Fixed Point Theory and Applications
a SpringerOpen Journal

RESEARCH **Open Access**

Some new theorems of expanding mappings without continuity in cone metric spaces

Yan Han[1] and Shaoyuan Xu[2*]

*Correspondence:
xushaoyuan@126.com
[2]School of Mathematics and Statistics, Hubei Normal University, Huangshi, 435002, P.R. China
Full list of author information is available at the end of the article

Abstract
In this paper, fixed point theorems for one mapping and common fixed point theorems for two mappings satisfying generalized expansive conditions are obtained. The mappings are not necessarily continuous and the cone is not normal. These results improve and generalize several well-known comparable results in (Aage and Salunke, in Acta Mathematica Sinica, English Series 27(6):1101-1106, 2011). Moreover, examples are given to support our new results.

MSC: 54H25; 47H10

Keywords: cone metric space; expanding mapping; common fixed point

1 Introduction and preliminaries

Recently, Huang and Zhang [1] introduced the concept of a cone metric space as a generalization of a metric space. They proved the properties of sequences in cone metric spaces and obtained various fixed point theorems for contractive mappings. Afterwords, Abbas and Jungck [2] established the common fixed points for two mappings without exploiting the notion of continuity. Since then, common fixed point theorems in cone metric spaces have been proved for mappings satisfying different contractive conditions by many authors (see [3–8]). But there are few results about expanding mappings. Chintaman and Jagannath [9] introduced several meaningful fixed point theorems for one expanding mapping. However, the mapping depended strongly on continuity. In this paper, we delete the continuity of the mappings and obtain some fixed point theorems for one expanding mapping and introduce common fixed point theorems for two expanding mappings, which satisfy generalized expansive conditions in nonnormal cone metric spaces. These results improve and generalize some important known results in [9, 10].

We recall some definitions of cone metric spaces and some of their properties [1]. Let E be a real Banach space and P be a subset of E. θ denotes the zero element of E and $\operatorname{int} P$ denotes the interior of P. The subset P is called a cone if and only if:

(i) P is closed, nonempty and $P \neq \{\theta\}$,
(ii) $a, b \in \mathbb{R}$, $a, b \geq 0$, $x, y \in P \Rightarrow ax + by \in P$,
(iii) $x \in P$ and $-x \in P \Rightarrow x = \theta$.

Given a cone $P \subset E$, we define a partial ordering $\leq$ with respect to P by $x \leq y$ if and only if $y - x \in P$. We will write $x < y$ if $x \leq y$ and $x \neq y$, while $x \ll y$ will stand for $y - x \in \operatorname{int} P$. A cone P is called normal if there is a number $K > 0$ such that for all $x, y \in P$,

$$\theta \leq x \leq y \quad \text{implies} \quad \|x\| \leq K\|y\|.$$

The least positive number satisfying the above inequality is called the normal constant of P.

Definition 1.1 ([1]) Let X be a nonempty set. Suppose that the mapping $d : X \times X \to E$ satisfies the following:

(d1) $\theta \leq d(x,y)$ for all $x,y \in X$ and $d(x,y) = \theta$ if and only if $x = y$;

(d2) $d(x,y) = d(y,x)$ for all $x,y \in X$;

(d3) $d(x,y) \leq d(x,z) + d(z,y)$ for all $x,y,z \in X$.

Then d is called a cone metric on X and (X,d) is called a cone metric space. It is clear that the cone metric space is more general than a metric space.

Definition 1.2 ([1]) Let (X,d) be a cone metric space. Then we say that $\{x_n\}$ is

(i) a Cauchy sequence if for every $c \in E$ with $c \gg \theta$, there is N such that for all $n,m > N$, $d(x_n,x_m) \ll c$;

(ii) a convergent sequence if for every $c \in E$ with $c \gg \theta$, there is N such that for all $m > N$, $d(x_m,x) \ll c$ for some fixed x in X.

A cone metric space X is said to be complete if every Cauchy sequence in X is convergent in X.

Lemma 1.3 ([11]) *The limit of a convergent sequence in a cone metric space is unique.*

2 Main results

In this section, we prove some fixed point theorems for expanding mappings without continuity in the following theorems.

