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Strong convergence for asymptotically nonexpansive mappings in the intermediate sense

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Abstract
In this paper, let C be a nonempty closed convex subset of a strictly convex Banach space. Then we prove strong convergence of the modified Ishikawa iteration process when T is an ANI self-mapping such that $T(C)$ is contained in a compact subset of C, which generalizes the result due to Takahashi and Kim (Math. Jpn. 48:1-9, 1998).

MSC: 47H05; 47H10

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1 Introduction

Let C be a nonempty closed convex subset of a Banach space E, and let T be a mapping of C into itself. Then T is said to be *asymptotically nonexpansive* [1] if there exists a sequence $\{k_n\}$, $k_n \geq 1$, with $\lim_{n\to\infty} k_n = 1$, such that

$$\left\| T^n x - T^n y \right\| \leq k_n \|x - y\|$$

for all $x, y \in C$ and $n \geq 1$. In particular, if $k_n = 1$ for all $n \geq 1$, T is said to be *nonexpansive*. T is said to be *uniformly L-Lipschitzian* if there exists a constant $L > 0$ such that

$$\left\| T^n x - T^n y \right\| \leq L \|x - y\|$$

for all $x, y \in C$ and $n \geq 1$. T is said to be *asymptotically nonexpansive in the intermediate sense* (in brief, ANI) [2] provided T is uniformly continuous and

$$\limsup_{n\to\infty} \sup_{x,y\in C} \left(\left\| T^n x - T^n y \right\| - \|x - y\| \right) \leq 0.$$

We denote by $F(T)$ the set of all fixed points of T, *i.e.*, $F(T) = \{x \in C : Tx = x\}$. We define the modulus of convexity for a convex subset of a Banach space; see also [3]. Let C be a nonempty bounded convex subset of a Banach space E with $d(C) > 0$, where $d(C)$ is the diameter of C. Then we define $\delta(C, \epsilon)$ with $0 \leq \epsilon \leq 1$ as follows:

$$\delta(C,\epsilon) = \frac{1}{r} \inf\left\{ \max\left(\|x - z\|, \|y - z\|\right) - \left\| z - \frac{x+y}{2} \right\| : x, y, z \in C, \|x - y\| \geq r\epsilon \right\},$$

where $r = d(C)$. When $\{x_n\}$ is a sequence in E, then $x_n \to x$ will denote strong convergence of the sequence $\{x_n\}$ to x. For a mappings T of C into itself, Rhoades [4] considered the following modified Ishikawa iteration process (*cf.* Ishikawa [5]) in C defined by $x_1 \in C$:

$$x_{n+1} = \alpha_n T^n y_n + (1-\alpha_n)x_n, \qquad y_n = \beta_n T^n x_n + (1-\beta_n)x_n, \tag{1.1}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are two real sequences in $[0,1]$. If $\beta_n = 0$ for all $n \ge 1$, then the iteration process (1.1) reduces to the modified Mann iteration process [6] (*cf.* Mann [7]).

Takahashi and Kim [8] proved the following result: Let E be a strictly convex Banach space and C be a nonempty closed convex subset of E and $T : C \to C$ be a nonexpansive mapping such that $T(C)$ is contained in a compact subset of C. Suppose $x_1 \in C$, and the sequence $\{x_n\}$ is defined by $x_{n+1} = \alpha_n T[\beta_n T x_n + (1-\beta_n)x_n] + (1-\alpha_n)x_n$, where $\{\alpha_n\}$ and $\{\beta_n\}$ are chosen so that $\alpha_n \in [a,b]$ and $\beta_n \in [0,b]$ or $\alpha_n \in [a,1]$ and $\beta_n \in [a,b]$ for some a, b with $0 < a \le b < 1$. Then $\{x_n\}$ converges strongly to a fixed point of T. In 2000, Tsukiyama and Takahashi [9] generalized the result due to Takahashi and Kim [8] to a nonexpansive mapping under much less restrictions on the iterative parameters $\{\alpha_n\}$ and $\{\beta_n\}$.

In this paper, let C be a nonempty closed convex subset of a strictly convex Banach space. We prove that if $T : C \to C$ is an ANI mapping such that $T(C)$ is contained in a compact subset of C, then the iteration $\{x_n\}$ defined by (1.1) converges strongly to a fixed point of T, which generalizes the result due to Takahashi and Kim [8].

