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Fixed and coupled fixed point theorems of omega-distance for nonlinear contraction

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Abstract

In this paper we utilize the notion of Ω-distance in the sense of Saadati *et al.* (Math. Comput. Model. 52:797-801, 2010) to construct and prove some fixed and coupled fixed point theorems in a complete G-metric space for a nonlinear contraction. Also, we provide an example to support our results.

MSC: 47H10; 54H25

Keywords: coupled fixed point; Ω-distance

1 Introduction

The concept of G-metric space was introduced by Mustafa and Sims [1]. After that, many authors constructed fixed point theorems in G-metric spaces. In [2] and [3], common fixed points results for mappings which satisfy the generalized (φ, ψ)-weak contraction are obtained. In [4], the author proves a common fixed point theorem for two self-mappings verifying a contractive condition of integral type in G-metric spaces. In [5, 6] and [7], tripled coincidence point results for a mixed monotone mapping in G-metric spaces are established; also see [8]. Some common fixed point results for two self-mappings, one of them being a generalized weakly G-contraction of type A and B with respect to the other mapping, are stated in [9]. Fixed point theorems for mappings with a contractive iterate at a point are formulated in [10] and in [11]. Papers [12] and [13] refer to common fixed point theorems for single-valued and multi-valued mappings which satisfy contractive conditions on G-metric spaces. In [14] and [15], theorems from G-metric spaces are used to obtain several results on complete D-metric spaces. Various contractive conditions on G-metric spaces which lead to fixed point results are stated in [16]. Paper [17] deals with the existence of fixed point results in G-metric spaces. In [18], common fixed point theorems with ϕ-maps on G-cone metric spaces are established. In [19], a general fixed point theorem for mappings satisfying an ϕ-implicit relation is proved. Paper [20] states fixed point theorems for mappings satisfying ϕ-maps in G-metric spaces. Mohamed Jleli and Bessem Samet [21] in their nice paper pointed out that the quasi-metric spaces play a major role to construct some known fixed point theorems in a G-metric space. For other recent results in G-metric spaces, please see [22–24].

The coupled fixed point is one of the most interesting subjects in metric spaces. The notion of coupled fixed point was introduced by Bhaskar and Lakshmikantham [25], and the notion of coincidence coupled fixed point was introduced by Lakshmikantham and Ćirić [26]. In recent years many authors established many nice coupled and coincidence coupled fixed point theorems in metric spaces, partial metric spaces and G-metric spaces. For some works on this subject, we refer the reader to [27–38].

2 Preliminaries

It is fundamental to recall the definition of G-metric spaces.

Definition 2.1 ([1]) Let X be a nonempty set. $G : X \times X \times X \to X$ is called *G-metric* if the following axioms are fulfilled:

(1) $G(x,y,z) = 0$ if $x = y = z$ (the coincidence);
(2) $G(x,x,y) > 0$ for all $x, y \in X$, $x \neq y$;
(3) $G(x,x,z) \leq G(x,y,z)$ for each triple (x,y,z) from $X \times X \times X$ with $z \neq y$;
(4) $G(x,y,z) = G(p\{x,y,z\})$ for each permutation of $\{x,y,z\}$ (the symmetry);
(5) $G(x,y,z) \leq G(x,a,a) + G(a,y,z)$ for each x, y, z and a in X (the rectangle inequality).

Definition 2.2 ([1]) Consider X to be a G-metric space and (x_n) to be a sequence in G.

(1) (x_n) is called a *G-Cauchy sequence* if for each $\varepsilon > 0$, there is a positive integer n_0 so that for all $m, n, l \geq n_0$, $G(x_n, x_m, x_l) < \varepsilon$.
(2) (x_n) is said to be *G-convergent* to $x \in X$ if for each $\varepsilon > 0$, there is a positive integer n_0 such that $G(x_m, x_n, x) < \varepsilon$ for each $m, n \geq n_0$.

Now, we recall the definitions of coupled and coincidence coupled fixed points.

Definition 2.3 ([25]) Consider X to be a nonempty set. The pair $(x,y) \in X \times X$ is called a *coupled fixed point* of the mapping $F : X \times X \to X$ if

$$F(x,y) = x, \qquad F(y,x) = y.$$

Definition 2.4 ([26]) Let X be a nonempty set. The element $(x,y) \in X \times X$ is a *coupled coincidence point* of mappings $F : X \times X \to X$ and $g : X \to X$ if

$$F(x,y) = gx, \qquad F(y,x) = gy.$$

In 2010, Saadati *et al.* [39] utilized the notion of G-metric spaces to introduce the concept of Ω-distance. Moreover, Saadati *et al.* [40] constructed some fixed point theorem in G-metric spaces by using the notion of Ω-distance.

Definition 2.5 ([39]) Consider (X,G) to be a G-metric space and $\Omega : X \times X \times X \to [0,+\infty)$. Ω is called an *Ω-distance* on X if it satisfies the three conditions as follows:

(1) $\Omega(x,y,z) \leq \Omega(x,a,a) + \Omega(a,y,z)$ for all x, y, z, a from X.
(2) For each x, y from X, $\Omega(x,y,\cdot), \Omega(x,\cdot,y) : X \to [0,+\infty)$ are lower semi-continuous.
(3) For each $\varepsilon > 0$, there is $\delta > 0$, so that $\Omega(x,a,a) \leq \delta$ and $\Omega(a,y,z) \leq \delta$ imply $G(x,y,z) \leq \varepsilon$.

The following lemma is very useful in this paper.