Theorem 2.1 *Let (X,d) be a complete cone metric space. Suppose the mapping $f : X \to X$ is onto and satisfies*

$$d(fx,fy) \geq a_1 d(x,y) + a_2 d(x,fx) + a_3 d(y,fy) + a_4 d(x,fy) + a_5 d(y,fx) \tag{2.1}$$

for all $x,y \in X$, where a_i $(i = 1,2,3,4,5)$ satisfies $a_1 + a_2 + a_3 > 1$ and $a_3 \leq 1 + a_4$. Then f has a fixed point.

Proof Since f is an onto mapping, for each $x_0 \in X$, there exists $fx_1 = x_0$. Continuing this process, we can define $\{x_n\}$ by $x_n = fx_{n+1}$, $n = 0,1,2,\ldots$. Without loss of generality, we suppose that $x_{n-1} \neq x_n$ for all $n \geq 1$. According to (2.1), we have

$$\begin{aligned} d(x_n,x_{n-1}) &= d(fx_{n+1},fx_n) \\ &\geq a_1 d(x_{n+1},x_n) + a_2 d(x_{n+1},fx_{n+1}) + a_3 d(x_n,fx_n) \\ &\quad + a_4 d(x_{n+1},fx_n) + a_5 d(x_n,fx_{n+1}) \\ &= a_1 d(x_{n+1},x_n) + a_2 d(x_{n+1},x_n) + a_3 d(x_n,x_{n-1}) + a_4 d(x_{n+1},x_{n-1}) + a_5 d(x_n,x_n). \end{aligned}$$

By $d(x_{n+1},x_{n-1}) \geq d(x_{n+1},x_n) - d(x_{n-1},x_n)$, the above inequality implies that

$$d(x_{n+1},x_n) \leq \frac{1 - a_3 + a_4}{a_1 + a_2 + a_4} d(x_n,x_{n-1}).$$

Let $h = \frac{1-a_3+a_4}{a_1+a_2+a_4}$. By $a_3 \le 1 + a_4$ and $a_1 + a_2 + a_3 > 1$, we know $a_1 + a_2 + a_4 > 1 - a_3 + a_4 \ge 0$ and $h \in [0,1)$. Hence, we get

$$d(x_{n+1}, x_n) \le hd(x_n, x_{n-1}).$$

So, by the triangle inequality, for any $n > m$, we see

$$\begin{aligned} d(x_n, x_m) &\le d(x_n, x_{n-1}) + d(x_{n-1}, x_{n-2}) + \cdots + d(x_{m+1}, x_m) \\ &\le \left(h^{n-1} + h^{n-2} + \cdots + h^m\right) d(x_1, x_0) \\ &\le \frac{h^m}{1-h} d(x_1, x_0). \end{aligned}$$

Thus, as $h \in [0,1)$, we can choose a natural number N_0 such that $\frac{h^m}{1-h} d(x_1, x_0) \ll c$ for each $c \gg \theta$ and $m > N_0$. Hence, we see

$$d(x_n, x_m) \ll c \quad \text{for all } n > m > N_0.$$

Therefore, $\{x_n\}$ is a Cauchy sequence in (X, d).

Since X is complete, there exists $q \in X$ such that $fx_{n+1} = x_n \to q$ as $n \to \infty$. Consequently, we can find a $p \in X$ such that $fp = q$. Now, we show that $p = q$. Substituting $x = p$, $y = x_{n+1}$ in (2.1), we get

$$\begin{aligned} d(q, x_n) &= d(fp, fx_{n+1}) \\ &\ge a_1 d(p, x_{n+1}) + a_2 d(p, fp) + a_3 d(x_{n+1}, fx_{n+1}) + a_4 d(p, fx_{n+1}) + a_5 d(x_{n+1}, fp). \end{aligned}$$

For the second and fourth term on the right-hand side, we have $d(p,q) \ge d(p, x_{n+1}) - d(q, x_{n+1})$ and $d(p, x_n) \ge d(p, x_{n+1}) - d(x_n, x_{n+1})$. For the left-hand side, $d(q, x_n) \le d(q, x_{n+1}) + d(x_{n+1}, x_n)$. It follows that

$$(a_1 + a_2 + a_4) d(p, x_{n+1}) \le (1 + a_2 - a_5) d(q, x_{n+1}) + (1 - a_3 + a_4) d(x_{n+1}, x_n).$$

