Nantadilok *Fixed Point Theory and Applications* (2015) 2015:30
DOI 10.1186/s13663-015-0280-y

Fixed Point Theory and Applications
a SpringerOpen Journal

RESEARCH **Open Access**

Coupled best proximity point theorems for α-ψ-proximal contractive multimaps

Jamnian Nantadilok*

*Correspondence:
jamnian52@lpru.ac.th
Department of Mathematics,
Faculty of Science, Lampang
Rajabhat University, Lampang,
52100, Thailand

Abstract
In this paper, we establish coupled best proximity point theorems for multivalued mappings. Our results extend some recent results by Ali *et al.* (Abstr. Appl. Anal. 2014:181598, 2014) as well as other results in the literature. We also give examples to support our main results.

MSC: 47H09; 47H10

Keywords: proximal contractive multivalued mapping; best proximity point; coupled fixed point; coupled best proximity point

1 Introduction and preliminaries

The Banach contraction principle is one of the most well-known and useful tools in analysis. This principle has been generalized by many authors in many different ways (see [1–6]). Recently, Samet *et al.* [7] introduced the notion of α-ψ-contractive type mappings and proved some fixed point theorems for such mappings within the framework of complete metric spaces. Karapınar and Samet [8] generalized α-ψ-contractive type mappings and obtained some fixed point theorems for generalized α-ψ-contractive type mappings. Some interesting multivalued generalizations of α-ψ-contractive type mappings are available in [9–18]. More recently, Jleli and Samet [19] introduced the notion of α-ψ-proximal contractive type mappings and proved certain best proximity point theorems. Many authors have obtained best proximity point theorems and have done so in a variety of settings; see, for example, [19–41]. Abkar and Gbeleh [22] and Al-Thagafi and Shahzad [24, 26] investigated best proximity points for multivalued mappings. Recently Ali *et al.* extended the results of Jleli and Samet [19] for nonself multivalued mappings. The concept of coupled best proximity point theorem was introduced by Sintunavarat and Kumam [36], and they proved the coupled best proximity theorem for cyclic contractions.

Inspired and motivated by the recent results of Ali *et al.* in [42] and by those of Sintunavarat and Kumam in [36], we establish the coupled best proximity points for α-ψ-proximal contractive multimaps. We also give examples to support our main results.

Let (X,d) be a metric space. For $A,B \subset X$, we use the following notations subsequently: $\operatorname{dist}(A,B) = \inf\{d(a,b) : a \in A, b \in B\}$, $D(x,B) = \inf\{d(x,b) : b \in B\}$, $A_0 = \{a \in A : d(a,b) = \operatorname{dist}(A,B)$ for some $b \in B\}$, $B_0 = \{b \in B : d(a,b) = \operatorname{dist}(A,B)$ for some $a \in A\}$, $2^X \setminus \emptyset$ is the set of all nonempty subsets of X, $CL(X)$ is the set of all nonempty closed subsets of X, and

$K(X)$ is the set of all nonempty compact subsets of X. For every $A, B \in CL(X)$, let

$$H(A,B) = \begin{cases} \max\{\sup_{x\in A} d(x,B), \sup_{y\in B} d(y,A)\} & \text{if the maximum exists;} \\ \infty & \text{otherwise.} \end{cases} \tag{1}$$

Such a map H is called the generalized Hausdorff metric induced by d. A point $x^* \in X$ is said to be the best proximity point of a mapping $T : A \to B$ if $d(x^*, Tx^*) = \operatorname{dist}(A,B)$. When $A = B$, the best proximity point is essentially the fixed point of the mapping T.

Definition 1.1 (see [34]) Let (A,B) be a pair of nonempty subsets of a metric space (X,d) with $A_0 \neq \emptyset$. Then the pair (A,B) is said to have the weak P-property if and only if, for any $x_1, x_2 \in A$ and $y_1, y_2 \in B$,

$$\left.\begin{aligned} d(x_1,y_1) &= \operatorname{dist}(A,B), \\ d(x_2,y_2) &= \operatorname{dist}(A,B) \end{aligned}\right\} \quad\Rightarrow\quad d(x_1,x_2) \le d(y_1,y_2). \tag{2}$$

Let Ψ denote the set of all functions $\psi : [0,\infty) \to [0,\infty)$ satisfying the following properties:

(a) ψ is monotone nondecreasing;
(b) $\sum_{n=1}^{\infty} \psi^n(t) < \infty$ for each $t > 0$.

Definition 1.2 (see [21]) An element $x^* \in A$ is said to be the best proximity point of a multivalued nonself mapping T if $D(x^*, Tx^*) = \operatorname{dist}(A,B)$.

Definition 1.3 (see [42]) Let A and B be two nonempty subsets of a metric space (X,d). A mapping $T : A \to 2^B \setminus \emptyset$ is called α-proximal admissible if there exists a mapping $\alpha : A \times A \to [0,\infty)$ such that

$$\left.\begin{aligned} \alpha(x_1,x_2) &\ge 1, \\ d(u_1,y_1) &= \operatorname{dist}(A,B), \\ d(u_2,y_2) &= \operatorname{dist}(A,B) \end{aligned}\right\} \quad\Rightarrow\quad \alpha(u_1,u_2) \ge 1, \tag{3}$$

where $x_1, x_2, u_1, u_2 \in A$, $y_1 \in Tx_1$ and $y_2 \in Tx_2$.

Definition 1.4 (see [42]) Let A and B be two nonempty subsets of a metric space (X,d). A mapping $T : A \to CL(B)$ is said to be an α-ψ-proximal contraction if there exist two functions $\psi \in \Psi$ and $\alpha : A \times A \to [0,\infty)$ such that

$$\alpha(x,y)H(Tx,Ty) \le \psi\bigl(d(x,y)\bigr), \quad \forall x,y \in A. \tag{4}$$

Lemma 1.5 (see [11]) *Let (X,d) be a metric space and $B \in CL(X)$. Then, for each $x \in X$ with $d(x,B) > 0$ and $q > 1$, there exists an element $b \in B$ such that*

$$d(x,b) < qd(x,B). \tag{5}$$

(C) If $\{x_n\}$ is a sequence in A such that $\alpha(x_n, x_{n+1}) \ge 1$ for all n and $x_n \to x \in A$ as $n \to \infty$, then there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $\alpha(x_{n_k}, x) \ge 1$ for all k.

The main results of Ali *et al.* in [42] are the following.