Lemma 2.1 ([39, 40]) *Let X be a metric space endowed with metric G, and let Ω be an Ω-distance on X. (x_n), (y_n) are sequences in X, (α_n) and (β_n) are sequences in* $[0,+\infty)$, *with* $\lim_{n\to+\infty} \alpha_n = \lim_{n\to+\infty} \beta_n = 0$. *If x, y, z and $a \in X$, then*

(1) *If* $\Omega(y,x_n,x_n) \le \alpha_n$ *and* $\Omega(x_n,y,z) \le \beta_n$*, for* $n \in \mathbb{N}$*, then* $G(y,y,z) < \varepsilon$*, and, by consequence,* $y = z$.
(2) *Inequalities* $\Omega(y_n,x_n,x_n) \le \alpha_n$ *and* $\Omega(x_n,y_m,z) \le \beta_n$*, for* $m > n$*, imply* $G(y_n,y_m,z) \to 0$*, hence* $y_n \to z$.
(3) *If* $\Omega(x_n,x_m,x_l) \le \alpha_n$ *for* $l,m,n \in \mathbb{N}$ *with* $n \le m \le l$*, then* (x_n) *is a G-Cauchy sequence.*
(4) *If* $\Omega(x_n,a,a) \le \alpha_n$*,* $n \in \mathbb{N}$*, then* (x_n) *is a G-Cauchy sequence.*

The following two sets are very useful to build our nonlinear contraction in this paper:

$$\Phi = \{\varphi : [0,+\infty) \to [0,+\infty) | \varphi \text{ is continuous, increasing}, \varphi(t) = 0 \text{ if and only if } t = 0\},$$
$$\Psi = \{\psi : [0,+\infty) \to [0,+\infty) | \psi \text{ is lower semi-continuous},$$
$$\psi(t) = 0 \text{ if and only if } t = 0\}.$$

For some works on fixed point theorems based on the above sets, see, for example, [1, 7, 14–20, 23, 33–39, 41–44].

In the present paper, we utilize the concept of Ω-distance and the sets Φ, Ψ to establish some fixed and coupled fixed point theorems. Also, we introduce an example as an application of our results.

3 Main results

In the first part of the section, we introduce and prove the following fixed point theorem.

Theorem 3.1 *Let* (X,G) *be a G-metric space and* Ω *be an* Ω*-distance on* X*. Consider* $\varphi \in \Phi$*,* $\psi \in \Psi$ *and* $T : X \to X$ *such that*

$$\varphi\Omega(Tx,Ty,Tz) \le \varphi\Omega(x,y,z) - \psi\Omega(x,y,z) \tag{1}$$

holds for each $(x,y,z) \in X \times X \times X$.

Suppose that if $u \ne Tu$*, then*

$$\inf\{\Omega(x,Tx,u) : x \in X\} > 0.$$

Then T *has a unique fixed point.*

Proof Let $x_0 \in X$ and $x_{n+1} = Tx_n$ for each $n \in \mathbb{N}$. If there is $n \in \mathbb{N}$ for which $x_{n+1} = x_n$, then x_n is a fixed point of T.

In the following, we assume $x_{n+1} \ne x_n$ for each $n \in \mathbb{N}$.

First we shall prove that $\lim_{n\to+\infty} \Omega(x_n,x_{n+1},x_{n+1}) = 0$.

For $n \in \mathbb{N}$, $n \ge 1$, we have

$$\begin{aligned}\varphi\Omega(x_n,x_{n+1},x_{n+1}) &= \varphi\Omega(Tx_{n-1},Tx_n,Tx_n)\\ &\le \varphi\Omega(x_{n-1},x_n,x_n) - \psi\Omega(x_{n-1},x_n,x_n)\\ &\le \varphi\Omega(x_{n-1},x_n,x_n).\end{aligned} \tag{2}$$

φ is a nondecreasing function, hence $\Omega(x_n,x_{n+1},x_{n+1}) \le \Omega(x_{n-1},x_n,x_n)$, $n \ge 1$. It follows that $(\Omega(x_n,x_{n+1},x_{n+1}))$ is a nondecreasing sequence, therefore there exists $\lim_{n\to+\infty} \Omega(x_n,x_{n+1},x_{n+1}) = r \ge 0$.

Taking $n \to +\infty$ in inequality (2) and using the continuity of φ and the lower semi-continuity of ψ, we get

$$\varphi r \leq \varphi r - \liminf_{n\to+\infty} \psi\Omega(x_{n-1}, x_n, x_n) \leq \varphi r - \psi r,$$

imposing $\psi r = 0$, that is, $r = 0$.

Analogously, it can be proved that $\lim_{n\to+\infty} \Omega(x_{n+1}, x_n, x_n) = 0$ and also that

$$\lim_{n\to+\infty} \Omega(x_n, x_n, x_{n+1}) = 0.$$

The next step is to prove that $\lim_{m,n\to+\infty} \Omega(x_n, x_m, x_m) = 0$, $m > n$.

By *reductio ad absurdum*, suppose the contrary. Hence, there exist $\varepsilon > 0$ and two sequences (n_k) and (m_k) such that

$$\Omega(x_{n_k}, x_{m_k}, x_{m_k}) \geq \varepsilon, \qquad \Omega(x_{n_k}, x_{m_k-1}, x_{m_k-1}) < \varepsilon, \quad m_k > n_k.$$

As $\lim_{n\to+\infty} \Omega(x_n, x_{n+1}, x_{n+1}) = 0$, it follows

$$\begin{aligned}\varepsilon &\leq \Omega(x_{n_k}, x_{m_k}, x_{m_k}) \leq \Omega(x_{n_k}, x_{m_k-1}, x_{m_k-1}) + \Omega(x_{m_k-1}, x_{m_k}, x_{m_k}) \\ &< \varepsilon + \Omega(x_{m_k-1}, x_{m_k}, x_{m_k}) \to \varepsilon \quad \text{as } k \to +\infty.\end{aligned}$$

Therefore, $\lim_{k\to+\infty} \Omega(x_{n_k}, x_{m_k}, x_{m_k}) = \varepsilon$.