Now, we have

$$\begin{aligned} d(p, x_{n+1}) &\le \frac{1 + a_2 - a_5}{a_1 + a_2 + a_4} d(q, x_{n+1}) + \frac{1 - a_3 + a_4}{a_1 + a_2 + a_4} d(x_{n+1}, x_n) \\ &\le \frac{1 + a_2 - a_5}{a_1 + a_2 + a_4} d(q, x_{n+1}) + d(x_{n+1}, x_n). \end{aligned}$$

If $1 + a_2 - a_5 > 0$ for each $c \gg \theta$, we can choose a natural number N_1 such that $d(x_{n+1}, x_n) \ll \frac{c}{2}$ and $d(q, x_{n+1}) \ll \frac{(a_1+a_2+a_4)c}{2(1+a_2-a_5)}$ for $n \ge N_1$. Thus, we obtain

$$d(p, x_{n+1}) \ll \frac{c}{2} + \frac{c}{2} = c.$$

If $1 + a_2 - a_5 \le 0$ for $n \ge N_1$,

$$d(p, x_{n+1}) \le d(x_{n+1}, x_n) \ll \frac{c}{2} \ll c.$$

Therefore, $x_{n+1} \to p$. From Lemma 1.3, we see $p = q$. The conclusion is true. □

Taking some particular value of a_i ($i = 1, 2, 3, 4, 5$) in Theorem 2.1, we obtain several new results in the following.

Corollary 2.2 *Let (X, d) be a complete cone metric space. Suppose the mapping $f : X \to X$ is onto and satisfies*

$$d(fx, fy) \geq kd(x, y) + ld(x, fx) + pd(y, fy)$$

for all $x, y \in X$, where $p \leq 1$ and $k + l + p > 1$. Then f has a fixed point.

Corollary 2.3 *Let (X, d) be a complete cone metric space. Suppose the mapping $f : X \to X$ is onto and satisfies*

$$d(fx, fy) \geq kd(x, y) + ld(fx, y)$$

for all $x, y \in X$, where k, l are constants and $k > 1$. Then f has a fixed point.

Remark 2.4 Obviously, in our theorem and its corollaries above, we delete the continuity of the mappings which is essential in the results of [9]. Moreover, in Corollary 2.2 we delete $k \geq -1$, $l > 1$, which is essential in Theorem 2.6 in [9]. In Corollary 2.3 we delete $l \geq 0$, which is essential in Theorem 2.5 in [9]. Theorem 2.3 in [9] is a special case of Theorem 2.1 with $a_1 = a_4 = a_5 = 0$, $a_2 = a_3 = K$, and f is continuous.

Now, we introduce some common fixed point theorems for two expanding mappings which satisfy generalized expansive conditions without continuity of the mappings.

Theorem 2.5 *Let (X, d) be a complete cone metric space. Suppose mappings $f, g : X \to X$ are onto and satisfy*

$$d(fx, gy) \geq a_1 d(x, y) + a_2 d(x, fx) + a_3 d(y, gy) + a_4 d(x, gy) + a_5 d(y, fx) \tag{2.2}$$

for all $x, y \in X$, where a_i ($i = 1, 2, 3, 4, 5$) satisfies $a_1 + a_2 + a_3 > 1$ and $a_2 \leq 1 + a_5$, $a_3 \leq 1 + a_4$. Then f and g have a common fixed point.