Theorem 1.6 (see [42]) *Let A and B be two nonempty closed subsets of a complete metric space (X,d) such that A_0 is nonempty. Let $\alpha : A \times A \to [0,\infty)$ and $\psi \in \Psi$ be a strictly increasing map. Suppose that $T : A \to \mathrm{CL}(B)$ is a mapping satisfying the following conditions*:

(i) *$Tx \subseteq B_0$ for each $x \in A_0$ and (A,B) satisfies the weak P-property*;
(ii) *T is an α-proximal admissible map*;
(iii) *there exist elements x_0, x_1 in A_0 and $y_1 \in Tx_0$ such that*

$$d(x_1,y_1) = d(A,B), \qquad \alpha(x_0,x_1) \geq 1; \tag{6}$$

(iv) *T is a continuous α-ψ-proximal contraction.*

Then there exists an element $x^* \in A_0$ such that

$$D\bigl(x^*, Tx^*\bigr) = \operatorname{dist}(A,B).$$

Theorem 1.7 (see [42]) *Let A and B be two nonempty closed subsets of a complete metric space (X,d) such that A_0 is nonempty. Let $\alpha : A \times A \to [0,\infty)$ and let $\psi \in \Psi$ be a strictly increasing map. Suppose that $T : A \to \mathrm{CL}(B)$ is a mapping satisfying the following conditions*:

(i) *$Tx \subseteq B_0$ for each $x \in A_0$ and (A,B) satisfies the weak P-property*;
(ii) *T is an α-proximal admissible map*;
(iii) *there exist elements x_0, x_1 in A_0 and $y_1 \in Tx_0$ such that*

$$d(x_1,y_1) = d(A,B), \qquad \alpha(x_0,x_1) \geq 1; \tag{7}$$

(iv) *property* (C) *holds and T is an α-ψ-proximal contraction.*

Then there exists an element $x^* \in A_0$ such that

$$D\bigl(x^*, Tx^*\bigr) = \operatorname{dist}(A,B).$$

The purpose of this paper is to extend the recent results of Ali *et al.* [42] to a coupled best proximity point of nonself multivalued mappings.

2 Main results

We begin this section by introducing the following definitions.

Definition 2.1 Let A and B be two nonempty subsets of a metric space (X,d). A mapping $T : A \times A \to 2^B \backslash \emptyset$ is called α-proximal admissible if there exists a mapping $\alpha : A \times A \to [0,\infty)$ such that

$$\left.\begin{aligned} \alpha(x_1,x_2) \geq 1, \\ d(w_1,u_1) = \operatorname{dist}(A,B), \\ d(w_2,u_2) = \operatorname{dist}(A,B) \end{aligned}\right\} \quad \Rightarrow \quad \alpha(w_1,w_2) \geq 1, \tag{8}$$

where $x_1, x_2, w_1, w_2, y_1, y_2 \in A$, $u_1 \in T(x_1, y_1)$ and $u_2 \in T(x_2, y_2)$, and

$$\left.\begin{aligned} \alpha(y_1, y_2) \geq 1, \\ d(w_1', v_1) = \operatorname{dist}(A, B), \\ d(w_2', v_2) = \operatorname{dist}(A, B) \end{aligned}\right\} \quad \Rightarrow \quad \alpha\left(w_1', w_2'\right) \geq 1, \tag{9}$$

where $y_1, y_2, w_1', w_2', x_1, x_2 \in A$, $v_1 \in T(y_1, x_1)$ and $v_2 \in T(y_2, x_2)$.

Definition 2.2 Let A and B be two nonempty subsets of a metric space (X, d). A mapping $T : A \times A \to \mathrm{CL}(B)$ is said to be an α-ψ-proximal contraction if there exist two functions $\psi \in \Psi$ and $\alpha : A \times A \to [0, \infty)$ such that

$$\alpha(x, y) H\left(T\left(x, x'\right), T\left(y, y'\right)\right) \leq \psi\left(d(x, y)\right), \quad \forall x, x', y, y' \in A. \tag{10}$$

Definition 2.3 An element $(x^*, y^*) \in A \times A$ is said to be the coupled best proximity point of a multivalued nonself mapping T if $D(x^*, T(x^*, y^*)) = \operatorname{dist}(A, B)$ and $D(y^*, T(y^*, x^*)) = \operatorname{dist}(A, B)$.

The following are our main results.

Theorem 2.4 *Let A and B be two nonempty closed subsets of a complete metric space (X, d) such that A_0 is nonempty. Let $\alpha : A \times A \to [0, \infty)$ and let $\psi \in \Psi$ be a strictly increasing map. Suppose that $T : A \times A \to \mathrm{CL}(B)$ is a mapping satisfying the following conditions*:

(i) *$T(x, y) \subseteq B_0$ for each $x, y \in A_0$ and (A, B) satisfies the weak P-property*;

(ii) *T is an α-proximal admissible map*;

(iii) *there exist elements (x_0, y_0), (x_1, y_1) in $A_0 \times A_0$ and $u_1 \in T(x_0, y_0)$, $v_1 \in T(y_0, x_0)$ such that*

$$\begin{aligned} &d(x_1, u_1) = d(A, B), \qquad \alpha(x_0, x_1) \geq 1 \quad \textit{and} \\ &d(y_1, v_1) = d(A, B), \qquad \alpha(y_0, y_1) \geq 1; \end{aligned} \tag{11}$$

(iv) *T is a continuous α-ψ-proximal contraction.*

Then there exists an element $(x^*, y^*) \in A_0 \times A_0$ such that

$$\begin{aligned} &D\left(x^*, T\left(x^*, y^*\right)\right) = \operatorname{dist}(A, B) \quad \textit{and} \\ &D\left(y^*, T\left(y^*, x^*\right)\right) = \operatorname{dist}(A, B). \end{aligned}$$

Proof From condition (iii), there exist elements (x_0, y_0), (x_1, y_1) in $A_0 \times A_0$ and $u_1 \in T(x_0, y_0)$, $v_1 \in T(y_0, x_0)$ such that

$$\begin{aligned} &d(x_1, u_1) = \operatorname{dist}(A, B), \qquad \alpha(x_0, x_1) \geq 1 \quad \text{and} \\ &d(y_1, v_1) = \operatorname{dist}(A, B), \qquad \alpha(y_0, y_1) \geq 1. \end{aligned} \tag{12}$$