2 Strong convergence theorem

We first begin with the following lemma.

Lemma 2.1 [9] *Let C be a nonempty compact convex subset of a Banach space E with $r = d(C) > 0$. Let $x, y, z \in C$ and suppose $\|x-y\| \ge \epsilon r$ for some ϵ with $0 \le \epsilon \le 1$. Then, for all λ with $0 \le \lambda \le 1$,*

$$\bigl\|\lambda(x-z) + (1-\lambda)(y-z)\bigr\| \le \max\bigl(\|x-z\|, \|y-z\|\bigr) - 2\lambda(1-\lambda)r\delta(C,\epsilon).$$

Lemma 2.2 [9] *Let C be a nonempty compact convex subset of a strictly convex Banach space E with $r = d(C) > 0$. If $\lim_{n\to\infty} \delta(C,\epsilon_n) = 0$, then $\lim_{n\to\infty} \epsilon_n = 0$.*

Lemma 2.3 [10] *Let $\{a_n\}$ and $\{b_n\}$ be two sequences of nonnegative real numbers such that $\sum_{n=1}^{\infty} b_n < \infty$ and*

$$a_{n+1} \le a_n + b_n$$

for all $n \ge 1$. Then $\lim_{n\to\infty} a_n$ exists.

Lemma 2.4 *Let C be a nonempty compact convex subset of a Banach space E, and let $T : C \to C$ be an ANI mapping. Put*

$$c_n = \sup_{x,y\in C} \bigl(\bigl\|T^n x - T^n y\bigr\| - \|x-y\|\bigr) \vee 0,$$

so that $\sum_{n=1}^{\infty} c_n < \infty$. Suppose that the sequence $\{x_n\}$ is defined by (1.1). Then $\lim_{n\to\infty} \|x_n - z\|$ exists for any $z \in F(T)$.

Proof The existence of a fixed point of T follows from Schauder's fixed theorem [11]. For a fixed $z \in F(T)$, since

$$\begin{aligned}
\|T^n y_n - z\| &\le \|y_n - z\| + c_n \\
&= \|\beta_n T^n x_n + (1-\beta_n)x_n - z\| + c_n \\
&\le \beta_n \|T^n x_n - z\| + (1-\beta_n)\|x_n - z\| + c_n \\
&\le \beta_n \|x_n - z\| + c_n + (1-\beta_n)\|x_n - z\| + c_n \\
&\le \|x_n - z\| + 2c_n,
\end{aligned}$$

we obtain

$$\begin{aligned}
\|x_{n+1} - z\| &= \|\alpha_n T^n y_n + (1-\alpha_n)x_n - z\| \\
&\le \alpha_n \|T^n y_n - z\| + (1-\alpha_n)\|x_n - z\| \\
&\le \alpha_n \bigl(\|x_n - z\| + 2c_n\bigr) + (1-\alpha_n)\|x_n - z\| \\
&\le \|x_n - z\| + 2c_n.
\end{aligned}$$

By Lemma 2.3, we readily see that $\lim_{n\to\infty} \|x_n - z\|$ exists. □

Theorem 2.5 *Let C be a nonempty compact convex subset of a strictly convex Banach space E with $r = d(C) > 0$. Let $T : C \to C$ be an ANI mapping. Put*

$$c_n = \sup_{x,y\in C} \bigl(\|T^n x - T^n y\| - \|x - y\|\bigr) \vee 0,$$

so that $\sum_{n=1}^{\infty} c_n < \infty$. Suppose $x_1 \in C$, and the sequence $\{x_n\}$ defined by (1.1) *satisfies $\alpha_n \in [a,b]$ and $\limsup_{n\to\infty} \beta_n = b < 1$ or $\liminf_{n\to\infty} \alpha_n > 0$ and $\beta_n \in [a,b]$ for some a, b with $0 < a \le b < 1$. Then $\lim_{n\to\infty} \|x_n - Tx_n\| = 0$.*

