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Iteration scheme for common fixed points of hemicontractive and nonexpansive operators in Banach spaces

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Dedicated to Professor Wataru Takahashi on the occasion of his seventieth birthday

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Abstract
The purpose of this paper is to characterize the conditions for the convergence of the iterative scheme in the sense of Agarwal *et al.* (J. Nonlinear Convex. Anal. 8(1): 61-79, 2007), associated with nonexpansive and ϕ-hemicontractive mappings in a nonempty convex subset of an arbitrary Banach space.

Keywords: modified iterative scheme; nonexpansive mappings; ϕ-hemicontractive mappings; Banach spaces

1 Preliminaries

Let K be a nonempty subset of an arbitrary Banach space X, and let X^* be its dual space. Let $T : X \to X$ be an operator. The symbols $D(T)$ and $R(T)$ stand for the domain and the range of T, respectively. We denote $F(T)$ by the set of fixed points of a single-valued mapping $T : K \to K$. We denote by J the normalized duality mapping from X to 2^{X^*} defined by

$$J(x) = \{f^* \in X^* : \langle x, f^* \rangle = \|x\|^2 = \|f^*\|^2\}.$$

Let $T : D(T) \subseteq X \to X$ be an operator.

Definition 1 T is called *L-Lipschitzian* if there exists $L \geq 0$ such that

$$\|Tx - Ty\| \leq L\|x - y\|$$

for all $x, y \in D(T)$. If $L = 1$, then T is called *non-expansive*, and if $0 \leq L < 1$, T is called *contraction*.

Definition 2 [1–3]

(i) T is said to be strongly pseudocontractive if there exists a $t > 1$ such that for each $x, y \in D(T)$, there exists $j(x - y) \in J(x - y)$ satisfying

$$\operatorname{Re}\langle Tx - Ty, j(x - y)\rangle \leq \frac{1}{t}\|x - y\|^2.$$

(ii) T is said to be strictly hemicontractive if $F(T) \neq \emptyset$ and if there exists a $t > 1$ such that for each $x \in D(T)$ and $q \in F(T)$, there exists $j(x-y) \in J(x-y)$ satisfying

$$\operatorname{Re}\langle Tx - q, j(x-q)\rangle \leq \frac{1}{t}\|x-q\|^2.$$

(iii) T is said to be ϕ-strongly pseudocontractive if there exists a strictly increasing function $\phi : [0,\infty) \to [0,\infty)$ with $\phi(0) = 0$ such that for each $x, y \in D(T)$, there exists $j(x-y) \in J(x-y)$ satisfying

$$\operatorname{Re}\langle Tx - Ty, j(x-y)\rangle \leq \|x-y\|^2 - \phi\big(\|x-y\|\big)\|x-y\|.$$

(iv) T is said to be ϕ-hemicontractive if $F(T) \neq \emptyset$ and if there exists a strictly increasing function $\phi : [0,\infty) \to [0,\infty)$ with $\phi(0) = 0$ such that for each $x \in D(T)$ and $q \in F(T)$, there exists $j(x-y) \in J(x-y)$ satisfying

$$\operatorname{Re}\langle Tx - q, j(x-q)\rangle \leq \|x-q\|^2 - \phi\big(\|x-q\|\big)\|x-q\|.$$

Clearly, each strictly hemicontractive operator is ϕ-hemicontractive.
For a nonempty convex subset K of a normed space $X, S : K \to K$ and $T : K \to K$,

(a) the Mann iteration scheme [4] is defined by the following sequence $\{x_n\}$:

$$\begin{cases} x_1 \in C, \\ x_{n+1} = (1-b_n)x_n + b_n Tx_n, \quad n \geq 1, \end{cases} \tag{M$_n$}$$

where $\{b_n\}$ is a sequence in $[0,1]$;

(b) the sequence $\{x_n\}$ defined by

$$\begin{cases} x_1 \in C, \\ y_n = (1-b'_n)x_n + b'_n Tx_n, \\ x_{n+1} = (1-b_n)x_n + b_n Ty_n, \quad n \geq 1, \end{cases} \tag{I$_n$}$$

where $\{b_n\}$, $\{b'_n\}$ are sequences in $[0,1]$ is known as the Ishikawa [2] iteration scheme;