On the other hand,

$$\begin{aligned}\varepsilon &\leq \Omega(x_{n_k}, x_{m_k}, x_{m_k}) \leq \Omega(x_{n_k}, x_{n_k+1}, x_{n_k+1}) + \Omega(x_{n_k+1}, x_{m_k}, x_{m_k}) \\ &\leq \Omega(x_{n_k}, x_{n_k+1}, x_{n_k+1}) + \Omega(x_{n_k+1}, x_{m_k+1}, x_{m_k+1}) + \Omega(x_{m_k+1}, x_{m_k}, x_{m_k}).\end{aligned} \tag{3}$$

The contraction condition (1) yields

$$\begin{aligned}\varphi\Omega(x_{n_k+1}, x_{m_k+1}, x_{m_k+1}) &\leq \varphi\Omega(x_{n_k}, x_{m_k}, x_{m_k}) - \psi\Omega(x_{n_k}, x_{m_k}, x_{m_k}) \\ &\leq \varphi\Omega(x_{n_k}, x_{m_k}, x_{m_k}),\end{aligned}$$

so $\Omega(x_{n_k+1}, x_{m_k+1}, x_{m_k+1}) \leq \Omega(x_{n_k}, x_{m_k}, x_{m_k})$, and relation (3) becomes

$$\begin{aligned}\varepsilon &\leq \Omega(x_{n_k}, x_{n_k+1}, x_{n_k+1}) + \Omega(x_{n_k+1}, x_{m_k+1}, x_{m_k+1}) + \Omega(x_{m_k+1}, x_{m_k}, x_{m_k}) \\ &\leq \Omega(x_{n_k}, x_{n_k+1}, x_{n_k+1}) + \Omega(x_{n_k}, x_{m_k}, x_{m_k}) + \Omega(x_{m_k+1}, x_{m_k}, x_{m_k}).\end{aligned}$$

Letting $k \to +\infty$, we get $\lim_{k\to+\infty} \Omega(x_{n_k+1}, x_{m_k+1}, x_{m_k+1}) = \varepsilon$.

Having in mind the continuity of φ and the lower semi-continuity of ψ, we obtain

$$\varphi\varepsilon \leq \varphi\varepsilon - \liminf_{k\to+\infty} \Omega(x_{n_k}, x_{m_k}, x_{m_k}) \leq \varphi\varepsilon - \psi\varepsilon,$$

which is impossible, since $\varepsilon > 0$.

It follows that $\lim_{m,n\to+\infty}\Omega(x_n,x_m,x_m)=0$, $m>n$.

In a similar manner, it can be proved that $\lim_{m,n\to+\infty}\Omega(x_n,x_n,x_m)=0$, $m>n$.

Consider now $l>m>n$, $l,m,n\in\mathbb{N}$. Since

$$\Omega(x_n,x_m,x_l)\le\Omega(x_n,x_m,x_m)+\Omega(x_m,x_m,x_l)\to 0$$

as $l,m,n\to+\infty$, we conclude that $\lim_{l,m,n\to+\infty}\Omega(x_n,x_m,x_l)=0$. By Lemma 2.1, (x_n) is a G-Cauchy sequence in the G-complete space (X,G), so it converges to $u\in X$.

Suppose $u\neq Tu$. Consider $\varepsilon>0$. As (x_n) is a Cauchy sequence, there is $n_0\in\mathbb{N}$ such that

$$\Omega(x_n,x_m,x_l)<\varepsilon,\quad \forall n,m,l\ge n_0.$$

Thus

$$\liminf_{l\to+\infty}\Omega(x_n,x_m,x_l)\le\liminf_{l\to+\infty}\varepsilon=\varepsilon,\quad \forall n,m\ge n_0.$$

From the lower semi-continuity of Ω in its third variables, we have

$$\Omega(x_n,x_m,u)\le\liminf_{l\to+\infty}\Omega(x_n,x_m,x_l)\le\varepsilon,\quad \forall n,m\ge n_0. \tag{4}$$

Considering $m=n+1$ in inequality (4), we get

$$\Omega(x_n,x_{n+1},u)\le\varepsilon.$$

On the other hand, we have

$$\begin{aligned}0 &< \inf\bigl\{\Omega(x,Tx,u):x\in X\bigr\}\\ &\le\inf\bigl\{\Omega(x_n,x_{n+1},u):n\ge n_0\bigr\}<\varepsilon,\end{aligned}$$

which contradicts the hypotheses.

Therefore, $u=Tu$ and hence u is a fixed point of T.

We shall deal now with the uniqueness of the fixed point of T.

Suppose that there are u and v in X fixed points of the mapping T.

It follows that

$$\varphi\Omega(v,u,u)=\varphi\Omega(Tv,Tu,Tu)\le\varphi\Omega(v,u,u)-\psi\Omega(v,u,u),$$

which is possible only for $\psi\Omega(v,u,u)=0$, that is, $\Omega(v,u,u)=0$.

Similarly, it can be proved that $\Omega(u,v,u)=0$.

According to the definition of an Ω-distance, $\Omega(v,u,u)=0$ and $\Omega(u,v,u)=0$ imply $u=v$. Hence, T has a unique fixed point. □

Haghi *et al.* [45] in their interesting paper showed that some common fixed point theorems can be obtained from the known fixed point theorems; for other interesting article by Haghi *et al.*, please see [46]. By using the same method of Haghi *et al.* [45], we get the following result.

Theorem 3.2 *Let (X, G) be a G-metric space and Ω be an Ω-distance on X. Consider $\varphi \in \Phi$, $\psi \in \Psi$ and $T, S : X \to X$ such that*

$$\varphi\Omega(Tx, Ty, Tz) \leq \varphi\Omega(Sx, Sy, Sz) - \psi\Omega(Sx, Sy, Sz)$$

holds for each $(x, y, z) \in X \times X \times X$.

Suppose the following hypotheses:

(1) $TX \subseteq SX$.

(2) *If $Su \neq Tu$, then*

$$\inf\{\Omega(Sx, Tx, Su) : x \in X\} > 0.$$

Then T and S have a unique common fixed point.

As consequent results of Theorem 3.1 and Theorem 3.2, we have the following.

Corollary 3.1 *Let (X, G) be a G-metric space and Ω be an Ω-distance on X. Consider $\psi \in \Psi$ and $T : X \to X$ such that*

$$\Omega(Tx, Ty, Tz) \leq \Omega(x, y, z) - \psi\Omega(x, y, z)$$

holds for each $(x, y, z) \in X \times X \times X$.