Proof The existence of a fixed point of T follows from Schauder's fixed theorem [11]. For any fixed $z \in F(T)$, we first show that if $\alpha_n \in [a,b]$ and $\limsup_{n\to\infty} \beta_n = b < 1$ for some $a, b \in (0,1)$, then we obtain $\lim_{n\to\infty} \|Tx_n - x_n\| = 0$. In fact, let $\epsilon_n = \frac{\|T^n y_n - x_n\|}{r}$. Then we have $0 \le \epsilon_n \le 1$ since $\|T^n y_n - x_n\| \le r$. As in the proof of Lemma 2.4, we obtain

$$\|T^n y_n - z\| \le \|x_n - z\| + 2c_n. \tag{2.1}$$

Since

$$\|T^n y_n - x_n\| = r\epsilon_n,$$

and by (2.1) and Lemma 2.1, we have

$$\begin{aligned}
\|x_{n+1} - z\| &= \|\alpha_n\bigl(T^n y_n - z\bigr) + (1-\alpha_n)(x_n - z)\| \\
&\le \|x_n - z\| + 2c_n - 2\alpha_n(1-\alpha_n) r\delta(C,\epsilon_n).
\end{aligned}$$

Thus

$$2\alpha_n(1-\alpha_n)r\delta(C,\epsilon_n) \le \|x_n - z\| - \|x_{n+1} - z\| + 2c_n.$$

Since

$$2r\sum_{n=1}^{\infty} a(1-b)\delta\left(C, \frac{\|T^n y_n - x_n\|}{r}\right) < \infty,$$

we obtain

$$\lim_{n\to\infty} \delta\left(C, \frac{\|T^n y_n - x_n\|}{r}\right) = 0.$$

By using Lemma 2.2, we obtain

$$\lim_{n\to\infty} \left\| T^n y_n - x_n \right\| = 0. \tag{2.2}$$

Since

$$\begin{aligned} \left\| T^n x_n - x_n \right\| &\le \left\| T^n x_n - T^n y_n \right\| + \left\| T^n y_n - x_n \right\| \\ &\le \|x_n - y_n\| + c_n + \left\| T^n y_n - x_n \right\| \\ &= \beta_n \left\| T^n x_n - x_n \right\| + c_n + \left\| T^n y_n - x_n \right\|, \end{aligned}$$

we obtain

$$(1-\beta_n)\left\| T^n x_n - x_n \right\| \le c_n + \left\| T^n y_n - x_n \right\|. \tag{2.3}$$

Since $\limsup_{n\to\infty} \beta_n = b < 1$, we have

$$\liminf_{n\to\infty}(1-\beta_n) = 1 - b > 0. \tag{2.4}$$

From (2.2), (2.3) and (2.4), we obtain

$$\lim_{n\to\infty} \left\| T^n x_n - x_n \right\| = 0. \tag{2.5}$$

Since

$$\begin{aligned} \|x_{n+1} - x_n\| &= \left\| (1-\alpha_n)x_n + \alpha_n T^n y_n - x_n \right\| \\ &= \alpha_n \left\| T^n y_n - x_n \right\| \\ &\le b \left\| T^n y_n - x_n \right\|, \end{aligned}$$

and by (2.2), we obtain

$$\lim_{n\to\infty} \|x_{n+1} - x_n\| = 0. \tag{2.6}$$

Since

$$\begin{aligned}
&\|x_n - Tx_n\| \\
&\quad \le \|x_n - x_{n+1}\| + \|x_{n+1} - T^{n+1}x_{n+1}\| + \|T^{n+1}x_{n+1} - T^{n+1}x_n\| + \|T^{n+1}x_n - Tx_n\| \\
&\quad \le 2\|x_n - x_{n+1}\| + c_{n+1} + \|x_{n+1} - T^{n+1}x_{n+1}\| + \|T^{n+1}x_n - Tx_n\|
\end{aligned}$$

and by the uniform continuity of T, (2.5) and (2.6), we have

$$\lim_{n\to\infty} \|x_n - Tx_n\| = 0. \tag{2.7}$$

Next, we show that if $\liminf_{n\to\infty} \alpha_n > 0$ and $\beta_n \in [a, b]$, then we also obtain (2.7). In fact, let $\epsilon_n = \frac{\|T^n x_n - x_n\|}{r}$. Then we have $0 \le \epsilon_n \le 1$. From $\liminf_{n\to\infty} \alpha_n > 0$, there are some positive integer n_0 and a positive number a such that $\alpha_n > a > 0$ for all $n \ge n_0$. Since

$$\begin{aligned}
\|x_{n+1} - z\| &= \|\alpha_n(T^n y_n - z) + (1-\alpha_n)(x_n - z)\| \\
&\le \alpha_n\|T^n y_n - z\| + (1-\alpha_n)\|x_n - z\| \\
&\le \alpha_n\|y_n - z\| + \alpha_n c_n + (1-\alpha_n)\|x_n - z\|,
\end{aligned}$$

and hence

$$\frac{\|x_{n+1} - z\| - \|x_n - z\|}{\alpha_n} \le \|y_n - z\| - \|x_n - z\| + c_n.$$

