Kutbi and Sintunavarat *Fixed Point Theory and Applications* 2014, 2014:139
http://www.fixedpointtheoryandapplications.com/content/2014/1/139

Fixed Point Theory and Applications
a SpringerOpen Journal

RESEARCH Open Access

Fixed point theorems for generalized w_α-contraction multivalued mappings in α-complete metric spaces

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Abstract

In this paper, using the concept of a w-distance on a metric space, we first prove the existence of a fixed point theorem for generalized w_α-contraction multivalued mappings without completeness in metric spaces. Our presented results generalize, extend, and improve the result of Kutbi and Sintunavarat (Abstr. Appl. Anal. 2013:165434, 2013) and various well-known results on the topic in the literature. Also, we give some examples to which the results of Kutbi and Sintunavarat (Abstr. Appl. Anal. 2013:165434, 2013) are not applied, but our results are.

MSC: 47H10; 54H25

Keywords: α-admissible mappings; α-complete metric spaces; w-distances; fixed points

1 Introduction

In 1996, Kada *et al.* [1] introduced the concept of w-distance on a metric space, which is a real generalization of a metric. By using this concept, they extended and improved Caristi's fixed point theorem, Ekland's variational principle, and Takahashi's existence theorem from the metric version to a w-distance version. Later, Suzuki and Takahashi [2] using the concept of w-distance to established the fixed point result for multivalued mapping. This result is an improvement of the famous Nadler fixed point theorem.

In 2013, Kutbi [3] improved a useful lemma given in [4] for the w-distance version and established the fixed point results via this lemma. Recently, Kutbi and Sintunavarat [5] introduced the notion of generalized w_α-contraction mapping and proved a fixed point theorem for such a mapping in complete metric spaces via the concept of α-admissible mapping due to Mohammadi *et al.* [6]. On the other hand, Hussain *et al.* [7] introduced the concepts of α-complete metric spaces and also established fixed point results in such spaces.

The purpose of this work is to weaken the condition of completeness of the metric space in the result of Kutbi and Sintunavarat [5] by using the concept of α-completeness of the metric space. We also give the example of a nonlinear contraction mapping which is not applied by the results of Kutbi and Sintunavarat [5], but can be applied to our results. The presented results extend and complement recent results of Kutbi and Sintunavarat [5] and many known existence results from the literature.

2 Preliminaries

Throughout this paper, we denote by $\mathbb{N}$ and $\mathbb{R}$ the sets of positive integers and real numbers, respectively.

For a metric space (X,d), we denote by 2^X, $Cl(X)$, and $CB(X)$ the collection of nonempty subsets of X, nonempty closed subsets of X and nonempty closed bounded subsets of X, respectively.

For $A,B \in CB(X)$, we define the Hausdorff distance with respect to d by

$$H(A,B) = \max\Big\{\sup_{x\in A} d(x,B), \sup_{y\in B} d(y,A)\Big\},$$

where $d(x,B) := \inf_{y\in B} d(x,y)$. It is well known that $(CB(X),H)$ is a metric space and $(CB(X),H)$ is complete if (X,d) is complete.

Definition 2.1 Let (X,d) be a metric space and $T : X \to 2^X$ be a multivalued mapping. A point $x \in X$ is called a fixed point of T if $x \in T(x)$ and the set of fixed points of T is denoted by $\mathcal{F}(T)$.

Definition 2.2 ([8]) Let (X,d) be a metric space and let $T : X \to CB(X)$ be a multivalued mapping. T is said to be a *contraction* if there exists a constant $\lambda \in (0,1)$ such that for each $x,y \in X$,

$$H\big(T(x), T(y)\big) \leq \lambda d(x,y).$$

Definition 2.3 ([1]) Let (X,d) be a metric space. A function $\omega : X \times X \to [0,\infty)$ is called a w-distance on X if it satisfies the following conditions for each $x,y,z \in X$:

(w_1) $\omega(x,z) \leq \omega(x,y) + \omega(y,z)$;
(w_2) a mapping $\omega(x,\cdot) : X \to [0,\infty)$ is lower semicontinuous;
(w_3) for any $\epsilon > 0$, there exists $\delta > 0$ such that $\omega(z,x) \leq \delta$ and $\omega(z,y) \leq \delta$ imply $d(x,y) \leq \epsilon$.