Suppose that if $u \neq Tu$, then

$$\inf\{\Omega(x, Tx, u) : x \in X\} > 0.$$

Then T has a unique fixed point.

Corollary 3.2 *Let (X, G) be a G-metric space and Ω be an Ω-distance on X. Consider $\psi \in \Psi$ and $T, S : X \to X$ such that*

$$\Omega(Tx, Ty, Tz) \leq \Omega(Sx, Sy, Sz) - \psi\Omega(Sx, Sy, Sz)$$

holds for each $(x, y, z) \in X \times X \times X$.

Suppose the following hypotheses:

(1) $TX \subseteq SX$.

(2) *If $Su \neq Tu$, then*

$$\inf\{\Omega(Sx, Tx, Su) : x \in X\} > 0.$$

Then T and S have a unique common fixed point.

In the second part of the section, we introduce and prove the following coincidence coupled fixed point theorem.

Theorem 3.3 *Consider (X, G) to be a G-metric space endowed with an Ω-distance called Ω. Let $F : X \times X \to X$ and $g : X \to X$ be two mappings with the properties*

$F(X \times X) \subseteq gX$, and gX is a complete subspace of X with respect to the topology induced by G.

Suppose that there exist $\varphi \in \Phi$ and $\psi \in \Psi$ such that

$$\begin{aligned}&\varphi\big(\Omega\big(F(x,y),F(x^*,y^*),F(z,z^*)\big) + \Omega\big(F(y,x),F(y^*,x^*),F(z^*,z)\big)\big)\\&\quad\le \varphi\big(\Omega\big(gx,gx^*,gz\big) + \Omega\big(gy,gy^*,gz^*\big)\big) - \psi\big(\Omega\big(gx,gx^*,gz\big) + \Omega\big(gy,gy^*,gz^*\big)\big)\end{aligned} \tag{5}$$

for each $(x,y),(x^*,y^*),(z,z^*) \in X \times X$.

Additionally, suppose that if $F(u,v) \neq gu$ or $F(v,u) \neq gv$, then

$$\inf\big\{\Omega\big(gx,F(x,y),gu\big) + \Omega\big(gy,F(y,x),gv\big) : x,y \in X\big\} > 0.$$

Then F and g have a unique coupled coincidence point (u,v), with $F(u,v) = gu = gv = F(v,u)$.

Proof Let $(x_0,y_0) \in X \times X$. Having in mind that $F(X \times X) \subseteq gX$, for each $n \in \mathbb{N}$, there is a pair $(x_{n+1},y_{n+1}) \in X \times X$ such that

$$gx_{n+1} = F(x_n,y_n), \qquad gy_{n+1} = F(y_n,x_n).$$

First, we prove that

$$\lim_{n\to+\infty} \Omega(gx_n,gx_{n+1},gx_{n+1}) = 0 \quad \text{and} \quad \lim_{n\to+\infty} \Omega(gy_n,gy_{n+1},gy_{n+1}) = 0.$$

Using inequality (5), we get

$$\begin{aligned}&\varphi\big(\Omega(gx_n,gx_{n+1},gx_{n+1}) + \Omega(gy_n,gy_{n+1},gy_{n+1})\big)\\&\quad= \varphi\big(\Omega\big(F(x_{n-1},y_{n-1}),F(x_n,y_n),F(x_n,y_n)\big)\\&\qquad+ \Omega\big(F(y_{n-1},x_{n-1}),F(y_n,x_n),F(y_n,x_n)\big)\big)\\&\quad\le \varphi\big(\Omega(gx_{n-1},gx_n,gx_n) + \Omega(gy_{n-1},gy_n,gy_n)\big)\\&\qquad- \psi\big(\Omega(gx_{n-1},gx_n,gx_n) + \Omega(gy_{n-1},gy_n,gy_n)\big)\\&\quad\le \varphi\big(\Omega(gx_{n-1},gx_n,gx_n) + \Omega(gy_{n-1},gy_n,gy_n)\big).\end{aligned} \tag{6}$$

Since φ is a nondecreasing function, we obtain

$$\begin{aligned}&\Omega(gx_n,gx_{n+1},gx_{n+1}) + \Omega(gy_n,gy_{n+1},gy_{n+1})\\&\quad\le \Omega(gx_{n-1},gx_n,gx_n) + \Omega(gy_{n-1},gy_n,gy_n), \quad n \in \mathbb{N}, n \ge 1,\end{aligned}$$

that is, $(\Omega(gx_n,gx_{n+1},gx_{n+1}) + \Omega(gy_n,gy_{n+1},gy_{n+1}))$ is a nondecreasing sequence. Denote by $r \ge 0$ its limit.

Letting $n \to +\infty$ in relation (6), the continuity of φ and the lower semi-continuity of ψ imply

$$\varphi r \le \varphi r - \liminf_{n\to+\infty} \psi\big(\Omega(gx_n,gx_{n+1},gx_{n+1}) + \Omega(gy_n,gy_{n+1},gy_{n+1})\big) \le \varphi r - \psi r,$$

which forces $\varphi r = 0$, that is, $r = 0$.

Since Ω takes nonnegative values,

$$\lim_{n\to+\infty}\Omega(gx_n,gx_{n+1},gx_{n+1})=0 \quad\text{and}\quad \lim_{n\to+\infty}\Omega(gy_n,gy_{n+1},gy_{n+1})=0.$$

A similar procedure leads us to

$$\begin{aligned}&\lim_{n\to+\infty}\Omega(gx_{n+1},gx_n,gx_n)=0, \qquad \lim_{n\to+\infty}\Omega(gy_{n+1},gy_n,gy_n)=0;\\ &\lim_{n\to+\infty}\Omega(gx_n,gx_n,gx_{n+1})=0, \qquad \lim_{n\to+\infty}\Omega(gy_n,gy_n,gy_{n+1})=0.\end{aligned}$$