So, we obtain

$$\begin{aligned}
\|x_n - z\| - \|y_n - z\| &\le \frac{\|x_n - z\| - \|x_{n+1} - z\|}{\alpha_n} + c_n \\
&\le \frac{\|x_n - z\| - \|x_{n+1} - z\|}{a} + c_n.
\end{aligned} \tag{2.8}$$

Since

$$\|T^n x_n - z\| \le \|x_n - z\| + c_n,$$

from Lemma 2.1, we obtain

$$\begin{aligned}
\|y_n - z\| &= \|\beta_n T^n x_n + (1-\beta_n)x_n - z\| \\
&= \|\beta_n(T^n x_n - z) + (1-\beta_n)(x_n - z)\| \\
&\le \|x_n - z\| + c_n - 2\beta_n(1-\beta_n)r\delta(C, \epsilon_n).
\end{aligned} \tag{2.9}$$

By using (2.8) and (2.9), we obtain

$$\begin{aligned}
2\beta_n(1-\beta_n)r\delta(C, \epsilon_n) &\le \|x_n - z\| - \|y_n - z\| + c_n \\
&\le \frac{\|x_n - z\| - \|x_{n+1} - z\|}{a} + 2c_n.
\end{aligned}$$

Hence

$$2r\sum_{n=1}^{\infty} a(1-b)\delta\left(C, \frac{\|T^n x_n - x_n\|}{r}\right) < \infty.$$

We also obtain

$$\lim_{n\to\infty} \|x_n - T^n x_n\| = 0 \tag{2.10}$$

similarly to the argument above. Since

$$\begin{aligned}\|y_n - x_n\| &= \|\beta_n T^n x_n + (1-\beta_n)x_n - x_n\| \\ &\le \beta_n \|T^n x_n - x_n\| \\ &\le b\|T^n x_n - x_n\|,\end{aligned}$$

and by using (2.10), we obtain

$$\lim_{n\to\infty} \|x_n - y_n\| = 0. \tag{2.11}$$

Since

$$\begin{aligned}\|T^n y_n - x_n\| &\le \|T^n y_n - T^n x_n\| + \|T^n x_n - x_n\| \\ &\le \|y_n - x_n\| + c_n + \|T^n x_n - x_n\|,\end{aligned}$$

by using (2.10) and (2.11), we obtain

$$\lim_{n\to\infty} \|T^n y_n - x_n\| = 0. \tag{2.12}$$

Since

$$\|T^n y_n - y_n\| \le \|T^n y_n - x_n\| + \|x_n - y_n\|,$$

by using (2.11) and (2.12), we obtain

$$\lim_{n\to\infty} \|T^n y_n - y_n\| = 0. \tag{2.13}$$

Since

$$\begin{aligned}\|x_n - x_{n-1}\| &= \|(1-\alpha_{n-1})x_{n-1} + \alpha_{n-1}T^{n-1}y_{n-1} - x_{n-1}\| \\ &= \alpha_{n-1}\|T^{n-1}y_{n-1} - x_{n-1}\| \\ &\le \|T^{n-1}y_{n-1} - y_{n-1}\| + \|y_{n-1} - x_{n-1}\|,\end{aligned}$$

by (2.11) and (2.13), we get

$$\lim_{n\to\infty} \|x_n - x_{n-1}\| = 0. \tag{2.14}$$

From

$$\begin{aligned}&\left\|T^{n-1}x_n - x_n\right\|\\&\quad\le \left\|T^{n-1}x_n - T^{n-1}x_{n-1}\right\| + \left\|T^{n-1}x_{n-1} - x_{n-1}\right\| + \|x_{n-1} - x_n\|\\&\quad\le 2\|x_n - x_{n-1}\| + c_{n-1} + \left\|T^{n-1}x_{n-1} - x_{n-1}\right\|\end{aligned}$$