For a metric space (X,d), it is easy to see that the metric d is a w-distance on X. But the converse is not true in the general case (see Examples 2.4 and 2.5). Therefore, the w-distance is a real generalization of the metric.

Example 2.4 Let (X,d) be a metric space. For a fixed positive real number c, define a function $\omega : X \times X \to [0,\infty)$ by $\omega(x,y) = c$ for all $x,y \in X$. Then ω is a w-distance on X.

Example 2.5 Let $(X,\|\cdot\|)$ be a normed linear space.

1. A function $\omega : X \times X \to [0,\infty)$ defined by $\omega(x,y) = \|x\| + \|y\|$ for all $x,y \in X$ is a w-distance on X.
2. A function $\omega : X \times X \to [0,\infty)$ defined by $\omega(x,y) = \|y\|$ for all $x,y \in X$ is a w-distance on X.

Remark 2.6 From Example 2.5, we obtain in general for $x,y \in X$, $\omega(x,y) \neq \omega(y,x)$ and neither of the implications $\omega(x,y) = 0 \Leftrightarrow x = y$ necessarily holds.

Definition 2.7 ([9]) Let (X,d) be a metric space. The w-distance $\omega : X \times X \to [0,\infty)$ on X is said to be a w_0-distance if $\omega(x,x) = 0$ for all $x \in X$.

For more details of other examples and properties of the w-distance, one can refer to [1, 2, 9]. The following lemmas are useful for the main results in this paper.

Lemma 2.8 ([1]) *Let (X,d) be a metric space and $\omega : X \times X \to [0,\infty)$ be a w-distance on X. Suppose that $\{x_n\}$ and $\{y_n\}$ are sequences in X and $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in $[0,\infty)$ converging to 0. Then the following hold for $x, y, z \in X$:*

1. *if $\omega(x_n, y) \leq \alpha_n$ and $\omega(x_n, z) \leq \beta_n$ for any $n \in \mathbb{N}$, then $y = z$; in particular, if $\omega(x, y) = 0$ and $\omega(x, z) = 0$, then $y = z$;*
2. *if $\omega(x_n, y_n) \leq \alpha_n$ and $\omega(x_n, z) \leq \beta_n$ for any $n \in \mathbb{N}$, then $\{y_n\}$ converges to z;*
3. *if $\omega(x_n, x_m) \leq \alpha_n$ for any $n, m \in \mathbb{N}$ with $m > n$, then $\{x_n\}$ is a Cauchy sequence;*
4. *if $\omega(y, x_n) \leq \alpha_n$ for any $n \in \mathbb{N}$, then $\{x_n\}$ is a Cauchy sequence.*

Next, we give the definition of some type of mapping. Before giving the next definition, we give the following notation. Let (X,d) be a metric space and $\omega : X \times X \to [0,\infty)$ be a w-distance on X. For $x \in X$ and $A \in 2^X$, we denote $\omega(x, A) := \inf_{y \in A} \omega(x, y)$.

Definition 2.9 ([2]) Let (X,d) be a metric space. The multivalued mapping $T : X \to Cl(X)$ is said to be a *w-contraction* if there exist a w-distance $\omega : X \times X \to [0,\infty)$ on X and $\lambda \in (0,1)$ such that for any $x, y \in X$ and $u \in T(x)$ there is $v \in T(y)$ with

$$\omega(u, v) \leq \lambda \omega(x, y).$$

Definition 2.10 ([5]) Let (X,d) be a metric space and $\alpha : X \times X \to [0,\infty)$ be a given mapping. The multivalued mapping $T : X \to Cl(X)$ is said to be a w_α-contraction if there exist a w-distance $\omega : X \times X \to [0,\infty)$ on X and $\lambda \in (0,1)$ such that for any $x, y \in X$ and $u \in T(x)$ there is $v \in T(y)$ with

$$\alpha(u, v)\omega(u, v) \leq \lambda \omega(x, y).$$

Definition 2.11 ([5]) Let (X,d) be a metric space and $\alpha : X \times X \to [0,\infty)$ be a given mapping. The multivalued mapping $T : X \to Cl(X)$ is said to be a generalized w_α-contraction if there exist a w_0-distance ω on X and $\lambda \in (0,1)$ such that for any $x, y \in X$ and $u \in T(x)$ there is $v \in T(y)$ with

$$\alpha(u, v)\omega(u, v) \leq \lambda \max\left\{\omega(x, y), \omega\big(x, T(x)\big), \omega\big(y, T(y)\big), \frac{1}{2}\big[\omega\big(x, T(y)\big) + \omega\big(y, T(x)\big)\big]\right\}.$$

Next, we give the concepts of an α-admissible multivalued mapping and α-completeness of metric spaces.