Assume that $u_1 \notin T(x_1, y_1)$, $v_1 \notin T(y_1, x_1)$; for otherwise (x_1, y_1) is the coupled best proximity point. From condition (iv), we have

$$\begin{aligned} 0 &< d\bigl(u_1, T(x_1, y_1)\bigr) \leq H\bigl(T(x_0, y_0), T(x_1, y_1)\bigr) \\ &\leq \alpha(x_0, x_1) H\bigl(T(x_0, y_0), T(x_1, y_1)\bigr) \\ &\leq \psi\bigl(d(x_0, x_1)\bigr) \end{aligned} \tag{13}$$

and

$$\begin{aligned} 0 &< d\bigl(v_1, T(y_1, x_1)\bigr) \leq H\bigl(T(y_0, x_0), T(y_1, x_1)\bigr) \\ &\leq \alpha(y_0, y_1) H\bigl(T(y_0, x_0), T(y_1, x_1)\bigr) \\ &\leq \psi\bigl(d(y_0, y_1)\bigr). \end{aligned} \tag{14}$$

For $q, q' > 1$, it follows from Lemma 1.5 that there exist $u_2 \in T(x_1, y_1)$ and $v_2 \in T(y_1, x_1)$ such that

$$\begin{aligned} &0 < d(u_1, u_2) < qd\bigl(u_1, T(x_1, y_1)\bigr) \quad \text{and} \\ &0 < d(v_1, v_2) < q' d\bigl(v_1, T(y_1, x_1)\bigr). \end{aligned} \tag{15}$$

From (13), (14) and (15), we have

$$0 < d(u_1, u_2) < qd\bigl(u_1, T(x_1, y_1)\bigr) \leq q\psi\bigl(d(x_0, x_1)\bigr) \tag{16}$$

and

$$0 < d(v_1, v_2) < q' d\bigl(v_1, T(y_1, x_1)\bigr) \leq q' \psi\bigl(d(y_0, y_1)\bigr). \tag{17}$$

As $u_2 \in T(x_1, y_1) \subseteq B_0$, there exists $x_2 \neq x_1 \in A_0$ such that

$$d(x_2, u_2) = \operatorname{dist}(A, B), \tag{18}$$

and as $v_2 \in T(y_1, x_1) \subseteq B_0$, there exists $y_2 \neq y_1 \in A_0$ such that

$$d(y_2, v_2) = \operatorname{dist}(A, B); \tag{19}$$

for otherwise (x_1, y_1) is the coupled best proximity point. As (A, B) satisfies the weak P-property, from (12), (18) and (19) we have

$$\begin{aligned} &0 < d(x_1, x_2) \leq d(u_1, u_2) \quad \text{and} \\ &0 < d(y_1, y_2) \leq d(v_1, v_2). \end{aligned} \tag{20}$$

From (16), (17) and (20) we have

$$\begin{aligned} &0 < d(x_1, x_2) \leq d(u_1, u_2) < qd\bigl(u_1, T(x_1, y_1)\bigr) \leq q\psi\bigl(d(x_0, x_1)\bigr) \quad \text{and} \\ &0 < d(y_1, y_2) \leq d(v_1, v_2) < q' d\bigl(v_1, T(y_1, x_1)\bigr) \leq q' \psi\bigl(d(y_0, y_1)\bigr). \end{aligned} \tag{21}$$

Since ψ is strictly increasing, we have

$$\psi\big(d(x_1,x_2)\big) < \psi\big(q\psi\big(d(x_0,x_1)\big)\big) \quad \text{and}$$
$$\psi\big(d(y_1,y_2)\big) < \psi\big(q'\psi\big(d(y_0,y_1)\big)\big).$$

Put

$$q_1 = \psi\big(q\psi\big(d(x_0,x_1)\big)\big)/\psi\big(d(x_1,x_2)\big),$$
$$q_1' = \psi\big(q'\psi\big(d(y_0,y_1)\big)\big)/\psi\big(d(y_1,y_2)\big).$$

We also have

$$\alpha(x_0,x_1) \geq 1, \qquad d(x_1,u_1) = \operatorname{dist}(A,B) \quad \text{and} \quad d(x_2,u_2) = \operatorname{dist}(A,B)$$

and

$$\alpha(y_0,y_1) \geq 1, \qquad d(y_1,v_1) = \operatorname{dist}(A,B) \quad \text{and} \quad d(y_2,v_2) = \operatorname{dist}(A,B).$$

Since T is an α-proximal admissible, then $\alpha(x_1,x_2) \geq 1$ and $\alpha(y_1,y_2) \geq 1$. Thus we have

$$\begin{aligned} &d(x_2,u_2) = \operatorname{dist}(A,B), \qquad \alpha(x_1,x_2) \geq 1 \quad \text{and} \\ &d(y_2,v_2) = \operatorname{dist}(A,B), \qquad \alpha(y_1,y_2) \geq 1. \end{aligned} \tag{22}$$

Assume that $u_2 \notin T(x_2,y_2)$ and $v_2 \notin T(y_2,x_2)$; for otherwise (x_2,y_2) is the coupled best proximity point. From condition (iv) we have

$$\begin{aligned} 0 < d\big(u_2, T(x_2,y_2)\big) &\leq H\big(T(x_1,y_1), T(x_2,y_2)\big) \\ &\leq \alpha(x_1,x_2)H\big(T(x_1,y_1), T(x_2,y_2)\big) \\ &\leq \psi\big(d(x_1,x_2)\big) \end{aligned} \tag{23}$$

and

$$\begin{aligned} 0 < d\big(v_2, T(y_2,x_2)\big) &\leq H\big(T(y_1,x_1), T(y_2,x_2)\big) \\ &\leq \alpha(y_1,y_2)H\big(T(y_1,x_1), T(y_2,x_2)\big) \\ &\leq \psi\big(d(y_1,y_2)\big). \end{aligned} \tag{24}$$

For $q_1, q_1' > 1$, it follows from Lemma 1.5 that there exist $u_3 \in T(x_2,y_2)$ and $v_3 \in T(y_2,x_2)$ such that

$$\begin{aligned} &0 < d(u_2,u_3) < q_1 d\big(u_2, T(x_2,y_2)\big), \\ &0 < d(v_2,v_3) < q_1' d\big(v_2, T(y_2,x_2)\big). \end{aligned} \tag{25}$$

From (23), (24) and (25) we have

$$\begin{aligned} 0 < d(u_2, u_3) &< q_1 d\bigl(u_2, T(x_2, y_2)\bigr) \\ &\le q_1 \psi\bigl(d(x_1, x_2)\bigr) \\ &= \psi\bigl(q\psi\bigl(d(x_0, x_1)\bigr)\bigr) \end{aligned} \tag{26}$$

and

$$\begin{aligned} 0 < d(v_2, v_3) &< q_1' d\bigl(v_2, T(y_2, x_2)\bigr) \\ &\le q_1' \psi\bigl(d(y_1, y_2)\bigr) \\ &= \psi\bigl(q' \psi\bigl(d(y_0, y_1)\bigr)\bigr). \end{aligned} \tag{27}$$