and by (2.10) and (2.14), we obtain

$$\lim_{n\to\infty}\left\|T^{n-1}x_n - x_n\right\| = 0. \tag{2.15}$$

Since

$$\begin{aligned}&\left\|x_n - Tx_n\right\|\\&\quad\le \|x_n - y_n\| + \left\|y_n - T^n y_n\right\| + \left\|T^n y_n - T^n x_n\right\| + \left\|T^n x_n - Tx_n\right\|\\&\quad\le \left\|y_n - T^n y_n\right\| + 2\|x_n - y_n\| + c_n + \left\|T^n x_n - Tx_n\right\|\end{aligned}$$

and by the uniform continuity of T, (2.11), (2.13) and (2.15), we have

$$\lim_{n\to\infty}\|x_n - Tx_n\| = 0. \qquad \square$$

Our Theorem 2.6 carries over Theorem 3 of Takahashi and Kim [8] to an ANI mapping.

Theorem 2.6 *Let C be a nonempty closed convex subset of a strictly convex Banach space E, and let $T : C \to C$ be an ANI mapping, and let $T(C)$ be contained in a compact subset of C. Put*

$$c_n = \sup_{x,y\in C}\left(\left\|T^n x - T^n y\right\| - \|x - y\|\right) \vee 0,$$

so that $\sum_{n=1}^{\infty} c_n < \infty$. Suppose $x_1 \in C$, and the sequence $\{x_n\}$ defined by (1.1) *satisfies $\alpha_n \in [a,b]$ and $\limsup_{n\to\infty}\beta_n = b < 1$ or $\liminf_{n\to\infty}\alpha_n > 0$ and $\beta_n \in [a,b]$ for some a, b with $0 < a \le b < 1$. Then $\{x_n\}$ converges strongly to a fixed point of T.*

Proof By Mazur's theorem [12], $A := \overline{co}(\{x_1\} \cup T(C))$ is a compact subset of C containing $\{x_n\}$ which is invariant under T. So, without loss of generality, we may assume that C is compact and $\{x_n\}$ is well defined. The existence of a fixed point of T follows from Schauder's fixed theorem [11]. If $d(C) = 0$, then the conclusion is obvious. So, we assume $d(C) > 0$. From Theorem 2.5, we obtain

$$\lim_{n\to\infty}\|x_n - Tx_n\| = 0. \tag{2.16}$$

Since C is compact, there exist a subsequence $\{x_{n_k}\}$ of the sequence $\{x_n\}$ and a point $p \in C$ such that $x_{n_k} \to p$. Thus we obtain $p \in F(T)$ by the continuity of T and (2.16). Hence we obtain $\lim_{n\to\infty}\|x_n - p\| = 0$ by Lemma 2.4. $\square$

Corollary 2.7 *Let C be a nonempty closed convex subset of a strictly convex Banach space E, and let $T: C \to C$ be an asymptotically nonexpansive mapping with $\{k_n\}$ satisfying $k_n \geq 1$, $\sum_{n=1}^{\infty}(k_n - 1) < \infty$, and let $T(C)$ be contained in a compact subset of C. Suppose $x_1 \in C$, and the sequence $\{x_n\}$ defined by* (1.1) *satisfies $\alpha_n \in [a,b]$ and $\limsup_{n\to\infty} \beta_n = b < 1$ or $\liminf_{n\to\infty} \alpha_n > 0$ and $\beta_n \in [a,b]$ for some a, b with $0 < a \leq b < 1$. Then $\{x_n\}$ converges strongly to a fixed point of T.*

Proof Note that

$$\sum_{n=1}^{\infty} c_n = \sum_{n=1}^{\infty}(k_n - 1)\operatorname{diam}(C) < \infty,$$

where $\operatorname{diam}(C) = \sup_{x,y\in C}\|x - y\| < \infty$. The conclusion now follows easily from Theorem 2.6. □

We give an example which satisfies all assumptions of T in Theorem 2.6, *i.e.*, $T: C \to C$ is an ANI mapping which is not Lipschitzian and hence not asymptotically nonexpansive.