Definition 2.12 ([6]) Let X be a nonempty set, $T : X \to 2^X$ and $\alpha : X \times X \to [0,\infty)$ be a given mapping. We say that T is an *α-admissible* whenever, for each $x \in X$ and $y \in T(x)$ with $\alpha(x, y) \geq 1$, we have $\alpha(y, z) \geq 1$ for all $z \in T(y)$.

Remark 2.13 The concept of α-admissible multivalued mapping is extension of concept of α_*-admissible multivalued mapping due to Asl *et al.* [10].

Many fixed point results via the concepts of α-admissible mappings occupy a prominent place in many aspects (see [5, 11–17] and references therein).

Definition 2.14 ([7]) Let (X,d) be a metric space and $\alpha : X \times X \to [0,\infty)$ be a given mapping. The metric space X is said to be α-complete if and only if every Cauchy sequence $\{x_n\}$ in X with $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$, converges in X.

Example 2.15 Let $X = (0,\infty)$ and define metric $d : X \times X \to [0,\infty)$ by $d(x,y) = |x-y|$ for all $x, y \in X$. Let A be a closed subset of X. Define $\alpha : X \times X \to [0,\infty)$ by

$$\alpha(x,y) = \begin{cases} \frac{x^3+4x^2y+5xy^2+y^3}{(x+y)^3}, & x,y \in A, \\ \frac{|x-y|}{x+y}, & \text{otherwise.} \end{cases}$$

Clearly, (X,d) is not a complete metric space, but (X,d) is an α-complete metric space. Indeed, if $\{x_n\}$ is a Cauchy sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$, then $x_n \in A$ for all $n \in \mathbb{N}$. Now, since (A,d) is a complete metric space, there exists $x^* \in A$ such that $x_n \to x^*$ as $n \to \infty$.

3 Main results

In this section, we prove a fixed point theorem for generalized w_α-contraction multivalued mappings in α-complete metric space.

Theorem 3.1 *Let (X,d) be a metric space, $\alpha : X \times X \to [0,\infty)$ and $T : X \to Cl(X)$ be a generalized w_α-contraction multivalued mapping. Suppose that (X,d) is an α-complete metric space and the following conditions hold*:

(a) *T is an α-admissible mapping*;
(b) *there exist $x_0 \in X$ and $x_1 \in T(x_0)$ such that $\alpha(x_0, x_1) \geq 1$*;
(c) *if for every $y \in X$ with $y \notin T(y)$, we have*

$$\inf\{\omega(x,y) + \omega(x, T(x)) : x \in X\} > 0.$$

Then $\mathcal{F}(T) \neq \emptyset$.

Proof We start from $x_0 \in X$ and $x_1 \in T(x_0)$ in (b). From the definition of a generalized w_α-contraction of T, we can find $x_2 \in T(x_1)$ such that

$$\begin{aligned} \alpha(x_1,x_2)\omega(x_1,x_2) \leq \lambda \max\biggl\{ & \omega(x_0,x_1), \omega\bigl(x_0, T(x_0)\bigr), \omega\bigl(x_1, T(x_1)\bigr), \\ & \frac{1}{2}\bigl[\omega\bigl(x_0, T(x_1)\bigr) + \omega\bigl(x_1, T(x_0)\bigr)\bigr]\biggr\}. \end{aligned} \tag{3.1}$$

Since T is an α-admissible mapping and $x_1 \in T(x_0)$ such that $\alpha(x_0,x_1) \geq 1$, we have

$$\alpha(x_1,x_2) \geq 1. \tag{3.2}$$

From (3.1) and (3.2), we obtain

$$\omega(x_1,x_2) \leq \alpha(x_1,x_2)\omega(x_1,x_2) \leq \lambda \max\biggl\{\omega(x_0,x_1), \omega\bigl(x_0, T(x_0)\bigr), \omega\bigl(x_1, T(x_1)\bigr), \frac{1}{2}\bigl[\omega\bigl(x_0, T(x_1)\bigr) + \omega\bigl(x_1, T(x_0)\bigr)\bigr]\biggr\}.$$