Now, our purpose is to show that

$$\lim_{m,n\to+\infty}\Omega(gx_n,gx_m,gx_m)=0 \quad\text{and}\quad \lim_{m,n\to+\infty}\Omega(gx_n,gx_m,gx_m)=0, \quad m>n.$$

Supposing the contrary, there exist $\varepsilon>0$ and two subsequences (n_k) and (m_k) for which

$$\begin{aligned}&\Omega(gx_{n_k},gx_{m_k},gx_{m_k})+\Omega(gy_{n_k},gy_{m_k},gy_{m_k})\geq\varepsilon,\\ &\Omega(gx_{n_k},gx_{m_k-1},gx_{m_k-1})+\Omega(gy_{n_k},gy_{m_k-1},gy_{m_k-1})<\varepsilon, \quad m_k>n_k.\end{aligned}$$

We obtain

$$\begin{aligned}\varepsilon &\leq \Omega(gx_{n_k},gx_{m_k},gx_{m_k})+\Omega(gy_{n_k},gy_{m_k},gy_{m_k})\\ &\leq \Omega(gx_{n_k},gx_{m_k-1},gx_{m_k-1})+\Omega(gy_{n_k},gy_{m_k-1},gy_{m_k-1})\\ &\quad+\Omega(gx_{m_k-1},gx_{m_k},gx_{m_k})+\Omega(gy_{m_k-1},gy_{m_k},gy_{m_k})\\ &< \varepsilon+\Omega(gx_{m_k-1},gx_{m_k},gx_{m_k})+\Omega(gy_{m_k-1},gy_{m_k},gy_{m_k}).\end{aligned}$$

As $k\to+\infty$ and $\lim_{n\to+\infty}(\Omega(gx_n,gx_{n+1},gx_{n+1})+\Omega(gy_n,gy_{n+1},gy_{n+1}))=0$, we get

$$\lim_{k\to+\infty}\bigl(\Omega(gx_{n_k},gx_{m_k},gx_{m_k})+\Omega(gy_{n_k},gy_{m_k},gy_{m_k})\bigr)=0.$$

Also, using the properties of Ω, we have

$$\begin{aligned}\varepsilon &\leq \Omega(gx_{n_k},gx_{m_k},gx_{m_k})+\Omega(gy_{n_k},gy_{m_k},gy_{m_k})\\ &\leq \Omega(gx_{n_k},gx_{n_k+1},gx_{n_k+1})+\Omega(gx_{n_k+1},gx_{m_k},gx_{m_k})\\ &\quad+\Omega(gy_{n_k},gy_{n_k+1},gy_{n_k+1})+\Omega(gy_{n_k+1},gy_{m_k},gy_{m_k})\\ &\leq \Omega(gx_{n_k},gx_{n_k+1},gx_{n_k+1})+\Omega(gx_{n_k+1},gx_{m_k+1},gx_{m_k+1})\\ &\quad+\Omega(gx_{m_k+1},gx_{m_k},gx_{m_k})+\Omega(gy_{n_k},gy_{n_k+1},gy_{n_k+1})\\ &\quad+\Omega(gy_{n_k+1},gy_{m_k+1},gy_{m_k+1})+\Omega(gy_{m_k+1},gy_{m_k},gy_{m_k}).\end{aligned}\tag{7}$$

Taking advantage of the contraction condition, it follows

$$\begin{aligned}&\varphi\bigl(\Omega(gx_{n_k+1},gx_{m_k+1},gx_{m_k+1})+\Omega(gy_{n_k+1},gy_{m_k+1},gy_{m_k+1})\bigr)\\ &\quad\leq\varphi\bigl(\Omega(gx_{n_k},gx_{m_k},gx_{m_k})+\Omega(gy_{n_k},gy_{m_k},gy_{m_k})\bigr)\end{aligned}$$

$$\begin{aligned}&- \psi\left(\Omega(gx_{n_k}, gx_{m_k}, gx_{m_k}) + \Omega(gy_{n_k}, gy_{m_k}, gy_{m_k})\right)\\
&\le \varphi\left(\Omega(gx_{n_k}, gx_{m_k}, gx_{m_k}) + \Omega(gy_{n_k}, gy_{m_k}, gy_{m_k})\right).\end{aligned}$$

Hence

$$\begin{aligned}&\Omega(gx_{n_k+1}, gx_{m_k+1}, gx_{m_k+1}) + \Omega(gy_{n_k+1}, gy_{m_k+1}, gy_{m_k+1})\\
&\quad \le \Omega(gx_{n_k}, gx_{m_k}, gx_{m_k}) + \Omega(gy_{n_k}, gy_{m_k}, gy_{m_k}),\end{aligned}$$

and relation (7) becomes

$$\begin{aligned}\varepsilon &\le \Omega(gx_{n_k}, gx_{n_k+1}, gx_{n_k+1}) + \Omega(gx_{n_k+1}, gx_{m_k+1}, gx_{m_k+1})\\
&\quad + \Omega(gx_{m_k+1}, gx_{m_k}, gx_{m_k}) + \Omega(gy_{n_k}, gy_{n_k+1}, gy_{n_k+1})\\
&\quad + \Omega(gy_{n_k+1}, gy_{m_k+1}, gy_{m_k+1}) + \Omega(gy_{m_k+1}, gy_{m_k}, gy_{m_k})\\
&\le \Omega(gx_{n_k}, gx_{n_k+1}, gx_{n_k+1}) + \Omega(gx_{n_k}, gx_{m_k}, gx_{m_k})\\
&\quad + \Omega(gx_{m_k+1}, gx_{m_k}, gx_{m_k}) + \Omega(gy_{n_k}, gy_{n_k+1}, gy_{n_k+1})\\
&\quad + \Omega(gy_{n_k}, gy_{m_k}, gy_{m_k}) + \Omega(gy_{m_k+1}, gy_{m_k}, gy_{m_k}).\end{aligned}$$

For $k \to +\infty$, $\lim_{k\to+\infty}(\Omega(gx_{n_k+1}, gx_{m_k+1}, gx_{m_k+1}) + \Omega(gy_{n_k+1}, gy_{m_k+1}, gy_{m_k+1})) = \varepsilon$.