(c) the sequence $\{x_n\}$ defined by

$$\begin{cases} x_1 \in C, \\ y_n = b'_n x_n + (1-b'_n)Tx_n, \\ x_{n+1} = b_n Tx_n + (1-b_n)Ty_n, \quad n \geq 1, \end{cases} \tag{ARS$_n$}$$

where $\{b_n\}$, $\{b'_n\}$ are sequences in $[0,1]$, is known as the Agarwal-O'Regan-Sahu [5] iteration scheme;

(d) the sequence $\{x_n\}$ defined by

$$\begin{cases} x_1 \in C, \\ y_n = b'_n x_n + (1-b'_n)Tx_n, \\ x_{n+1} = b_n Sx_n + (1-b_n)Ty_n, \quad n \geq 1, \end{cases} \tag{ARS$_n$}$$

where $\{b_n\}$, $\{b'_n\}$ are sequences in $[0,1]$, is known as the modified Agarwal-O'Regan-Sahu iteration scheme.

Chidume [1] established that the Mann iteration sequence converges strongly to the unique fixed point of T in case T is a Lipschitz strongly pseudo-contractive mapping from a bounded closed convex subset of L_p (or l_p) into itself. Afterwards, several authors generalized this result of Chidume in various directions [3, 6–12].

The purpose of this paper is to characterize conditions for the convergence of the iterative scheme in the sense of Agarwal *et al.* [5] associated with nonexpansive and ϕ-hemicontractive mappings in a nonempty convex subset of an arbitrary Banach space. Our results improve and generalize most results in recent literature [1, 3, 5, 6, 8, 9, 11, 12].

2 Main result

The following result is now well known.

Lemma 3 [13] *For all $x, y \in X$ and $j(x+y) \in J(x+y)$,*

$$\|x+y\|^2 \le \|x\|^2 + 2\operatorname{Re}\langle y, j(x+y)\rangle.$$

Now, we prove our main result.

Theorem 4 *Let K be a nonempty closed and convex subset of an arbitrary Banach space X, let $S: K \to K$ be nonexpansive, and let $T: K \to K$ be a uniformly continuous* ϕ-hemicontractive *mapping such that S and T have the common fixed point. Suppose that $\{b_n\}_{n=1}^{\infty}$ and $\{b'_n\}_{n=1}^{\infty}$ are sequences in $[0,1]$ satisfying conditions*

(i) $\lim_{n\to\infty}(1-b_n) = \lim_{n\to\infty} b'_n = 0$,

(ii) $\sum_{n=1}^{\infty}(1-b_n) = \infty$.

For any $x_1 \in K$, define the sequence $\{x_n\}_{n=1}^{\infty}$ inductively as follows:

$$\begin{cases} y_n = b'_n x_n + (1-b'_n)Tx_n, \\ x_{n+1} = b_n S x_n + (1-b_n)Ty_n, \quad n \ge 1. \end{cases} \tag{2.1}$$

Then the following conditions are equivalent:

(a) *$\{x_n\}_{n=1}^{\infty}$ converges strongly to the common fixed point q of S and T.*

(b) *$\{Sx_n\}_{n=1}^{\infty}$, $\{Tx_n\}_{n=1}^{\infty}$ and $\{Ty_n\}_{n=1}^{\infty}$ are bounded.*

Proof First, we prove that (a) implies (b).

Since T is ϕ-hemicontractive, it follows that $F(T)$ is a singleton. Let $F(S) \cap F(T) = \{q\}$ for some $q \in K$.

Suppose that $\lim_{n\to\infty} x_n = q$. Then the continuity of S and T yields that

$$\lim_{n\to\infty} Sx_n = q = \lim_{n\to\infty} Tx_n$$

and

$$\lim_{n\to\infty} y_n = \lim_{n\to\infty}\left[b'_n x_n + \left(1-b'_n\right)Tx_n\right] = q.$$

Thus, $\lim_{n\to\infty} Ty_n = q$. Therefore, $\{Sx_n\}_{n=1}^{\infty}$, $\{Tx_n\}_{n=1}^{\infty}$ and $\{Ty_n\}_{n=1}^{\infty}$ are bounded.

Second, we need to show that (b) implies (a). Suppose that $\{Sx_n\}_{n=1}^{\infty}$, $\{Tx_n\}_{n=1}^{\infty}$ and $\{Ty_n\}_{n=1}^{\infty}$ are bounded.