Again, using the definition of a generalized w_α-contraction of T, there exists $x_3 \in T(x_2)$ such that

$$\alpha(x_2,x_3)\omega(x_2,x_3) \leq \lambda \max\biggl\{\omega(x_1,x_2), \omega\bigl(x_1, T(x_1)\bigr), \omega\bigl(x_2, T(x_2)\bigr), \frac{1}{2}\bigl[\omega\bigl(x_1, T(x_2)\bigr) + \omega\bigl(x_2, T(x_1)\bigr)\bigr]\biggr\}. \tag{3.3}$$

Since $\alpha(x_1,x_2) \geq 1$ and T is an α-admissible mapping, we get

$$\alpha(x_2,x_3) \geq 1. \tag{3.4}$$

From (3.3) and (3.4), we have

$$\omega(x_2,x_3) \leq \alpha(x_2,x_3)\omega(x_2,x_3) \leq \lambda \max\biggl\{\omega(x_1,x_2), \omega\bigl(x_1, T(x_1)\bigr), \omega\bigl(x_2, T(x_2)\bigr), \frac{1}{2}\bigl[\omega\bigl(x_1, T(x_2)\bigr) + \omega\bigl(x_2, T(x_1)\bigr)\bigr]\biggr\}.$$

Continuing this process, we can construct the sequence $\{x_n\}$ in X such that $x_n \in T(x_{n-1})$,

$$\alpha(x_n,x_{n+1}) \geq 1 \tag{3.5}$$

and

$$\omega(x_n,x_{n+1}) \leq \lambda \max\biggl\{\omega(x_{n-1},x_{n-2}), \omega\bigl(x_{n-1}, T(x_{n-1})\bigr), \omega\bigl(x_{n-2}, T(x_{n-2})\bigr), \frac{1}{2}\bigl[\omega\bigl(x_{n-1}, T(x_{n-2})\bigr) + \omega\bigl(x_{n-2}, T(x_{n-1})\bigr)\bigr]\biggr\} \tag{3.6}$$

for all $n \in \mathbb{N}$. Now, for each $n \in \mathbb{N}$, we have

$$\begin{aligned}
\omega(x_n,x_{n+1}) &\leq \lambda \max\biggl\{\omega(x_{n-1},x_n), \omega\bigl(x_{n-1}, T(x_{n-1})\bigr), \omega\bigl(x_n, T(x_n)\bigr), \\
&\qquad \frac{1}{2}\bigl[\omega\bigl(x_{n-1}, T(x_n)\bigr) + \omega\bigl(x_n, T(x_{n-1})\bigr)\bigr]\biggr\} \\
&\leq \lambda \max\biggl\{\omega(x_{n-1},x_n), \omega(x_{n-1},x_n), \omega(x_n,x_{n+1}), \frac{1}{2}\bigl[\omega(x_{n-1},x_{n+1}) + \omega(x_n,x_n)\bigr]\biggr\} \\
&= \lambda \max\biggl\{\omega(x_{n-1},x_n), \omega(x_n,x_{n+1}), \frac{1}{2}\bigl[\omega(x_{n-1},x_{n+1})\bigr]\biggr\}
\end{aligned}$$

$$\le \lambda \max\left\{\omega(x_{n-1},x_n),\omega(x_n,x_{n+1}),\frac{1}{2}\left[\omega(x_{n-1},x_n)+\omega(x_n,x_{n+1})\right]\right\}$$
$$\le \lambda \max\left\{\omega(x_{n-1},x_n),\omega(x_n,x_{n+1})\right\}. \tag{3.7}$$

If $\max\{\omega(x_{n'-1},x_{n'}),\omega(x_{n'},x_{n'+1})\} = \omega(x_{n'},x_{n'+1})$ for some $n' \in \mathbb{N}$, then we have $\omega(x_{n'},x_{n'+1}) = 0$ and hence $\omega(x_{n'-1},x_{n'}) = 0$. By the property of the w-distance, we get

$$\omega(x_{n'-1},x_{n'+1}) \le \omega(x_{n'-1},x_{n'}) + \omega(x_{n'},x_{n'+1}) = 0.$$

We find from Lemma 2.8, $\omega(x_{n'-1},x_{n'}) = 0$, and $\omega(x_{n'-1},x_{n'+1}) = 0$ that $x_{n'} = x_{n'+1}$. This implies that $x_{n'} \in T(x_{n'})$ and so $x_{n'}$ is a fixed point of T.