The properties of φ, ψ lead us to

$$\begin{aligned}\varphi\varepsilon &= \lim_{k\to+\infty} \varphi\left(\Omega(gx_{n_k+1}, gx_{m_k+1}, gx_{m_k+1}) + \Omega(gy_{n_k+1}, gy_{m_k+1}, gy_{m_k+1})\right)\\
&\le \varphi\varepsilon - \liminf_{k\to+\infty} \psi\left(\Omega(gx_{n_k}, gx_{m_k}, gx_{m_k}) + \Omega(gy_{n_k}, gy_{m_k}, gy_{m_k})\right) \le \varphi\varepsilon - \psi\varepsilon.\end{aligned}$$

Since $\varepsilon > 0$, we obtain a contradiction. Therefore, $\lim_{m,n\to+\infty} \Omega(gx_n, gx_m, gx_m) = 0$ and $\lim_{m,n\to+\infty} \Omega(gy_n, gy_m, gy_m) = 0$, $m > n$.

Analogously, it can be proved that $\lim_{m,n\to+\infty} \Omega(gx_n, gx_n, gx_m) = 0$ and also

$$\lim_{m,n\to+\infty} \Omega(gy_n, gy_n, gy_m) = 0, \quad m > n.$$

Consider $l > m > n$. Then

$$\Omega(gx_n, gx_m, gx_l) \le \Omega(gx_n, gx_m, gx_m) + \Omega(gx_m, gx_m, gx_l) \to 0 \quad \text{as } n, m, l \to +\infty.$$

By Lemma 2.1, we get $\lim_{n,m,l\to+\infty} \Omega(gx_n, gx_m, gx_l) = 0$, $l > m > n$. Hence, (gx_n) is a G-Cauchy sequence in gX, which is complete. Similarly, (gy_n) converges in gX. Let $gu = \lim_{n\to+\infty} gx_n$ and $gv = \lim_{n\to+\infty} gy_n$, $u, v \in X$.

Let us show now that (u, v) is a coupled coincidence point of F and g. In that respect, consider $\varepsilon > 0$. Since (gx_n) is a Cauchy sequence, then there exists $n_0 \in \mathbb{N}$ such that for each $n, m, l \ge n_0$, $\Omega(gx_n, gx_m, gx_l) < \varepsilon$. The properties of lower semi-continuity of Ω imply

$$\Omega(gx_n, gx_m, gu) \le \liminf_{p\to+\infty} \Omega(gx_n, gx_m, gx_p) \le \varepsilon, \tag{8}$$

$$\Omega(gy_n, gy_m, gv) \le \liminf_{p\to+\infty} \Omega(gy_n, gy_m, gy_p) \le \varepsilon. \tag{9}$$

Considering $m = n + 1$ in (8) and (9), we obtain

$$\Omega\big(gx_n, F(x_n, y_n), gu\big) + \Omega\big(gy_n, F(y_n, x_n), gv\big) \le 2\varepsilon.$$

On the other hand, we get

$$\begin{aligned} 0 &< \inf\big\{\Omega\big(gx, F(x,y), gu\big) + \Omega\big(gy, F(y,x), gv\big) : x, y \in X\big\} \\ &\le \inf\big\{\Omega\big(gx_n, F(x_n, y_n), gu\big) + \Omega\big(gy_n, F(y_n, x_n), gv\big) : n \ge n_0\big\} \le 2\varepsilon, \end{aligned}$$

which is a contradiction.

Therefore, $F(u,v) = gu$ and $F(v,u) = gv$.

In the following, we refer to the uniqueness of the coupled coincidence point of F and g.

Consider (u,v) and (u^*, v^*) to be two coupled coincidence points of F and g.

By using the contraction condition, we obtain

$$\begin{aligned} &\varphi\big(\Omega\big(gu^*, gu, gu\big) + \Omega\big(gv^*, gv, gv\big)\big) \\ &\quad = \varphi\big(\Omega\big(F\big(u^*, v^*\big), F(u,v), F(u,v)\big) + \Omega\big(F\big(v^*, u^*\big), F(v,u), F(v,u)\big)\big) \\ &\quad \le \varphi\big(\Omega\big(gu^*, gu, gu\big) + \Omega\big(gv^*, gv, gv\big)\big) - \psi\big(\Omega\big(gu^*, gu, gu\big) + \Omega\big(gv^*, gv, gv\big)\big) \\ &\quad \le \varphi\big(\Omega\big(gu^*, gu, gu\big) + \Omega\big(gv^*, gv, gv\big)\big), \end{aligned}$$

which leads us to $\psi(\Omega(gu^*, gu, gu) + \Omega(gv^*, gv, gv)) = 0$, or $\Omega(gu^*, gu, gu) = \Omega(gv^*, gv, gv) = 0$.

In a similar manner, we prove that $\Omega(gu, gu^*, gu) = \Omega(gv, gv^*, gv) = 0$.

Lemma 2.1 implies that $gu = gu^*$ and $gv = gv^*$.

Having in mind that $gu = F(u,v)$ and $gv = F(v,u)$, we get

$$\begin{aligned} &\varphi\big(\Omega(gu, gv, gv) + \Omega(gv, gu, gv)\big) \\ &\quad = \varphi\big(\Omega\big(F(u,v), F(v,u), F(v,u)\big) + \Omega\big(F(v,u), F(u,v), F(u,v)\big)\big) \\ &\quad \le \varphi\big(\Omega(gu, gv, gv) + \Omega(gv, gu, gv)\big) - \psi\big(\Omega(gu, gv, gv) + \Omega(gv, gu, gv)\big), \end{aligned}$$

hence $\psi(\Omega(gu, gv, gv) + \Omega(gv, gu, gv)) = 0$, or $\Omega(gu, gv, gv) = 0$ and $\Omega(gv, gu, gv) = 0$. Applying Lemma 2.1, it follows that $gu = gv$. □

Taking $g = Id_X$, the identity mapping, in Theorem 3.3 we obtain a theorem of coupled fixed points.