As $u_3 \in T(x_2, y_2) \in B_0$, there exists $x_3 \neq x_2 \in A_0$ such that

$$d(x_3, u_3) = \operatorname{dist}(A, B); \tag{28}$$

and as $v_3 \in T(y_2, x_2) \in B_0$, there exists $y_3 \neq y_2 \in A_0$ such that

$$d(y_3, v_3) = \operatorname{dist}(A, B); \tag{29}$$

for otherwise (x_2, y_2) is the coupled best proximity point. As (A, B) satisfies the weak P-property, from (22), (28) and (29) we have

$$\begin{aligned} &0 < d(x_2, x_3) \le d(u_2, u_3), \\ &0 < d(y_2, y_3) \le d(v_2, v_3). \end{aligned} \tag{30}$$

From (26), (27) and (30) we have

$$\begin{aligned} 0 < d(x_2, x_3) &< q_1 d\bigl(u_2, T(x_2, y_2)\bigr) \\ &\le q_1 \psi\bigl(d(x_1, x_2)\bigr) \\ &= \psi\bigl(q\psi\bigl(d(x_0, x_1)\bigr)\bigr) \end{aligned} \tag{31}$$

and

$$\begin{aligned} 0 < d(y_2, y_3) &< q_1' d\bigl(v_2, T(y_2, x_2)\bigr) \\ &\le q_1' \psi\bigl(d(y_1, y_2)\bigr) \\ &= \psi\bigl(q' \psi\bigl(d(y_0, y_1)\bigr)\bigr). \end{aligned} \tag{32}$$

Since ψ is strictly increasing, we have

$$\psi\bigl(d(x_2, x_3)\bigr) < \psi^2\bigl(q\psi\bigl(d(x_0, x_1)\bigr)\bigr) \quad \text{and} \quad \psi\bigl(d(y_2, y_3)\bigr) < \psi^2\bigl(q' \psi\bigl(d(y_0, y_1)\bigr)\bigr). \tag{33}$$

Put

$$q_2 = \psi^2\big(q\psi\big(d(x_0,x_1)\big)\big)/\psi\big(d(x_2,x_3)\big),$$
$$q_2' = \psi^2\big(q'\psi\big(d(y_0,y_1)\big)\big)/\psi\big(d(y_2,y_3)\big).$$

We also have

$$\alpha(x_1,x_2) \geq 1, \qquad d(x_2,u_2) = \operatorname{dist}(A,B) \quad \text{and} \quad d(x_3,u_3) = \operatorname{dist}(A,B)$$

and

$$\alpha(y_1,y_2) \geq 1, \qquad d(y_2,v_2) = \operatorname{dist}(A,B) \quad \text{and} \quad d(y_3,v_3) = \operatorname{dist}(A,B).$$

Since T is an α-proximal admissible, then $\alpha(x_2,x_3) \geq 1$ and $\alpha(y_2,y_3) \geq 1$, respectively. Thus we have

$$\begin{aligned} &d(x_3,u_3) = \operatorname{dist}(A,B), \qquad \alpha(x_2,x_3) \geq 1 \quad \text{and} \\ &d(y_3,v_3) = \operatorname{dist}(A,B), \qquad \alpha(y_2,y_3) \geq 1. \end{aligned} \tag{34}$$

Continuing in the same process, we get sequences $\{x_n\}$, $\{y_n\}$ in A_0 and $\{u_n\}$, $\{v_n\}$ in B_0, where $u_n \in T(x_{n-1},y_{n-1})$ and $v_n \in T(y_{n-1},x_{n-1})$ for each $n \in \mathbb{N}$, such that

$$\begin{aligned} &d(x_{n+1},u_{n+1}) = \operatorname{dist}(A,B), \qquad \alpha(x_n,x_{n+1}) \geq 1 \quad \text{and} \\ &d(y_{n+1},v_{n+1}) = \operatorname{dist}(A,B), \qquad \alpha(y_n,y_{n+1}) \geq 1, \end{aligned} \tag{35}$$

and

$$\begin{aligned} &d(u_{n+1},u_{n+2}) < \psi^n\big(q\psi\big(d(x_0,x_1)\big)\big) \quad \text{and} \\ &d(v_{n+1},v_{n+2}) < \psi^n\big(q'\psi\big(d(y_0,y_1)\big)\big). \end{aligned} \tag{36}$$

As $u_{n+2} \in T(x_{n+1},y_{n+1}) \in B_0$, there exists $x_{n+2} \neq x_{n+1} \in A_0$ such that

$$d(x_{n+2},u_{n+2}) = \operatorname{dist}(A,B) \tag{37}$$

and as $v_{n+2} \in T(y_{n+1},x_{n+1}) \in B_0$, there exists $y_{n+2} \neq y_{n+1} \in A_0$ such that

$$d(y_{n+2},v_{n+2}) = \operatorname{dist}(A,B). \tag{38}$$

Since (A,B) satisfies the weak P-property, from (35), (37) and (38) we have

$$d(x_{n+1},x_{n+2}) \leq d(u_{n+1},u_{n+2}) \quad \text{and} \quad d(y_{n+1},y_{n+2}) \leq d(v_{n+1},v_{n+2}).$$

Thus, from (36) we have

$$\begin{aligned} &d(x_{n+1},x_{n+2}) < \psi^n\big(q\psi\big(d(x_0,x_1)\big)\big) \quad \text{and} \\ &d(y_{n+1},y_{n+2}) < \psi^n\big(q'\psi\big(d(y_0,y_1)\big)\big). \end{aligned} \tag{39}$$