Next, we assume that $\max\{\omega(x_{n-1},x_n),\omega(x_n,x_{n+1})\} = \omega(x_{n-1},x_n)$ for all $n \in \mathbb{N}$. We obtain from (3.7)

$$\omega(x_n,x_{n+1}) \le \lambda\omega(x_{n-1},x_n) \tag{3.8}$$

for all $n \in \mathbb{N}$.

By repeating (3.8), we get

$$\omega(x_n,x_{n+1}) \le \lambda^n\omega(x_0,x_1)$$

for all $n \in \mathbb{N}$.

For $m,n \in \mathbb{N}$ with $m > n$, we obtain

$$\begin{aligned}\omega(x_n,x_m) &\le \omega(x_n,x_{n+1}) + \omega(x_{n+1},x_{n+2}) + \cdots + \omega(x_{m-1},x_m)\\ &\le \lambda^n\omega(x_0,x_1) + \lambda^{n+1}\omega(x_0,x_1) + \cdots + \lambda^{m-1}\omega(x_0,x_1)\\ &\le \frac{\lambda^n}{1-\lambda}\omega(x_0,x_1).\end{aligned}$$

Since $0 < \lambda < 1$, we get $\frac{\lambda^n}{1-\lambda}\omega(x_0,x_1) \to 0$ as $n \to \infty$. By Lemma 2.8, we find that $\{x_n\}$ is a Cauchy sequence in X. From (3.5) we know that $\alpha(x_n,x_{n+1}) \ge 1$ for all $n \in \mathbb{N}$. Using α-completeness of X, we obtain $x_n \to z$ as $n \to \infty$ for some $z \in X$. Since $\omega(x_n,\cdot)$ is lower semicontinuous, we have

$$\begin{aligned}\omega(x_n,z) &\le \liminf_{m\to\infty}\omega(x_n,x_m)\\ &\le \frac{\lambda^n}{1-\lambda}\omega(x_0,x_1).\end{aligned}$$

Finally, we will assume that $z \notin T(z)$. By hypothesis, we get

$$\begin{aligned}0 &< \inf\left\{\omega(x,z) + \omega\left(x,T(x)\right) : x \in X\right\}\\ &\le \inf\left\{\omega(x_n,z) + \omega\left(x_n,T(x_n)\right) : n \in \mathbb{N}\right\}\\ &\le \inf\left\{\omega(x_n,z) + \omega(x_n,x_{n+1}) : n \in \mathbb{N}\right\}\\ &\le \inf\left\{\frac{\lambda^n}{1-\lambda}\omega(x_0,x_1) + \lambda^n\omega(x_0,x_1) : n \in \mathbb{N}\right\}\end{aligned}$$

$$\begin{aligned}
&= \left(\left\{\frac{2-\lambda}{1-\lambda}\right\}\omega(x_0,x_1)\right)\inf\{\lambda^n : n \in \mathbb{N}\} \\
&= 0,
\end{aligned}$$

which is a contradiction. Consequently, $z \in T(z)$, that is, z is a fixed point of T as required. This completes the proof. □

Corollary 3.2 (Theorem 3.1 in [5]) *Let* (X,d) *be a complete metric space,* $\alpha : X \times X \to [0,\infty)$ *and* $T : X \to Cl(X)$ *be a generalized* w_α*-contraction mapping. Suppose that the following conditions hold*:

(a) T *is an* α*-admissible mapping*;
(b) *there exist* $x_0 \in X$ *and* $x_1 \in T(x_0)$ *such that* $\alpha(x_0,x_1) \geq 1$;
(c) *if for every* $y \in X$ *with* $y \notin T(y)$, *we have*

$$\inf\{\omega(x,y) + \omega(x,T(x)) : x \in X\} > 0.$$

Then $\mathcal{F}(T) \neq \emptyset$.