Proof Suppose x_0 is an arbitrary point in X. Since f, g are onto, there exist $x_1, x_2 \in X$ such that $x_0 = gx_1$, $x_1 = fx_2$. Continuing this process, we can define $\{x_n\}$ by $x_{2n} = gx_{2n+1}$, $x_{2n+1} = fx_{2n+2}$, $n = 0, 1, 2, \ldots$. By (2.2), we have

$$\begin{aligned}
d(x_{2n+1}, x_{2n}) &= d(fx_{2n+2}, gx_{2n+1}) \\
&\geq a_1 d(x_{2n+2}, x_{2n+1}) + a_2 d(x_{2n+2}, fx_{2n+2}) + a_3 d(x_{2n+1}, gx_{2n+1}) \\
&\quad + a_4 d(x_{2n+2}, gx_{2n+1}) + a_5 d(x_{2n+1}, fx_{2n+2}) \\
&= a_1 d(x_{2n+2}, x_{2n+1}) + a_2 d(x_{2n+2}, x_{2n+1}) + a_3 d(x_{2n+1}, x_{2n}) \\
&\quad + a_4 d(x_{2n+2}, x_{2n}) + a_5 d(x_{2n+1}, x_{2n+1}).
\end{aligned}$$

Since $d(x_{2n+2}, x_{2n}) \geq d(x_{2n+2}, x_{2n+1}) - d(x_{2n}, x_{2n+1})$, the above inequality implies that

$$(1 - a_3 + a_4) d(x_{2n}, x_{2n+1}) \geq (a_1 + a_2 + a_4) d(x_{2n+1}, x_{2n+2}).$$

Similarly, it can be shown that

$$\begin{aligned} d(x_{2n-1},x_{2n}) &= d(fx_{2n},gx_{2n+1}) \\ &\geq a_1 d(x_{2n},x_{2n+1}) + a_2 d(x_{2n},fx_{2n}) + a_3 d(x_{2n+1},gx_{2n+1}) \\ &\quad + a_4 d(x_{2n},gx_{2n+1}) + a_5 d(x_{2n+1},fx_{2n}) \\ &= a_1 d(x_{2n},x_{2n+1}) + a_2 d(x_{2n},x_{2n-1}) + a_3 d(x_{2n+1},x_{2n}) \\ &\quad + a_4 d(x_{2n},x_{2n}) + a_5 d(x_{2n+1},x_{2n-1}), \end{aligned}$$

which also implies that

$$(1 - a_2 + a_5) d(x_{2n-1},x_{2n}) \geq (a_1 + a_3 + a_5) d(x_{2n},x_{2n+1}).$$

Let $M = \frac{1-a_3+a_4}{a_1+a_2+a_4}$, $N = \frac{1-a_2+a_5}{a_1+a_3+a_5}$. From $a_1 + a_2 + a_3 > 1$ and $a_2 \leq 1 + a_5$, $a_3 \leq 1 + a_4$, we see $a_1 + a_2 + a_4 > 1 - a_3 + a_4 \geq 0$ and $a_1 + a_3 + a_5 > 1 - a_2 + a_5 \geq 0$. Thus, $h = MN \in [0,1)$. Now, by induction we have

$$\begin{aligned} d(x_{2n+2},x_{2n+1}) &\leq M d(x_{2n+1},x_{2n}) \leq MN d(x_{2n},x_{2n-1}) \\ &\leq M^2 N d(x_{2n-1},x_{2n-2}) \leq \cdots \leq M h^n d(x_1,x_0) \end{aligned}$$

and

$$d(x_{2n+1},x_{2n}) \leq N d(x_{2n},x_{2n-1}) \leq \cdots \leq h^n d(x_1,x_0).$$

Hence, for any $n > m$, we deduce

$$\begin{aligned} d(x_{2n+1},x_{2m+1}) &\leq d(x_{2n+1},x_{2n}) + d(x_{2n},x_{2n-1}) + \cdots + d(x_{2m+2},x_{2m+1}) \\ &\leq \left(\sum_{i=m+1}^{n} h^i + M \sum_{i=m}^{n-1} h^i \right) d(x_1,x_0) \\ &\leq \left(\frac{h^{m+1}}{1-h} + \frac{M h^m}{1-h} \right) d(x_1,x_0) \\ &= (N+1) \frac{M h^m}{1-h} d(x_1,x_0). \end{aligned}$$

In an analogous way, we gain

$$\begin{aligned} d(x_{2n},x_{2m+1}) &\leq (N+1) \frac{M h^m}{1-h} d(x_1,x_0), \\ d(x_{2n},x_{2m}) &\leq (M+1) \frac{h^m}{1-h} d(x_1,x_0) \end{aligned}$$

and

$$d(x_{2n+1},x_{2m}) \leq (M+1) \frac{h^m}{1-h} d(x_1,x_0).$$

Thus, for $n > m > 0$,

$$d(x_n, x_m) \leq \max\left\{(N+1)\frac{Mh^m}{1-h}, (M+1)\frac{h^m}{1-h}\right\} d(x_1, x_0) = \lambda_m d(x_1, x_0),$$

where $\lambda_m \to 0$ as $m \to \infty$.