Example 2.8 Let $E := \mathbb{R}$ and $C := [0,2]$. Define $T: C \to C$ by

$$Tx = \begin{cases} 1, & x \in [0,1]; \\ \sqrt{2-x}, & x \in [1,2]. \end{cases}$$

Note that $T^n x = 1$ for all $x \in C$ and $n \geq 2$ and $F(T) = \{1\}$. Clearly, T is uniformly continuous, ANI on C, but T is not Lipschitzian. Indeed, suppose not, *i.e.*, there exists $L > 0$ such that

$$|Tx - Ty| \leq L|x - y|$$

for all $x, y \in C$. If we take $y := 2$ and $x := 2 - \frac{1}{(L+1)^2} > 1$, then

$$\sqrt{2-x} \leq L(2-x) \quad \Leftrightarrow \quad \frac{1}{L^2} \leq 2 - x = \frac{1}{(L+1)^2} \quad \Leftrightarrow \quad L + 1 \leq L.$$

This is a contradiction.

We also give an example of an ANI mapping which is not a Lipschitz function.

Example 2.9 Let $E = \mathbb{R}$ and $C = [-3\pi, 3\pi]$ and let $|h| < 1$. Let $T: C \to C$ be defined by

$$Tx = hx \sin nx$$

for each $x \in C$ and for all $n \in \mathbb{N}$, where $\mathbb{N}$ denotes the set of all positive integers. Clearly $F(T) = \{0\}$. Since

$$T(x) = hx \sin nx,$$

$$T^2 x = h^2 x \sin nx \sin nhx \sin n(\sin nx) \cdots,$$

we obtain $\{T^n x\} \to 0$ uniformly on C as $n \to \infty$. Thus

$$\limsup_{n\to\infty}\left\{\left\|T^n x - T^n y\right\| - \|x-y\| \vee 0\right\} = 0$$

for all $x, y \in C$. Hence T is an ANI mapping, but it is not a Lipschitz function. In fact, suppose that there exists $h > 0$ such that $|Tx - Ty| \leq h|x-y|$ for all $x, y \in C$. If we take $x = \frac{5\pi}{2n}$ and $y = \frac{3\pi}{2n}$, then

$$|Tx - Ty| = \left|h\frac{5\pi}{2n}\sin n\frac{5\pi}{2n} - h\frac{3\pi}{2n}\sin n\frac{3\pi}{2n}\right| = \frac{4h\pi}{n},$$

whereas

$$h|x-y| = h\left|\frac{5\pi}{2n} - \frac{3\pi}{2n}\right| = \frac{h\pi}{n}.$$

Competing interests
The author declares that they have no competing interests.

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References

1. Goebel, K, Kirk, WA: A fixed point theorem for asymptotically nonexpansive mappings. Proc. Am. Math. Soc. **35**, 171-174 (1972)
2. Bruck, RE, Kuczumow, T, Reich, S: Convergence of iterates of asymptotically nonexpansive mappings in Banach spaces with the uniform Opial property. Colloq. Math. **65**, 169-179 (1993)
3. Ishihara, H, Takahashi, W: Modulus of convexity, characteristic of convexity and fixed point theorems. Kodai Math. J. **10**, 197-208 (1987)
4. Rhoades, BE: Fixed point iterations for certain nonlinear mappings. J. Math. Anal. Appl. **183**, 118-120 (1994)
5. Ishikawa, S: Fixed points by a new iteration method. Proc. Am. Math. Soc. **44**, 147-150 (1974)
6. Schu, J: Iterative contraction of fixed points of asymptotically nonexpansive mappings. J. Math. Anal. Appl. **158**, 407-413 (1991)
7. Mann, WR: Mean value methods in iteration. Proc. Am. Math. Soc. **4**, 506-510 (1953)
8. Takahashi, W, Kim, GE: Approximating fixed points of nonexpansive mappings in Banach spaces. Math. Jpn. **48**, 1-9 (1998)
9. Tsukiyama, N, Takahashi, W: Approximating fixed points of nonexpansive mappings with compact domains. Commun. Appl. Nonlinear Anal. **7**(4), 39-47 (2000)
10. Tan, KK, Xu, HK: Approximating fixed points of nonexpansive mappings by the Ishikawa Iteration process. J. Math. Anal. Appl. **178**, 301-308 (1993)
11. Schauder, J: Der Fixpunktsatz in Funktionalräumen. Stud. Math. **2**, 171-180 (1930)
12. Dunford, N, Schwartz, JT: Linear Operators, Part I. Interscience, New York (1958)

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