Proof We find that the completeness of the metric space (X,d) implies α-completeness. Therefore, by using Theorem 3.1, we obtain the desired result. □

Theorem 3.3 *Let* (X,d) *be a complete metric space,* $\alpha : X \times X \to [0,\infty)$ *and* $T : X \to Cl(X)$ *be a* w_α*-contraction mapping. Suppose that* (X,d) *is an* α*-complete metric space and the following conditions hold*:

(a) T *is* α*-admissible mapping*;
(b) *there exist* $x_0 \in X$ *and* $x_1 \in T(x_0)$ *such that* $\alpha(x_0,x_1) \geq 1$;
(c) *for every* $y \in X$ *with* $y \notin T(y)$, *we have*

$$\inf\{\omega(x,y) + \omega(x,T(x)) : x \in X\} > 0.$$

Then $\mathcal{F}(T) \neq \emptyset$.

Proof We see that this result can be proven by using a similar method to Theorem 3.1. In order to avoid repetition, the details are omitted. □

Example 3.4 Let $X = (-1,\infty)$ and define metric $d : X \times X \to [0,\infty)$ by $d(x,y) = |x-y|$ for all $x,y \in X$. Define $\alpha : X \times X \to [0,\infty)$ by

$$\alpha(x,y) = \begin{cases} x^2 + y^2 + 1, & x,y \in [0,1], \\ 0, & \text{otherwise.} \end{cases}$$

Let a multivalued mapping $T : X \to Cl(X)$ be defined by

$$T(x) = \begin{cases} \{\frac{x}{6}\}, & x \in [0,1], \\ \{x, 5|x|\}, & \text{otherwise.} \end{cases}$$

Now we show that T is a w_α-contraction multivalued mapping with $\lambda = \frac{1}{2}$ and w-distance $\omega : X \times X \to [0,\infty)$ defined by $\omega(x,y) = y$ for all $x,y \in X$. For $x,y \in [0,1]$, let $u \in T(x) = \{\frac{x}{6}\}$, that is, $u = \frac{x}{6}$, we can find $v = \frac{y}{6} \in T(y)$ such that

$$\begin{aligned}\alpha(u,v)\omega(u,v) &= \alpha\left(\frac{x}{6},\frac{y}{6}\right)\omega\left(\frac{x}{6},\frac{y}{6}\right)\\ &= \left(\frac{x^2}{36}+\frac{y^2}{36}+1\right)\frac{y}{6}\\ &\leq (1+1+1)\frac{y}{6}\\ &= \frac{1}{2}y\\ &= \lambda\omega(x,y).\end{aligned}$$

Otherwise, it is easy to see that the w_α-contractive condition holds. Therefore, T is a w_α-contraction multivalued mapping.

Clearly, (X,d) is not a complete metric space and then the main results of Kutbi and Sintunavarat [5] cannot be applied to this case.

Next, we show that our results in this paper can be used for this case. First, we claim that (X,d) is an α-complete metric space. Let $\{x_n\}$ be a Cauchy sequence in X such that $\alpha(x_n,x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$. So $x_n \in [0,1]$ for all $n \in \mathbb{N}$. Now, since $([0,1],d)$ is a complete metric space, there exists $x^* \in A$ such that $x_n \to x^*$ as $n \to \infty$. Consequently, (X,d) is an α-complete metric space. Also, it is easy to see that T is α-admissible and there exists $x_0 = 1$ such that $x_1 = 1/6 \in T(1)$ and $\alpha(x_0,x_1) = \alpha(1,1/6) \geq 1$. Finally, we see that for $y \in X$ with $y \notin T(y)$, we obtain $y \in (0,1]$ and hence $\inf\{\omega(x,y)+\omega(x,T(x)) : x \in X\} > 0$.

Therefore, all the conditions of Theorem 3.3 are satisfied and so T has a fixed point.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

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Acknowledgements
The first author gratefully acknowledges the support from the Deanship of Scientific Research (DSR) at King Abdulaziz University (KAU) during this research. The second author would like to thank the Thailand Research Fund and Thammasat University under Grant No. TRG5780013 for financial support during the preparation of this manuscript. Moreover, the authors thank the editors and referees for their insightful comments.

Received: 16 February 2014 Accepted: 27 May 2014 Published: 18 Jul 2014

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10.1186/1687-1812-2014-139
Cite this article as: Kutbi and Sintunavarat: **Fixed point theorems for generalized w_α-contraction multivalued mappings in α-complete metric spaces.** *Fixed Point Theory and Applications* 2014, 2014:139