For each $c \gg \theta$, choose $\delta > 0$ such that $c - x \in \operatorname{int} P$, where $\|x\| < \delta$, *i.e.*, $x \ll c$. For this δ, we can choose a natural number N_2 such that $\|\lambda_m d(x_1, x_0)\| < \delta$ for $m > N_2$. Thus, we get

$$d(x_n, x_m) \leq \lambda_m d(x_1, x_0) \ll c \quad \text{for all } n > m > N_2.$$

Therefore, $\{x_n\}$ is a Cauchy sequence in (X, d).

As X is complete, there exists $q \in X$ such that $x_n \to q$ as $n \to \infty$. It is equivalent to $x_{2n} = gx_{2n+1} \to q$, $x_{2n+1} = fx_{2n+2} \to q$ as $n \to \infty$. Since f, g are onto, there exist $u, p \in X$ such that $fu = gp = q$. Now, we show that $u = p = q$. By (2.2), we have

$$\begin{aligned} d(fx_{2n+2}, gp) \geq{}& a_1 d(x_{2n+2}, p) + a_2 d(x_{2n+2}, fx_{2n+2}) + a_3 d(p, gp) \\ &+ a_4 d(x_{2n+2}, gp) + a_5 d(p, fx_{2n+2}), \end{aligned}$$

that is,

$$d(x_{2n+1}, q) \geq a_1 d(x_{2n+2}, p) + a_2 d(x_{2n+2}, x_{2n+1}) + a_3 d(p, q) + a_4 d(x_{2n+2}, q) + a_5 d(p, x_{2n+1}).$$

From the fact that $d(p,q) \geq d(p, x_{2n+2}) - d(q, x_{2n+2})$, $d(p, x_{2n+1}) \geq d(p, x_{2n+2}) - d(x_{2n+1}, x_{2n+2})$ and $d(x_{2n+1}, q) \leq d(x_{2n+1}, x_{2n+2}) + d(x_{2n+2}, q)$, we get

$$(a_1 + a_3 + a_5) d(p, x_{2n+2}) \leq (1 + a_3 - a_4) d(x_{2n+2}, q) + (1 - a_2 + a_5) d(x_{2n+1}, x_{2n+2}).$$

Now, we have

$$\begin{aligned} d(p, x_{2n+2}) &\leq \frac{1 + a_3 - a_4}{a_1 + a_3 + a_5} d(x_{2n+2}, q) + \frac{1 - a_2 + a_5}{a_1 + a_3 + a_5} d(x_{2n+1}, x_{2n+2}) \\ &\leq \frac{2}{a_1 + a_3 + a_5} d(x_{2n+2}, q) + d(x_{2n+1}, x_{2n+2}). \end{aligned}$$

For each $c \gg \theta$, we can choose a natural number N_3 such that $d(x_{2n+1}, x_{2n+2}) \ll \frac{c}{2}$ and $d(x_{2n+2}, q) \ll \frac{(a_1+a_3+a_5)c}{4}$ for $n \geq N_3$. Hence, we obtain $d(p, x_{2n+2}) \ll \frac{c}{2} + \frac{c}{2} = c$, *i.e.*, $x_{2n+2} \to p$. By Lemma 1.3, we know $p = q$, $gq = q$. Similarly, we also have

$$\begin{aligned} d(fu, gx_{2n+1}) \geq{}& a_1 d(u, x_{2n+1}) + a_2 d(u, fu) + a_3 d(x_{2n+1}, gx_{2n+1}) \\ &+ a_4 d(u, gx_{2n+1}) + a_5 d(x_{2n+1}, fu). \end{aligned}$$

As in the previous proof, it is not difficult to get $q = u$, *i.e.*, $fq = q$. Therefore, $fq = gq = q$. □

Corollary 2.6 *Let (X, d) be a complete cone metric space. Suppose mappings $f, g : X \to X$ are onto and satisfy*

$$d(fx, gy) \geq \alpha d(x, y) + \beta\bigl[d(x, fx) + d(y, gy)\bigr] + \gamma\bigl[d(x, gy) + d(y, fx)\bigr]$$

for all $x, y \in X$, where $\beta \leq 1 + \gamma$ and $\alpha + 2\beta > 1$. Then f and g have a common fixed point.