Put

$$M_1 = \|x_1 - q\| + \sup_{n\geq 1} \|Sx_n - q\| + \sup_{n\geq 1} \|Tx_n - q\| + \sup_{n\geq 1} \|Ty_n - q\|.$$

It is clear that $\|x_1 - q\| \leq M_1$. Let $\|x_n - q\| \leq M_1$. Next, we will prove that $\|x_{n+1} - q\| \leq M_1$.

Note that

$$\begin{aligned}
\|x_{n+1} - q\| &= \big\|b_n Sx_n + (1-b_n)Ty_n - q\big\| \\
&= \big\|b_n(Sx_n - q) + (1-b_n)(Ty_n - q)\big\| \\
&\leq b_n\|Sx_n - q\| + (1-b_n)\|Ty_n - q\| \\
&\leq \big(b_n + (1-b_n)\big)M_1 \\
&= M_1.
\end{aligned}$$

Thus, we can conclude that the sequence $\{x_n - q\}_{n\geq 1}$ is bounded, and hence, there is a constant $M > 0$ satisfying

$$M = \sup_{n\geq 1} \|x_n - q\| + \sup_{n\geq 1} \|Sx_n - q\| + \sup_{n\geq 1} \|Tx_n - q\| + \sup_{n\geq 1} \|Ty_n - q\|. \tag{2.2}$$

Let $w_n = \|Ty_n - Tx_{n+1}\|$ for each $n \geq 1$. The uniform continuity of T ensures that

$$\lim_{n\to\infty} w_n = 0, \tag{2.3}$$

because

$$\begin{aligned}
\|y_n - x_{n+1}\| &= \big\|b_n'(x_n - Tx_n) + (1-b_n)(Sx_n - Ty_n)\big\| \\
&\leq b_n'\|x_n - Tx_n\| + (1-b_n)\|Sx_n - Ty_n\| \\
&\leq 2M\big(b_n' + (1-b_n)\big) \\
&\to 0 \quad \text{as } n \to \infty.
\end{aligned}$$

By virtue of Lemma 3 and (2.1), we infer that

$$\begin{aligned}
\|x_{n+1} - q\|^2 &= \big\|b_n Sx_n + (1-b_n)Ty_n - q\big\|^2 \\
&= \big\|b_n(Sx_n - q) + (1-b_n)(Ty_n - q)\big\|^2 \\
&\leq b_n^2\|Sx_n - q\|^2 + 2(1-b_n)\operatorname{Re}\big\langle Ty_n - q, j(x_{n+1} - q)\big\rangle \\
&\leq b_n^2\|x_n - q\|^2 + 2(1-b_n)\operatorname{Re}\big\langle Ty_n - Tx_{n+1}, j(x_{n+1} - q)\big\rangle \\
&\quad + 2(1-b_n)\operatorname{Re}\big\langle Tx_{n+1} - q, j(x_{n+1} - q)\big\rangle \\
&\leq b_n^2\|x_n - q\|^2 + 2(1-b_n)\|Ty_n - Tx_{n+1}\|\|x_{n+1} - q\| \\
&\quad + 2(1-b_n)\|x_{n+1} - q\|^2 - 2(1-b_n)\phi\big(\|x_{n+1} - q\|\big)\|x_{n+1} - q\|
\end{aligned}$$

$$\leq b_n^2\|x_n - q\|^2 + 2M(1-b_n)w_n + 2(1-b_n)\|x_{n+1} - q\|^2$$
$$- 2(1-b_n)\phi\big(\|x_{n+1} - q\|\big)\|x_{n+1} - q\|. \tag{2.4}$$

The real function $f : [0,\infty) \to [0,\infty)$, $f(t) = t^2$ is increasing and convex. For all $a \in [0,1]$ and $t_1, t_2 > 0$, we have

$$\big((1-a)t_1 + at_2\big)^2 \leq (1-a)t_1^2 + at_2^2.$$

Hence,

$$\begin{aligned}
\|x_{n+1} - q\|^2 &= \big\|b_nSx_n + (1-b_n)Ty_n - q\big\|^2 \\
&= \big\|b_n(Sx_n - q) + (1-b_n)(Ty_n - q)\big\|^2 \\
&\leq b_n\|Sx_n - q\|^2 + (1-b_n)\|Ty_n - q\|^2 \\
&\leq b_n\|x_n - q\|^2 + (1-b_n)M^2,
\end{aligned} \tag{2.5}$$

where the second inequality holds by the convexity of $\|\cdot\|^2$.