Now, we shall prove that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in A. Let $\epsilon > 0$ be fixed. Since $\sum_{n=1}^{\infty} \psi^n(q\psi(d(x_0,x_1))) < \infty$ and $\sum_{n=1}^{\infty} \psi^n(q'\psi(d(y_0,y_1))) < \infty$, there exist some positive integers $h = h(\epsilon)$ and $h' = h'(\epsilon)$ such that

$$\sum_{k\geq h}^{\infty} \psi^k\bigl(q\psi\bigl(d(x_0,x_1)\bigr)\bigr) < \epsilon$$

and

$$\sum_{k\geq h'}^{\infty} \psi^k\bigl(q'\psi\bigl(d(y_0,y_1)\bigr)\bigr) < \epsilon,$$

respectively. For $m > n > h$, using the triangular inequality, we obtain

$$\begin{aligned} d(x_n,x_m) &\leq \sum_{k=n}^{m-1} d(x_k,x_{k+1}) \leq \sum_{k=n}^{m-1} \psi^k\bigl(q\psi\bigl(d(x_0,x_1)\bigr)\bigr) \\ &\leq \sum_{k\geq h}^{\infty} \psi^k\bigl(q\psi\bigl(d(x_0,x_1)\bigr)\bigr) < \epsilon \end{aligned} \tag{40}$$

and

$$\begin{aligned} d(y_n,y_m) &\leq \sum_{k=n}^{m-1} d(y_k,y_{k+1}) \leq \sum_{k=n}^{m-1} \psi^k\bigl(q'\psi\bigl(d(y_0,y_1)\bigr)\bigr) \\ &\leq \sum_{k\geq h'}^{\infty} \psi^k\bigl(q'\psi\bigl(d(y_0,y_1)\bigr)\bigr) < \epsilon, \end{aligned} \tag{41}$$

respectively. Hence $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in A. Similarly, one can show that $\{u_n\}$ and $\{v_n\}$ are Cauchy sequences in B. Since A and B are closed subsets of a complete metric space, there exists (x^*,y^*) in $A \times A$ such that $x_n \to x^*$, $y_n \to y^*$ as $n \to \infty$ and there exist u^*, v^* in B such that $u_n \to u^*$, $v_n \to v^*$ as $n \to \infty$. By (37) and (38) we conclude that

$$\begin{aligned} &d\bigl(x^*,u^*\bigr) = \operatorname{dist}(A,B) \quad \text{as } n \to \infty \quad \text{and} \\ &d\bigl(y^*,v^*\bigr) = \operatorname{dist}(A,B) \quad \text{as } n \to \infty. \end{aligned}$$

Since T is continuous and $u_n \in T(x_{n-1},y_{n-1})$, we have $u^* \in T(x^*,y^*)$ and $v_n \in T(y_{n-1},x_{n-1})$, we have $v^* \in T(y^*,x^*)$. Hence,

$$\operatorname{dist}(A,B) \leq D\bigl(x^*,T\bigl(x^*,y^*\bigr)\bigr) \leq d\bigl(x^*,u^*\bigr) = \operatorname{dist}(A,B)$$

and

$$\operatorname{dist}(A,B) \leq D\bigl(y^*,T\bigl(y^*,x^*\bigr)\bigr) \leq d\bigl(y^*,v^*\bigr) = \operatorname{dist}(A,B).$$

Therefore, (x^*,y^*) is the coupled best proximity point of the mapping T. □

Theorem 2.5 *Let A and B be two nonempty closed subsets of a complete metric space (X,d) such that A_0 is nonempty. Let $\alpha : A \times A \to [0,\infty)$ and let $T : A \times A \to \mathrm{K}(B)$ be a mapping satisfying the following conditions*:

(i) *$T(x,y) \subseteq B_0$ for each $(x,y) \in A_0 \times A_0$ and (A,B) satisfies the weak P-property*;

(ii) *T is an α-proximal admissible map*;

(iii) *there exist elements (x_0,y_0), (x_1,y_1) in $A_0 \times A_0$ and $u_1 \in T(x_0,y_0)$, $v_1 \in T(y_0,x_0)$ such that*

$$\begin{aligned} &d(x_1,u_1) = \operatorname{dist}(A,B), \qquad \alpha(x_0,x_1) \geq 1 \quad \textit{and} \\ &d(y_1,v_1) = \operatorname{dist}(A,B), \qquad \alpha(y_0,y_1) \geq 1; \end{aligned} \tag{42}$$

(iv) *T is a continuous α-ψ-proximal contraction.*

Then there exists an element $(x^*,y^*) \in A_0 \times A_0$ such that

$$\begin{aligned} &D\bigl(x^*, T\bigl(x^*,y^*\bigr)\bigr) = \operatorname{dist}(A,B) \quad \textit{and} \\ &D\bigl(y^*, T\bigl(y^*,x^*\bigr)\bigr) = \operatorname{dist}(A,B). \end{aligned}$$

Theorem 2.6 *Let A and B be two nonempty closed subsets of a complete metric space (X,d) such that A_0 is nonempty. Let $\alpha : A \times A \to [0,\infty)$ and let $\psi \in \Psi$ be a strictly increasing map. Suppose that $T : A \times A \to \mathrm{CL}(B)$ is a mapping satisfying the following conditions*:

(i) *$T(x,y) \subseteq B_0$ for each $(x,y) \in A_0 \times A_0$ and (A,B) satisfies the weak P-property*;

(ii) *T is an α-proximal admissible map*;

(iii) *there exist elements (x_0,y_0), (x_1,y_1) in $A_0 \times A_0$ and $u_1 \in T(x_0,y_0)$, $v_1 \in T(y_0,x_0)$ such that*

$$\begin{aligned} &d(x_1,u_1) = d(A,B), \qquad \alpha(x_0,x_1) \geq 1 \quad \textit{and} \\ &d(y_1,v_1) = d(A,B), \qquad \alpha(y_0,y_1) \geq 1; \end{aligned} \tag{43}$$

(iv) *property* (C) *holds and T is an α-ψ-proximal contraction.*

Then there exists an element $(x^*,y^*) \in A_0 \times A_0$ such that

$$\begin{aligned} &D\bigl(x^*, T\bigl(x^*,y^*\bigr)\bigr) = \operatorname{dist}(A,B) \quad \textit{and} \\ &D\bigl(y^*, T\bigl(y^*,x^*\bigr)\bigr) = \operatorname{dist}(A,B). \end{aligned}$$

Proof Similar to the proof of Theorem 2.4, there exist Cauchy sequences $\{x_n\}$ and $\{y_n\}$ in A and Cauchy sequences $\{u_n\}$ and $\{v_n\}$ in B such that

$$\begin{aligned} &d(x_{n+1},u_{n+1}) = \operatorname{dist}(A,B), \qquad \alpha(x_n,x_{n+1}) \geq 1 \quad \text{and} \\ &d(y_{n+1},v_{n+1}) = \operatorname{dist}(A,B), \qquad \alpha(y_n,y_{n+1}) \geq 1; \end{aligned} \tag{44}$$

and $x_n \to x^* \in A$, $y_n \to y^* \in A$ as $n \to \infty$ and $u_n \to u^* \in B$, $v_n \to v^* \in B$ as $n \to \infty$.