Corollary 3.3 *Consider (X, G) to be a complete G-metric space endowed with an Ω-distance called Ω. Let $F : X \times X \to X$ be a mapping.*

Suppose that there exist $\varphi \in \Phi$ and $\psi \in \Psi$ such that

$$\begin{aligned} &\varphi\big(\Omega\big(F(x,y), F\big(x^*, y^*\big), F\big(z, z^*\big)\big) + \Omega\big(F(y,x), F\big(y^*, x^*\big), F\big(z^*, z\big)\big)\big) \\ &\quad \le \varphi\big(\Omega\big(x, x^*, z\big) + \Omega\big(y, y^*, z^*\big)\big) - \psi\big(\Omega\big(x, x^*, z\big) + \Omega\big(y, y^*, z^*\big)\big) \end{aligned}$$

holds for each $(x,y), (x^*, y^*), (z, z^*) \in X \times X$.

Additionally, suppose that if $F(u,v) \neq u$ *or* $F(v,u) \neq v$, *then*

$$\inf\{\Omega(x, F(x,y), u) + \Omega(y, F(y,x), v) : x, y \in X\} > 0.$$

Then F and g have a unique coupled coincidence point (u,v), *with* $F(u,v) = u = v = F(v,u)$.

Taking $\varphi = i_{[0,+\infty)}$, the identity function, in Theorem 3.3 and Corollary 3.3, we get the following results.

Corollary 3.4 *Consider* (X,G) *to be a G-metric space endowed with an* Ω*-distance called* Ω. *Let* $F : X \times X \to X$ *and* $g : X \to X$ *be two mappings with the properties* $F(X \times X) \subseteq gX$, *and gX is a complete subspace of X with respect to the topology induced by G.*

Suppose that there exists $\psi \in \Psi$ *such that*

$$\begin{aligned}&\Omega(F(x,y), F(x^*,y^*), F(z,z^*)) + \Omega(F(y,x), F(y^*,x^*), F(z^*,z)) \\ &\quad \leq \Omega(gx, gx^*, gz) + \Omega(gy, gy^*, gz^*) - \psi(\Omega(gx, gx^*, gz) + \Omega(gy, gy^*, gz^*))\end{aligned}$$

holds for each $(x,y), (x^*,y^*), (z,z^*) \in X \times X$.

Additionally, suppose that if $F(u,v) \neq gu$ *or* $F(v,u) \neq gv$, *then*

$$\inf\{\Omega(gx, F(x,y), gu) + \Omega(gy, F(y,x), gv) : x, y \in X\} > 0.$$

Then F and g have a unique coupled coincidence point (u,v), *with* $F(u,v) = gu = gv = F(v,u)$.

Corollary 3.5 *Consider* (X,G) *to be a complete G-metric space endowed with an* Ω*-distance called* Ω. *Let* $F : X \times X \to X$ *be a mapping.*

Suppose that there exist $\varphi \in \Phi$ *and* $\psi \in \Psi$ *such that*

$$\begin{aligned}&\Omega(F(x,y), F(x^*,y^*), F(z,z^*)) + \Omega(F(y,x), F(y^*,x^*), F(z^*,z)) \\ &\quad \leq \Omega(x, x^*, z) + \Omega(y, y^*, z^*) - \psi(\Omega(x, x^*, z) + \Omega(y, y^*, z^*))\end{aligned}$$

holds for each $(x,y), (x^*,y^*), (z,z^*) \in X \times X$.

Additionally, suppose that if $F(u,v) \neq u$ *or* $F(v,u) \neq v$, *then*

$$\inf\{\Omega(x, F(x,y), u) + \Omega(y, F(y,x), v) : x, y \in X\} > 0.$$

Then F and g have a unique coupled coincidence point (u,v), *with* $F(u,v) = u = v = F(v,u)$.

Corollary 3.6 *Consider* (X,G) *to be a G-metric space endowed with an* Ω*-distance called* Ω. *Let* $F : X \times X \to X$ *and* $g : X \to X$ *be two mappings with the properties* $F(X \times X) \subseteq gX$, *and gX is a complete subspace of X with respect to the topology induced by G.*

Suppose that there exists $k \in [0,1)$ *such that*

$$\begin{aligned}&\Omega(F(x,y), F(x^*,y^*), F(z,z^*)) + \Omega(F(y,x), F(y^*,x^*), F(z^*,z)) \\ &\quad \leq k(\Omega(gx, gx^*, gz) + \Omega(gy, gy^*, gz^*))\end{aligned}$$

holds for each $(x,y), (x^*,y^*), (z,z^*) \in X \times X$.

Additionally, suppose that if $F(u,v) \neq gu$ or $F(v,u) \neq gv$, then

$$\inf\{\Omega(gx, F(x,y), gu) + \Omega(gy, F(y,x), gv) : x, y \in X\} > 0.$$

Then F and g have a unique coupled coincidence point (u,v), with $F(u,v) = gu = gv = F(v,u)$.

Proof The proof follows from Corollary 3.4 by defining $\psi : [0,+\infty) \to [0,+\infty)$ via $\psi(t) = (1-k)t$. □

Corollary 3.7 *Consider (X,G) to be a complete G-metric space endowed with an Ω-distance called Ω. Let $F : X \times X \to X$ be a mapping.*

Suppose that there exists $k \in [0,1)$ such that

$$\begin{aligned}&\Omega(F(x,y), F(x^*,y^*), F(z,z^*)) + \Omega(F(y,x), F(y^*,x^*), F(z^*,z))\\&\quad \leq k(\Omega(x,x^*,z) + \Omega(y,y^*,z^*))\end{aligned}$$

holds for each $(x,y), (x^*,y^*), (z,z^*) \in X \times X$.

Additionally, suppose that if $F(u,v) \neq u$ or $F(v,u) \neq v$, then

$$\inf\{\Omega(x, F(x,y), u) + \Omega(y, F(y,x), v) : x, y \in X\} > 0.$$

Then F and g have a unique coupled coincidence point (u,v), with $F(u,v) = u = v = F(v,u)$.