By substituting (2.5) in (2.4), we get

$$\begin{aligned}
\|x_{n+1} - q\|^2 &\leq \big(b_n^2 + 2b_n(1-b_n)\big)\|x_n - q\|^2 \\
&\quad + 2M(1-b_n)\big(w_n + M(1-b_n)\big) \\
&\quad - 2(1-b_n)\phi\big(\|x_{n+1} - q\|\big)\|x_{n+1} - q\| \\
&= \big(1 - (1-b_n)^2\big)\|x_n - q\|^2 + 2M(1-b_n)\big(w_n + M(1-b_n)\big) \\
&\quad - 2(1-b_n)\phi\big(\|x_{n+1} - q\|\big)\|x_{n+1} - q\| \\
&\leq \|x_n - q\|^2 + 2M(1-b_n)\big(w_n + M(1-b_n)\big) \\
&\quad - 2(1-b_n)\phi\big(\|x_{n+1} - q\|\big)\|x_{n+1} - q\| \\
&= \|x_n - q\|^2 + (1-b_n)l_n - 2(1-b_n)\phi\big(\|x_{n+1} - q\|\big)\|x_{n+1} - q\|,
\end{aligned} \tag{2.6}$$

where

$$l_n = 2M\big(w_n + M(1-b_n)\big) \to 0, \tag{2.7}$$

as $n \to \infty$.

Let $\delta = \inf\{\|x_{n+1} - q\| : n \geq 0\}$. We claim that $\delta = 0$. Otherwise, $\delta > 0$. Thus, (2.7) implies that there exists a positive integer N_1 such that $l_n < \phi(\delta)\delta$ for each $n \geq N_1$. In view of (2.6), we conclude that

$$\|x_{n+1} - q\|^2 \leq \|x_n - q\|^2 - \phi(\delta)\delta(1-b_n), \quad n \geq N_1,$$

which implies that

$$\phi(\delta)\delta\sum_{n=N_1}^{\infty}(1-b_n) \leq \|x_{N_1} - q\|^2, \tag{2.8}$$

which contradicts (ii). Therefore, $\delta = 0$. Thus, there exists a subsequence $\{x_{n_i+1}\}_{n=1}^{\infty}$ of $\{x_{n+1}\}_{n=1}^{\infty}$ such that

$$\lim_{i\to\infty} x_{n_i+1} = q. \tag{2.9}$$

Let $\epsilon > 0$ be a fixed number. By virtue of (2.7) and (2.9), we can select a positive integer $i_0 > N_1$ such that

$$\|x_{n_{i_0}+1} - q\| < \epsilon, \qquad l_n < \phi(\epsilon)\epsilon, \quad n \geq n_{i_0}. \tag{2.10}$$

Let $p = n_{i_0}$. By induction, we show that

$$\|x_{p+m} - q\| < \epsilon, \quad m \geq 1. \tag{2.11}$$

Observe that (2.6) means that (2.11) is true for $m = 1$. Suppose that (2.11) is true for some $m \geq 1$. If $\|x_{p+m+1} - q\| \geq \epsilon$, by (2.6) and (2.10), we know that

$$\begin{aligned}
\epsilon^2 &\leq \|x_{p+m+1} - q\|^2 \\
&\leq \|x_{p+m} - q\|^2 + (1 - b_{p+m}) l_{p+m} \\
&\quad - 2(1 - b_{p+m})\phi(\|x_{p+m+1} - q\|)\|x_{p+m+1} - q\| \\
&< \epsilon^2 + (1 - b_{p+m})\phi(\epsilon)\epsilon - 2(1 - b_{p+m})\phi(\epsilon)\epsilon \\
&= \epsilon^2 - (1 - b_{p+m})\phi(\epsilon)\epsilon < \epsilon^2,
\end{aligned}$$

which is impossible. Hence, $\|x_{p+m+1} - q\| < \epsilon$. That is, (2.11) holds for all $m \geq 1$. Thus, (2.11) ensures that $\lim_{n\to\infty} x_n = q$. This completes the proof. □

Taking $S = I$ in Theorem 4, we get the following.