From condition (C), there exist subsequences $\{x_{n_k}\}$ of $\{x_n\}$, $\{y_{n_k}\}$ of $\{y_n\}$ such that $\alpha(x_{n_k},x^*) \geq 1$, $\alpha(y_{n_k},y^*) \geq 1$ for all k. Since T is an α-ψ-proximal contraction, we have

$$\begin{aligned} H\bigl(T(x_{n_k},y_{n_k}), T\bigl(x^*,y^*\bigr)\bigr) &\leq \alpha\bigl(x_{n_k},x^*\bigr) H\bigl(T(x_{n_k},y_{n_k}), T\bigl(x^*,y^*\bigr)\bigr) \\ &\leq \psi\bigl(d\bigl(x_{n_k},x^*\bigr)\bigr), \quad \forall k, \end{aligned}$$

and

$$\begin{aligned} H\big(T(y_{n_k},x_{n_k}),T\big(y^*,x^*\big)\big) &\le \alpha\big(y_{n_k},y^*\big)H\big(T(y_{n_k},x_{n_k}),T\big(y^*,x^*\big)\big) \\ &\le \psi\big(d\big(y_{n_k},y^*\big)\big), \quad \forall k. \end{aligned}$$

Letting $k \to \infty$ in the above inequality, we get $T(x_{n_k},y_{n_k}) \to T(x^*,y^*)$ and $T(y_{n_k},x_{n_k}) \to T(y^*,x^*)$, respectively. By the continuity of the metric d, we have

$$\begin{aligned} d\big(x^*,u^*\big) &= \lim_{k\to\infty} d(x_{n_k+1},u_{n_k+1}) = \operatorname{dist}(A,B), \\ d\big(y^*,v^*\big) &= \lim_{k\to\infty} d(y_{n_k+1},v_{n_k+1}) = \operatorname{dist}(A,B). \end{aligned} \tag{45}$$

Since $u_{n_k+1} \in T(x_{n_k},y_{n_k})$, $u_{n_k} \to u^*$ and $T(x_{n_k},y_{n_k}) \to T(x^*,y^*)$, then $u^* \in T(x^*,y^*)$ and since $v_{n_k+1} \in T(y_{n_k},x_{n_k})$, $v_{n_k} \to v^*$ and $T(y_{n_k},x_{n_k}) \to T(y^*,x^*)$, then $v^* \in T(y^*,x^*)$. Hence,

$$\operatorname{dist}(A,B) \le D\big(x^*,T\big(x^*,y^*\big)\big) \le d\big(x^*,u^*\big) = \operatorname{dist}(A,B)$$

and

$$\operatorname{dist}(A,B) \le D\big(y^*,T\big(y^*,x^*\big)\big) \le d\big(y^*,v^*\big) = \operatorname{dist}(A,B).$$

Therefore, (x^*,y^*) is the coupled best proximity point of the mapping T. □

Theorem 2.7 *Let A and B be two nonempty closed subsets of a complete metric space (X,d) such that A_0 is nonempty. Let $\alpha : A \times A \to [0,\infty)$ and let $T : A \times A \to \mathrm{K}(B)$ be a mapping satisfying the following conditions*:

(i) *$T(x,y) \subseteq B_0$ for each $(x,y) \in A_0 \times A_0$ and (A,B) satisfies the weak P-property*;

(ii) *T is an α-proximal admissible map*;

(iii) *there exist elements (x_0,y_0), (x_1,y_1) in $A_0 \times A_0$ and $u_1 \in T(x_0,y_0)$, $v_1 \in T(y_0,x_0)$ such that*

$$\begin{aligned} d(x_1,u_1) &= \operatorname{dist}(A,B), \qquad \alpha(x_0,x_1) \ge 1 \quad \textit{and} \\ d(y_1,v_1) &= \operatorname{dist}(A,B), \qquad \alpha(y_0,y_1) \ge 1; \end{aligned} \tag{46}$$

(iv) *property* (C) *holds and T is an α-ψ-proximal contraction.*

Then there exists an element $(x^*,y^*) \in A_0 \times A_0$ such that

$$\begin{aligned} &D\big(x^*,T\big(x^*,y^*\big)\big) = \operatorname{dist}(A,B) \quad \textit{and} \\ &D\big(y^*,T\big(y^*,x^*\big)\big) = \operatorname{dist}(A,B). \end{aligned}$$

With a similar idea to the examples in [42], we give the following examples to support our main results.

Example 2.8 Let $X = [0,\infty) \times [0,\infty)$ be a product space endowed with the usual metric d. Suppose that $A = \{(\frac{1}{2},x) : 0 \le x < \infty\}$ and $B = \{(0,x) : 0 \le x < \infty\}$.

Define $T : A \times A \to \mathrm{CL}(B)$ by

$$T\left(\left(\frac{1}{2},a\right),\left(\frac{1}{2},b\right)\right) = \begin{cases} \{(0,\frac{x}{2}) : 0 \leq x \leq \max\{a,b\}\} & \text{if } a,b \leq 1, \\ \{(0,x^2) : 0 \leq x \leq \max\{a^2,b^2\}\} & \text{if } a,b > 1, \end{cases} \tag{47}$$

and define $\alpha : A \times A \to [0,\infty)$ by

$$\alpha(x,y) = \begin{cases} 1 & \text{if } x,y \in \{(\frac{1}{2},a) : 0 \leq a \leq 1\}, \\ 0 & \text{otherwise.} \end{cases}$$

Let $\Psi(t) = \frac{t}{2}$ for all $t \geq 0$. Note that $A_0 = A$, $B_0 = B$, and $T(x,y) \subseteq B_0$ for each $(x,y) \in A_0 \times A_0$. Also, the pair (A,B) satisfies the weak P-property.

Let $(x_0,y_0),(x_1,y_1) \in \{(\frac{1}{2},x) : 0 \leq x \leq 1\}^2$; then $T(x_0,y_0), T(x_1,y_1) \subseteq \{(0,\frac{x}{2}) : 0 \leq x \leq 1\}$. Consider $u_1 \in T(x_0,y_0)$, $u_2 \in T(x_1,y_1)$ and $w_1,w_2 \in A$ such that $d(w_1,u_1) = \mathrm{dist}(A,B)$ and $d(w_2,u_2) = \mathrm{dist}(A,B)$. Then we have $w_1,w_2 \in \{(\frac{1}{2},x) : 0 \leq x \leq \frac{1}{2}\}$, so $\alpha(w_1,w_2) = 1$. And, for $v_1 \in T(y_0,x_0)$, $v_2 \in T(y_1,x_1)$ and $w_1',w_2' \in A$ such that $d(w_1',v_1) = \mathrm{dist}(A,B)$ and $d(w_2',v_2) = \mathrm{dist}(A,B)$. Then we have $w_1',w_2' \in \{(\frac{1}{2},x) : 0 \leq x \leq \frac{1}{2}\}$, so $\alpha(w_1',w_2') = 1$. Therefore, T is an α-proximal admissible map. For $(x_0,y_0) = ((\frac{1}{2},1),(\frac{1}{2},1)) \in A_0 \times A_0$ and $u_1 = (0,\frac{1}{2}) \in T(x_0,y_0)$, $v_1 = (0,\frac{1}{4}) \in T(y_0,x_0)$ in B_0, we have $(x_1,y_1) = ((\frac{1}{2},\frac{1}{2}),(\frac{1}{2},\frac{1}{4})) \in A_0 \times A_0$ such that