Proof The proof follows from Corollary 3.5 by defining $\psi : [0,+\infty) \to [0,+\infty)$ via $\psi(t) = (1-k)t$. □

The following example supports our results.

Example 3.1 Take $X = \{0,1,2,3,\ldots\}$. Define $G : X \times X \times X \to [0,+\infty)$ by the formula

$$G(x,y,z) = \begin{cases} 0 & \text{if } x = y = z; \\ x + y + z & \text{if } x \neq y, \text{ or } x \neq z, \text{ or } y \neq z. \end{cases}$$

Define

$$\Omega : X \times X \times X \to X, \qquad \Omega(x,y,z) = x + 2\max\{y,z\}$$

and

$$T : X \to X, \qquad Tx = \begin{cases} 0 & \text{if } x = 0,1; \\ x - 1 & \text{if } x \geq 2. \end{cases}$$

Also, define $\varphi : [0,+\infty) \to [0,+\infty)$ via $\varphi(t) = t^2$ and $\psi : [0,+\infty) \to [0,+\infty)$ via $\psi(t) = t$. Then:

(1) (X,G) is a complete G-metric space.

(2) $\varphi \in \Phi$ and $\psi \in \Psi$.

(3) Ω is an Ω-distance function.

(4) If $u \neq Tu$, then

$$\inf\{\Omega(x, Tx, u) : x \in X\} > 0.$$

(5) The following inequality:

$$\varphi\Omega(Tx, Ty, Tz) \leq \varphi\Omega(x, y, z) - \psi\Omega(x, y, z)$$

holds for all $x, y, z \in X$.

Proof The proofs of (1) and (2) are clear. To prove part (3), consider $x, y, z, a \in X$. Since

$$x + 2\max\{y, z\} \leq x + 2a + a + 2\max\{y, z\},$$

we get

$$\Omega(x, y, z) \leq \Omega(x, a, a) + \Omega(a, y, z).$$

This finishes the proof of the first item of the definition of Ω-distance.

To prove the second item of the definition of Ω-distance, let $x, y \in X$ and (z_n) be any sequence in X converging to z with respect to the topology induced by G in X. Thus $z_n = z$ for all $n \in \mathbb{N}$ except finitely many terms. Therefore

$$x + 2\max\{y, z_n\} \to x + 2\max\{y, x\} \quad \text{as } n \to +\infty.$$

So, $\Omega(x, y, z_n) \to \Omega(x, y, z)$ and hence $\Omega(x, y, \cdot) : X \to [0, +\infty)$ is lower semi-continuous.

Similarly, we can show that $\Omega(x, \cdot, z) : X \to [0, +\infty)$ is lower semi-continuous.

To prove the last item of the definition of Ω-distance, consider $\varepsilon > 0$. Take $\delta = \frac{\varepsilon}{2}$. Given $x, y, z \in X$ such that $\Omega(x, a, a) \leq \delta$ and $\Omega(a, y, z) \leq \delta$, by the definition of a G-metric space, we have

$$\begin{aligned} G(x, y, z) &\leq G(x, a, a) + G(a, y, z) \\ &\leq x + 2a + a + y + z \\ &\leq x + 2a + a + 2\max\{y, z\} \\ &= \Omega(x, a, a) + \Omega(a, y, z) \\ &\leq \varepsilon. \end{aligned}$$

This completes the proof of an Ω-distance.

To prove part (4), given $u \in X$ such that $u \neq Tu$, then $u \neq 0$. Note that

$$\begin{aligned} &\inf\{\Omega(x, Tx, u) : x \in X\} \\ &\quad \geq \inf\{x + 2u + : x \in X\} \\ &\quad \geq 2u > 0. \end{aligned}$$

To prove part (5), given $x, y, z \in X$, we divide the proof into the following four cases.

Case 1: $x = y = z = 0$. Here, $\Omega(x,y,z) = 0$ and $\Omega(Tx,Ty,Tz) = 0$. Thus

$$\varphi\Omega(Tx,Ty,Tz) \le \varphi\Omega(x,y,z) - \psi\Omega(x,y,z).$$

Case 2: $x > 0$ and $y = z = 0$. Here, $\Omega(x,y,z) = x$ and $\Omega(Tx,Ty,Tz) = x - 1$. Since $(x-1)^2 \le x^2 - x$, we have

$$\varphi\Omega(Tx,Ty,Tz) \le \varphi\Omega(x,y,z) - \psi\Omega(x,y,z).$$

Case 3: $x = 0$ and y or z are not equal to 0. Without loss of generality, we may assume that $y \ge z$. Thus $y \ne 0$. Here, $\Omega(x,y,z) = 2y$ and $\Omega(Tx,Ty,Tz) = 2(y-1)$. Since $4(y-1)^2 \le 4y^2 - 2y$, we have

$$\varphi\Omega(Tx,Ty,Tz) \le \varphi\Omega(x,y,z) - \psi\Omega(x,y,z).$$

Case 4: x, y, z are all different from 0. Without loss of generality, we assume that $y \ge z$. Then $\Omega(x,y,z) = x + 2y$ and $\Omega(Tx,Ty,Tz) = x - 1 + 2(y-1)$. Since $(x-1)^2 \le x^2 - x$ and $4(y-1)^2 \le 4y^2 - 2y$, we have

$$\begin{aligned}
\varphi\Omega(Tx,Ty,Tz) &= \bigl[x - 1 + 2(y-1)\bigr]^2 \\
&= (x-1)^2 + 4(x-1)(y-1) + 4(y-1)^2 \\
&\le x^2 - x + 4xy + 4y^2 - 2y \\
&= (x+2y)^2 - (x+2y) \\
&= \varphi\Omega(x,y,z) - \psi\Omega(x,y,z).
\end{aligned}$$

Note that Example 3.1 satisfies all the hypotheses of Theorem 3.1. Thus T has a unique fixed point. Here, 0 is the unique fixed point of T. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
Both authors contributed equally and significantly in writing this article. Both authors read and approved the final manuscript.

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