Corollary 5 *Let K be a nonempty closed and convex subset of an arbitrary Banach space X, and let $T : K \to K$ be a uniformly continuous ϕ-hemicontractive mapping. Suppose that $\{b_n\}_{n=1}^{\infty}$ and $\{b'_n\}_{n=1}^{\infty}$ are sequences in $[0,1]$ satisfying conditions (i)-(ii) of Theorem 4. For any $x_1 \in K$, define the sequence $\{x_n\}_{n=1}^{\infty}$ inductively as follows:*

$$\begin{cases} y_n = b'_n x_n + (1 - b'_n)Tx_n, \\ x_{n+1} = b_n x_n + (1 - b_n)Ty_n, \quad n \geq 1. \end{cases}$$

Then the following conditions are equivalent:

(a) $\{x_n\}_{n=1}^{\infty}$ *converges strongly to the unique fixed point q of T.*

(b) $\{Tx_n\}_{n=1}^{\infty}$ *is bounded.*

Remark 6

1. All the results can also be proved for the same iterative scheme with error terms.
2. The known results for strongly pseudocontractive mappings are weakened by the ϕ-hemicontractive mappings.

3. Our results hold in arbitrary Banach spaces, where as other known results are restricted for L_p (or l_p) spaces and q-uniformly smooth Banach spaces.
4. Theorem 4 is more general in comparison to the results of Agarwal *et al.* [5] in the context of the class of ϕ-hemicontractive mappings. Theorem 4 extends convergence results coercing ϕ-hemicontractive mappings in the literature in the framework of Agarwal-O'Regan-Sahu iteration process (see also [14–21]).

3 Applications

Theorem 7 *Let X be an arbitrary real Banach space, $S : X \to X$ be nonexpansive, and let $T : X \to X$ be uniformly continuous ϕ-strongly accretive operators, respectively. Suppose that $\{b_n\}_{n=1}^{\infty}$ and $\{b'_n\}_{n=1}^{\infty}$ are sequences in $[0,1]$ satisfying conditions (i)-(ii) of Theorem 4. For any $x_1 \in X$, define the sequence $\{x_n\}_{n=1}^{\infty}$ inductively as follows:*

$$\begin{cases} y_n = b'_n x_n + (1 - b'_n)(f + (I - T)x_n), \\ x_{n+1} = b_n(f + (I - S)x_n) + (1 - b_n)(f + (I - T)y_n), \quad n \geq 1, \end{cases}$$

where $f \in X$, and I is the identity operator. Then the following conditions are equivalent:

(a) *$\{x_n\}_{n=1}^{\infty}$ converges strongly to the solution of the system $Sx = f = Tx$.*
(b) *$\{(I - S)x_n\}_{n=1}^{\infty}$, $\{(I - T)x_n\}_{n=1}^{\infty}$ and $\{(I - T)y_n\}_{n=1}^{\infty}$ are bounded.*

Proof Suppose that x^* is the solution of the system $Sx = f = Tx$. Define $G, G' : X \to X$ by $Gx = f + (I - S)x$ and $G'x = f + (I - T)x$, respectively. Since S and T are nonexpansive and uniformly continuous ϕ-strongly accretive operators, respectively, so are G and G', then x^* is the common fixed point of G and G'. Thus, Theorem 7 follows from Theorem 4. □

Example 8 Let $X = \mathbb{R}$ be the reals with the usual norm and $K = [0,1]$. Define $S : K \to K$ by

$$Sx = \sin x \quad \text{for all } x \in K$$

and $T : K \to K$ by

$$Tx = x - \tan x \quad \text{for all } x \in K.$$

By the mean value theorem, we have

$$\left|T(x) - T(y)\right| \leq \sup_{c \in (0,1)} \left|T'(c)\right| |x - y| \quad \text{for all } x, y \in K.$$

Noticing that $T'(c) = 1 - \sec^2(c)$ and $1 < \sup_{c \in (0,1)} |T'(c)| = 2.4255$. Hence,

$$\left|T(x) - T(y)\right| \leq L|x - y| \quad \text{for all } x, y \in K,$$

where $L = 2.4255$. It is easy to verify that T is ϕ-hemicontractive mapping with $\phi : [0,\infty) \to [0,\infty)$ defined by $\phi(t) = \tan(t)$ for all $t \in [0,\infty)$. Moreover, 0 is the common fixed point of S and T. Let $\{b_n\}_{n=1}^{\infty}$ and $\{b'_n\}_{n=1}^{\infty}$ be sequences in $[0,1]$ defined by

$$b_n = 1 - \frac{1}{n} \quad \text{and} \quad b'_n = \frac{1}{n}, \quad n \geq 1.$$