$$d(x_1,u_1) = \mathrm{dist}(A,B), \qquad \alpha(x_0,x_1) = \alpha\left(\left(\frac{1}{2},1\right),\left(\frac{1}{2},\frac{1}{2}\right)\right) = 1$$

and

$$d(y_1,v_1) = \mathrm{dist}(A,B), \qquad \alpha(y_0,y_1) = \alpha\left(\left(\frac{1}{2},1\right),\left(\frac{1}{2},\frac{1}{4}\right)\right) = 1.$$

If $x,x',y,y' \in \{(\frac{1}{2},a) : 0 \leq a \leq 1\}^2$, then we have

$$\alpha(x,y)H\big(T(x,x'),T(y,y')\big) = \frac{|x-y|}{2} = \frac{1}{2}d(x,y) = \psi\big(d(x,y)\big),$$

for otherwise

$$\alpha(x,y)H\big(T(x,x'),T(y,y')\big) \leq \psi\big(d(x,y)\big).$$

Hence, T is an α-ψ-proximal contraction. Moreover, if $\{x_n\}$ is a sequence in A such that $\alpha(x_n,x_{n+1}) = 1$ for all n and $x_n \to x \in A$ as $n \to \infty$, then there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $\alpha(x_{n_k},x) = 1$ for all k. Therefore, all the conditions of Theorem 2.6 hold and T has the coupled best proximity point.

Example 2.9 Let $X = [0,\infty) \times [0,\infty)$ be endowed with the usual metric d. Let $a > 1$ be any fixed real number, $A = \{(a,x) : 0 \leq x < \infty\}$ and $B = \{(0,x) : 0 \leq x < \infty\}$. Define $T : A \times A \to \mathrm{CL}(B)$ by

$$T\big((a,x),(a,y)\big) = \big\{\big(0,b^2\big) : 0 \leq b \leq \max\{x,y\}\big\}, \tag{48}$$

and $\alpha : A \times A \to [0,\infty)$ by

$$\alpha\big((a,x),(a,y)\big) = \begin{cases} 1 & \text{if } x = y = 0, \\ \frac{1}{a(x+y)} & \text{otherwise.} \end{cases} \tag{49}$$

Let $\psi(t) = \frac{t}{a}$ for all $t \geq 0$. Note that $A_0 = A$, $B_0 = B$ and $T(x,y) \in B_0$ for each $x, y \in A_0$. If $w_1 = (a,y_1), w_1' = (a,y_1'), w_2 = (a,y_2), w_2' = (a,y_2') \in A$ with either $y_1 \neq 0$ or $y_2 \neq 0$ or both are nonzero, we have

$$\begin{aligned} \alpha(w_1,w_2)H\big(T\big(w_1,w_1'\big),T\big(w_2,w_2'\big)\big) &= \frac{1}{a(y_1+y_2)}\big|y_1^2 - y_2^2\big| \\ &= \frac{1}{a}|y_1 - y_2| \\ &= \psi\big(d(w_1,w_2)\big) \end{aligned}$$

for otherwise

$$\alpha(w_1,w_2)H\big(T\big(w_1,w_1'\big),T\big(w_2,w_2'\big)\big) = 0 = \psi\big(d(w_1,w_2)\big).$$

For $x_0 = (a, \frac{1}{2a}), y_0 = (a, \frac{1}{3a}) \in A_0$ and $u_1 = (0, \frac{1}{4a^2}) \in T(x_0,y_0)$ such that $d(x_1,u_1) = a = \operatorname{dist}(A,B)$ and $\alpha(x_0,x_1) = \frac{4a}{1+2a} > 1$. And for $x_1 = (a, \frac{1}{3a}), y_1 = (a, \frac{1}{9a^2}) \in A_0$ and $v_1 = (0, \frac{1}{9a^2}) \in T(x_1,y_1)$ such that $d(y_1,v_1) = a = \operatorname{dist}(A,B)$ and $\alpha(y_0,y_1) = \frac{9a}{1+3a} > 1$. Furthermore, one can see that the remaining conditions of Theorem 2.4 also hold. Therefore, T has the coupled best proximity point.

Competing interests
The author declares that he has no competing interests.

Acknowledgements
The author is grateful to Lampang Rajabhat University for financial support during the preparation of this manuscript and to the referees for useful suggestions.

Received: 11 November 2014 Accepted: 4 February 2015 Published online: 24 February 2015

References

1. Arvanitakis, AD: A proof of the generalized Banach contraction conjecture. Proc. Am. Math. Soc. **131**(12), 3647-3656 (2003)
2. Boyd, DW, Wong, JSW: On nonlinear contractions. Proc. Am. Math. Soc. **20**, 458-464 (1969)
3. Choudhury, BS, Das, KP: A new contraction principle in Menger spaces. Acta Math. Sin. **24**(8), 1379-1386 (2008)
4. Mongkolkeha, C, Sintunavarat, W, Kumam, P: Fixed point theorems for contraction mappings in modular metric spaces. Fixed Point Theory Appl. **2011**, Article ID 93 (2011)
5. Sintunavarat, W, Kumam, P: Gregus type fixed points for a tangential multi-valued mappings satisfying contractive conditions of integral type. J. Inequal. Appl. **2011**, Article ID 3 (2011)
6. Suzuki, T: A generalized Banach contraction principle that characterizes metric completeness. Proc. Am. Math. Soc. **136**(5), 1861-1869 (2008)
7. Samet, B, Vetro, C, Vetro, P: Fixed point theorems for α-ψ-contractive type mappings. Nonlinear Anal., Theory Methods Appl. **75**(4), 2154-2165 (2012)
8. Karapınar, E, Samet, B: Generalized α-ψ contractive type mappings and related fixed point theorems with applications. Abstr. Appl. Anal. **2012**, Article ID 793486 (2012)
9. Asl, JH, Rezapour, S, Shahzad, N: On fixed points of α-ψ-contractive multifunctions. Fixed Point Theory Appl. **2012**, Article ID 212 (2012)
10. Mohammadi, B, Rezapour, S, Shahzad, N: Some results on fixed points of (α-ψ)-Ćirić generalized multifunctions. Fixed Point Theory Appl. **2013**, Article ID 24 (2013)
11. Ali, MU, Kamran, T: On (α^*, ψ)-contractive multi-valued mappings. Fixed Point Theory Appl. **2013**, Article ID 137 (2013)