Corollary 2.7 *Let (X,d) be a complete cone metric space. Suppose mappings $f,g : X \to X$ are onto and satisfy*

$$d(fx,gy) \geq kd(x,y)$$

for all $x,y \in X$, where $k > 1$ is a constant. Then f and g have a unique common fixed point.

Corollary 2.8 *Let (X,d) be a complete cone metric space. Suppose the mapping $f : X \to X$ is onto and satisfies*

$$d\big(f^p x, f^q y\big) \geq kd(x,y)$$

for all $x,y \in X$, where p, q are positive integers and $k > 1$ is a constant. Then f has a unique fixed point.

Proof Let $f = f^p$, $g = f^q$. Since f is an onto mapping, $f = f^p$, $g = f^q$ are onto mappings, the conditions of Corollary 2.7 are satisfied. □

Remark 2.9 In Corollary 2.8, we obtain Corollary 2.2 in [9] when we take $p = q$.

Now, we present the following examples. In Example 1, we gain a fixed point for one expanding mapping of the situation when Corollary 2.2 can be applied, while the results in [9] cannot. In Example 2, we obtain the common fixed point for two expanding mappings in a cone metric space.

Example 1 Let $X = [1, +\infty)$, $E = C^2_{\mathbb{R}}([0,1])$ with $\|x\| = \|x\|_\infty + \|x'\|_\infty$ and $P = \{x \in E : x(t) \geq 0, t \in [0,1]\}$ (this cone is not normal). Define $d : X \times X \to E$ by $d(x,y) = |x-y|\varphi$, where $\varphi : [0,1] \to \mathbb{R}$ such that $\varphi(t) = e^t$. Consider the mapping

$$f(x) = \begin{cases} 2x^2 - x, & 1 \leq x < 2; \\ \frac{5}{2}x, & x \geq 2, \end{cases}$$

which implies that f is onto in X. Taking $k = 2$, $l = \frac{1}{4}$, $p = -\frac{1}{4}$, for $1 \leq x < 2$, all the conditions of Corollary 2.2 are fulfilled. Indeed, since $0 < \frac{1}{2}x + \frac{1}{2}y + \frac{3}{2} < 2x + 2y - 1$, we have

$$\begin{aligned} d(fx,fy) &= \big|2x^2 - x - \big(2y^2 - y\big)\big|e^t \\ &= \big|(x-y)(2x+2y-1)\big|e^t \\ &\geq |x-y|\left(\frac{1}{2}x + \frac{1}{2}y + \frac{3}{2}\right)e^t \\ &= 2|x-y|e^t + \frac{1}{2}\big|(x-y)(x+y-1)\big|e^t \\ &\geq 2|x-y|e^t + \frac{1}{4}\big|2x - 2x^2\big|e^t - \frac{1}{4}\big|2y - 2y^2\big|e^t. \end{aligned}$$

For $x \geq 2$, since $f(x)$ is increasing in x, we have

$$d(fx,fy) = \left|\frac{5}{2}x - \frac{5}{2}y\right|e^t \geq 2|x-y|e^t + \frac{1}{4}\left|x - \frac{5}{2}x\right|e^t - \frac{1}{4}\left|y - \frac{5}{2}y\right|e^t.$$

Therefore, we can apply Corollary 2.2 and conclude that f has a (unique) fixed point 0 in X. Since f is not continuous in X and $l < 1$, Theorem 2.6 in [9] is not applicable. Hence, our theorems have improved and generalized the main results in [9].