Then $\{x_n\}_{n=1}^{\infty}$ defined by (2.1) in Theorem 4 converges to 0, which is the common fixed point of S and T.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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References
1. Chidume, CE: Iterative approximation of fixed point of Lipschitz strictly pseudocontractive mappings. Proc. Am. Math. Soc. **99**, 283-288 (1987)
2. Ishikawa, S: Fixed point by a new iteration method. Proc. Am. Math. Soc. **44**, 147-150 (1974)
3. Liu, LW: Approximation of fixed points of a strictly pseudocontractive mapping. Proc. Am. Math. Soc. **125**, 1363-1366 (1997)
4. Mann, WR: Mean value methods in iteration. Proc. Am. Math. Soc. **26**, 506-510 (1953)
5. Agarwal, RP, O'Regan, D, Sahu, DR: Iterative construction of fixed points of nearly asymptotically nonexpansive mappings. J. Nonlinear Convex Anal. **8**(1), 61-79 (2007)
6. Ciric, LB, Ume, JS: Ishikawa iterative process for strongly pseudocontractive operators in Banach spaces. Math. Commun. **8**, 43-48 (2003)
7. Liu, LS: Ishikawa and Mann iterative process with errors for nonlinear strongly accretive mappings in Banach spaces. J. Math. Anal. Appl. **194**, 114-125 (1995)
8. Liu, Z, Kim, JK, Kang, SM: Necessary and sufficient conditions for convergence of Ishikawa iterative schemes with errors to ϕ-hemicontractive mappings. Commun. Korean Math. Soc. **18**(2), 251-261 (2003)
9. Liu, Z, Xu, Y, Kang, SM: Almost stable iteration schemes for local strongly pseudocontractive and local strongly accretive operators in real uniformly smooth Banach spaces. Acta Math. Univ. Comen. **LXXVII**(2), 285-298 (2008)
10. Schu, J: On a theorem of C.E. Chidume concerning the iterative approximation of fixed points. Math. Nachr. **153**, 313-319 (1991)
11. Xue, Z: Iterative approximation of fixed point for ϕ-hemicontractive mapping without Lipschitz assumption. Int. J. Math. Math. Sci. **17**, 2711-2718 (2005)
12. Zhou, HY, Cho, YJ: Ishikawa and Mann iterative processes with errors for nonlinear ϕ-strongly quasi-accretive mappings in normed linear spaces. J. Korean Math. Soc. **36**, 1061-1073 (1999)
13. Xu, HK: Inequality in Banach spaces with applications. Nonlinear Anal. **16**, 1127-1138 (1991)
14. Kato, T: Nonlinear semigroups and evolution equations. J. Math. Soc. Jpn. **19**, 508-520 (1967)
15. Takahashi, W: Nonlinear Functional Analysis - Fixed Point Theory and Its Applications. Yokohama Publishers, Yokohama (2000)
16. Takahashi, W, Yao, J-C: Weak and strong convergence theorems for positively homogeneous nonexpansive mappings in Banach spaces. Taiwan. J. Math. **15**, 961-980 (2011)
17. Tan, KK, Xu, HK: Iterative solutions to nonlinear equations of strongly accretive operators in Banach spaces. J. Math. Anal. Appl. **178**, 9-21 (1993)
18. Xu, Y: Ishikawa and Mann iterative processes with errors for nonlinear strongly accretive operator equations. J. Math. Anal. Appl. **224**, 91-101 (1998)
19. Hussain, N, Rafiq, A: On modified implicit Mann iteration method involving strictly hemicontractive mappings in smooth Banach spaces. J. Comput. Anal. Appl. **15**(5), 892-902 (2013)
20. Hussain, N, Rafiq, A, Ciric, LB: Stability of the Ishikawa iteration scheme with errors for two strictly hemicontractive operators in Banach spaces. Fixed Point Theory Appl. **2012**, 160 (2012)
21. Khan, SH, Rafiq, A, Hussain, N: A three-step iterative scheme for solving nonlinear ϕ-strongly accretive operator equations in Banach spaces. Fixed Point Theory Appl. **2012**, 149 (2012)

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