12. Amiri, P, Rezapour, S, Shahzad, N: Fixed points of generalized (α-ψ)-contractions. Rev. R. Acad. Cienc. Exactas Fís. Nat., Ser. A Mat. **108**(2), 519-526 (2014)
13. Minak, G, Altun, I: Some new generalizations of Mizoguchi-Takahashi type fixed point theorem. J. Inequal. Appl. **2013**, Article ID 493 (2013)
14. Ali, MU, Kamran, T, Sintunavarat, W, Katchang, P: Mizoguchi-Takahashi's fixed point theorem with α, η functions. Abstr. Appl. Anal. **2013**, Article ID 418798 (2013)
15. Chen, CM, Karapınar, E: Fixed point results for the α-Meir-Keeler contraction on partial Hausdorff metric spaces. J. Inequal. Appl. **2013**, Article ID 410 (2013)
16. Ali, MU, Kamran, T, Karapınar, E: (α, ψ, ξ)-Contractive multivalued mappings. Fixed Point Theory Appl. **2014**, Article ID 7 (2014)
17. Ali, MU, Kamran, T, Karapınar, E: A new approach to (α-ψ)-contractive nonself multivalued mappings. J. Inequal. Appl. **2014**, Article ID 71 (2014)
18. Ali, MU, Kiran, Q, Shahzad, N: Fixed point theorems for multi-valued mappings involving α-function. Abstr. Appl. Anal. **2014**, Article ID 409467 (2014)
19. Jleli, M, Samet, B: Best proximity points for (α-ψ)-proximal contractive type mappings and applications. Bull. Sci. Math. **137**(8), 977-995 (2013)
20. Abkar, A, Gabeleh, M: Best proximity points for asymptotic cyclic contraction mappings. Nonlinear Anal. **74**(18), 7261-7268 (2011)
21. Abkar, A, Gabeleh, M: Best proximity points for cyclic mappings in ordered metric spaces. J. Optim. Theory Appl. **151**(2), 418-424 (2011)
22. Abkar, A, Gabeleh, M: The existence of best proximity points for multivalued non-self mappings. Rev. R. Acad. Cienc. Exactas Fís. Nat., Ser. A Mat. **107**(2), 319-325 (2012)
23. Alghamdi, MA, Shahzad, N: Best proximity point results in geodesic metric spaces. Fixed Point Theory Appl. **2012**, Article ID 234 (2012)
24. Al-Thagafi, MA, Shahzad, N: Best proximity pairs and equilibrium pairs for Kakutani multimaps. Nonlinear Anal., Theory Methods Appl. **70**(3), 1209-1216 (2009)
25. Al-Thagafi, MA, Shahzad, N: Convergence and existence results for best proximity points. Nonlinear Anal., Theory Methods Appl. **70**(10), 3665-3671 (2009)
26. Al-Thagafi, MA, Shahzad, N: Best proximity sets and equilibrium pairs for a finite family of multimaps. Fixed Point Theory Appl. **2008**, Article ID 457069 (2008)
27. Derafshpour, M, Rezapour, S, Shahzad, N: Best proximity points of cyclic φ-contractions in ordered metric spaces. Topol. Methods Nonlinear Anal. **37**(1), 193-202 (2011)
28. Di Bari, C, Suzuki, T, Vetro, C: Best proximity points for cyclic Meir-Keeler contractions. Nonlinear Anal., Theory Methods Appl. **69**, 3790-3794 (2008)
29. Eldred, AA, Veeramani, P: Existence and convergence of best proximity points. J. Math. Anal. Appl. **323**, 1001-1006 (2006)
30. Markin, J, Shahzad, N: Best proximity points for relatively *u*-continuous mappings in Banach and hyperconvex spaces. Abstr. Appl. Anal. **2013**, Article ID 680186 (2013)
31. Rezapour, S, Derafshpour, M, Shahzad, N: Best proximity points of cyclic ϕ-contractions on reflexive Banach spaces. Fixed Point Theory Appl. **2010**, Article ID 946178 (2010)
32. Basha, SS, Shahzad, N, Jeyaraj, R: Best proximity point theorems for reckoning optimal approximate solutions. Fixed Point Theory Appl. **2012**, Article ID 202 (2012)
33. Vetro, C: Best proximity points: convergence and existence theorems for *p*-cyclic mappings. Nonlinear Anal., Theory Methods Appl. **73**, 2283-2291 (2010)
34. Zhang, J, Su, Y, Cheng, Q: A note on 'A best proximity point theorem for Geraghty-contractions'. Fixed Point Theory Appl. **2013**, Article ID 83 (2013)
35. Mongkolkeha, C, Kumam, P: Best proximity point theorems for generalized cyclic contractions in ordered metric spaces. J. Optim. Theory Appl. **155**, 215-226 (2012)
36. Sintunavarat, W, Kumam, P: Coupled best proximity point theorem in metric spaces. Fixed Point Theory Appl. **2012**, Article ID 93 (2012)
37. Nashine, HK, Vetro, C, Kumam, P: Best proximity point theorems for rational proximal contractions. Fixed Point Theory Appl. **2013**, Article ID 95 (2013)
38. Cho, YJ, Gupta, A, Karapınar, E, Kumam, P, Sintunavarat, W: Tripled best proximity point theorem in metric spaces. Math. Inequal. Appl. **16**, 1197-1216 (2013)
39. Mongkolkeha, C, Kongban, C, Kumam, P: Existence and uniqueness of best proximity points for generalized almost contractions. Abstr. Appl. Anal. **2014**, Article ID 813614 (2014)
40. Kumam, P, Salimi, P, Vetro, C: Best proximity point results for modified α-proximal *c*-contraction mappings. Fixed Point Theory Appl. **2014**, Article ID 99 (2014)
41. Pragadeeswarar, V, Marudai, M, Kumam, P, Sitthithakerngkiet, K: The existence and uniqueness of coupled best proximity point for proximally coupled contraction in a complete ordered metric space. Abstr. Appl. Anal. **2014**, Article ID 274062 (2014)
42. Ali, MU, Kamran, T, Shahzad, N: Best proximity point for α-ψ-proximal contractive multimap. Abstr. Appl. Anal. **2014**, Article ID 181598 (2014)