because it is almost the same as the counterpart in [15], Theorem 2.1, for b-metric spaces without using Lemma 2.8 and Lemma 2.9, however, many other papers have to use them.

Theorem 2.10 *Let (X, d) be a dislocated quasi-b-metric space with $s > 1$ and let $f, g : X \to X$ be two maps such that $f(X) \subseteq g(X)$ and one of these two subsets of X is dqb-complete. If for some function $\beta \in \mathcal{B}_s$ and for all $x, y \in X$ we have*

$$d(fx, fy) \leq \beta\bigl(d(gx, gy)\bigr)d(gx, gy),$$

then f and g have a unique point of coincidence ω. Moreover, for each $x_0 \in X$, a corresponding Jungck sequence $\{y_n\}$ can be chosen such that $\lim_{n\to\infty} y_n = \omega$. In addition, if f and g are weakly compatible, then they have a unique common fixed point.

Taking $g = I_X$ (identity mapping of X) in Theorem 2.10, we see that the following Geraghty type theorem in dislocated quasi-b-metric spaces.

Corollary 2.11 *Let (X, d) be a dqb-complete dislocated quasi-b-metric space with $s > 1$. Suppose that a mapping $f : X \to X$ satisfies*

$$d(fx, fy) \leq \beta\bigl(d(x, y)\bigr)d(x, y)$$

for all $x, y \in X$ and some $\beta \in \mathcal{B}_s$. Then f has a unique fixed point $z \in X$, and for each $x \in X$, the Picard sequence $\{f^n x\}$ converges to z in X.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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References

1. Banach, S: Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales. Fundam. Math. **3**, 133-181 (1922)
2. Kannan, R: Some results on fixed points - II. Am. Math. Mon. **76**, 405-408 (1969)
3. George, R, Rajagopalan, R, Vinayagam, S: Cyclic contractions and fixed points in dislocated metric spaces. Int. J. Math. Anal. **7**(9), 403-411 (2013)
4. Kirk, WA, Srinavasan, PS, Veeramani, P: Fixed points for mapping satisfying cyclical contractive conditions. Fixed Point Theory **4**, 79-89 (2003)
5. Klin-eam, C, Suanoom, C: Dislocated quasi-*b*-metric spaces and fixed point theorems for cyclic contractions. Fixed Point Theory Appl. **2015**, 74 (2015)
6. Petrić, MA: Some results concerning cyclical contractive mappings. Gen. Math. **18**(4), 213-226 (2010)
7. Radenović, S, Došenović, T, Aleksić-Lampert, T, Golubović, Z: A note on some recent fixed point results for cyclic contractions in *b*-metric spaces and an application to integral equations. Appl. Math. Comput. **273**, 155-164 (2016)
8. Roshan, JR, Parvaneh, V, Kadelburg, Z: Common fixed point theorems for weakly isotone increasing mappings in ordered *b*-metric spaces. J. Nonlinear Sci. Appl. **7**, 229-245 (2014)
9. Zoto, K, Hoxha, E, Isufati, A: Fixed point theorems for cyclic contraction in dislocated metric spaces. Adv. Res. Sci. Areas **2**, 359-363 (2013)
10. Zoto, K, Kumari, PS, Hoxha, E: Some fixed point theorems and cyclic contractions in dislocated and dislocated quasi-metric spaces. Am. J. Numer. Anal. **2**(3), 79-84 (2014)
11. Shukla, S, Radenović, S, Rajić, VĆ: Some common fixed point theorems in 0-σ-complete metric-like spaces. Vietnam J. Math. **41**, 341-352 (2013)
12. Karapınar, E, Erhan, IM: Best proximity on different type contraction. Appl. Math. Inf. Sci. **5**, 558-569 (2010)
13. Alghamdi, MA, Hussain, N, Salimi, P: Fixed point and coupled fixed point theorems on *b*-metric-like spaces. J. Inequal. Appl. **2013**, 142 (2013)
14. Amini-Harandi, A: Metric-like spaces, partial metric spaces and fixed points. Fixed Point Theory Appl. **2012**, 204 (2012)
15. Ding, H-S, Imdad, M, Radenović, S, Vujaković, J: On some fixed point results in *b*-metric, rectangular and *b*-rectangular metric spaces. Arab J. Math. Sci. (2015). doi:10.1016/j.ajmsc.2015.05.003
16. Hitzler, P: Generalized metrics and topology in logic programming semantics. PhD thesis, National University of Ireland, University College Cork (2001)
17. Wilson, WA: On quasi-metric spaces. Am. J. Math. **53**(3), 675-684 (1931)
18. Zhu, C-X, Chen, C-F, Zhang, X: Some results in quasi-*b*-metric-like spaces. J. Inequal. Appl. **2014**, 437 (2014)
19. Salimi, P, Hussain, N, Shukla, S, Fathollahi, S, Radenović, S: Fixed point results for cyclic α-ψ-ϕ-contractions with applications to integral equations. J. Comput. Appl. Math. **290**, 445-458 (2015)