Example 2 Let $X = \{1, 2, 3\}$ and $d : X \times X \to \mathbb{R}^2$ be defined by $d(x, y) = (0, 0)$ for $x = y$ and

$$d(2,3) = d(3,2) = (0,0), \qquad d(2,1) = d(1,2) = (1,1), \qquad d(1,3) = d(3,1) = (1,1).$$

Then (X, d) is a complete cone metric space. Further, define mappings $f, g : X \to X$ as follows:

$$f(x) = \begin{cases} 1, & x = 1, \\ 3, & x = 2, \\ 2, & x = 3; \end{cases} \qquad g(x) = \begin{cases} 1, & x = 1, \\ 2, & x = 2, \\ 3, & x = 3, \end{cases}$$

which implies that f, g are onto in X. Note that

$$d(fx, gy) \geq a_1 d(x, y) + a_2 d(x, fx) + a_3 d(y, gy) + a_4 d(x, gy) + a_5 d(y, fx)$$

for all $x, y \in X$ by taking $a_1 = -\frac{1}{7}$, $a_2 = -\frac{2}{7}$, $a_3 = \frac{11}{7}$, $a_4 = \frac{5}{7}$, $a_5 = \frac{3}{7}$. Thus, all the conditions of Theorem 2.5 are fulfilled. Then f and g have a unique common fixed point 1 in X.

Remark 2.10 Obviously, in the above two examples, we obtain the (common) fixed point which essentially needs the structure of a cone metric and not an ordinary metric on X. Then the results in a metric space in [10] cannot be applied to these examples.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

Author details
[1]Department of Mathematics, Zhaotong University, Zhaotong, 657000, P.R. China. [2]School of Mathematics and Statistics, Hubei Normal University, Huangshi, 435002, P.R. China.

Acknowledgements
The authors thank the editor and the referees for their valuable comments and suggestions which improve greatly the quality for this paper. The research was supported by the Foundation of Education Ministry, Hubei Province, China (No: D20102502).

Received: 28 December 2011 Accepted: 14 December 2012 Published: 7 January 2013

References

1. Huang, L-G, Zhang, X: Cone metric space and fixed point theorems of contractive mappings. J. Math. Anal. Appl. **332**, 1468-1476 (2007)
2. Abbas, M, Jungck, G: Common fixed point results for noncommuting mappings without continuity in cone metric spaces. J. Math. Anal. Appl. **341**, 416-420 (2008)
3. Abbas, M, Rhoades, BE: Fixed and periodic point results in cone metric spaces. Appl. Math. Lett. **22**, 511-515 (2009)
4. Radenovic, S: Common fixed points under contractive conditions in cone metric spaces. Comput. Math. Appl. **58**, 1273-1278 (2009)
5. Altun, I, Damjanovic, B, Djoric, D: Fixed point and common fixed point theorems on ordered cone metric spaces. Appl. Math. Lett. **23**, 310-316 (2010)
6. Rezapour, S, Hamlbarani, R: Some note on the paper "Cone metric spaces and fixed point theorems of contractive mappings". J. Math. Anal. Appl. **345**, 719-724 (2008)

7. Han, Y, Xu, S-Y: New common fixed point results for four maps on cone metric spaces. Appl. Math. **2**, 1114-1118 (2011)
8. Bari, CD, Vetro, P: φ-pairs and common fixed points in cone metric spaces. Rend. Circ. Mat. Palermo **57**, 279-285 (2008)
9. Aage, CT, Salunke, JN: Some fixed point theorems for expansion onto mappings on cone metric spaces. Acta Math. Sin. Engl. Ser. **27**(6), 1101-1106 (2011)
10. Wang, S-Z, Li, B-Y, Gao, Z-M, Lseki, K: Some fixed points of expansion mappings. Math. Jpn. **29**, 631-636 (1984)
11. Zhang, X: Common fixed point theorems of Lipschitz type mappings in cone metric spaces. Acta Math. Sin. **53**(6), 1139-1148 (2010) (in Chinese)

doi:10.1186/1687-1812-2013-3
Cite this article as: Han and Xu: Some new theorems of expanding mappings without continuity in cone metric spaces. *Fixed Point Theory and Applications* 2013 **2013**:3.