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Fixed point of a pair of contractive dominated mappings on a closed ball in an ordered dislocated metric space

Muhammad Arshad[1], Abdullah Shoaib[1] and Ismat Beg[2*]

*Correspondence: ibeg@lums.edu.pk
[2]Centre for Mathematics and Statistical Sciences, Lahore School of Economics, Lahore, Pakistan
Full list of author information is available at the end of the article

Abstract
Common fixed point results for mappings satisfying locally contractive conditions on a closed ball in an ordered complete dislocated metric space have been established. The notion of dominated mappings is applied to approximate the unique solution of nonlinear functional equations. Our results improve several well-known conventional results.
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1 Introduction and preliminaries

Let $T: X \longrightarrow X$ be a mapping. A point $x \in X$ is called a fixed point of T if $x = Tx$. Let x_0 be an arbitrarily chosen point in X. Define a sequence $\{x_n\}$ in X by a simple iterative method given by

$$x_{n+1} = Tx_n, \quad \text{where } n \in \{0, 1, 2, \ldots\}.$$

Such a sequence is called a Picard iterative sequence, and its convergence plays a very important role in proving the existence of a fixed point of a mapping T. A self-mapping T on a metric space X is said to be a Banach contraction mapping if

$$d(Tx, Ty) \leq kd(x, y)$$

holds for all $x, y \in X$, where $0 \leq k < 1$.

Fixed points results of mappings satisfying a certain contractive condition on the entire domain have been at the center of rigorous research activity (for example, see [1–12]) and they have a wide range of applications in different areas such as nonlinear and adaptive control systems, parameter estimation problems, computing magnetostatic fields in a nonlinear medium and convergence of recurrent networks (see [13–15]).

From the application point of view, the situation is not yet completely satisfactory because it frequently happens that a mapping T is a contraction not on the entire space X but merely on a subset Y of X. However, if Y is closed and a Picard iterative sequence $\{x_n\}$ in X converges to some x in X, then by imposing a subtle restriction on the choice of x_0, one may force the Picard iterative sequence to stay eventually in Y. In this case, the closedness of Y coupled with some suitable contractive condition establishes the existence of a fixed point of T. Azam *et al.* [16] proved a significant result concerning the existence of fixed points of a mapping satisfying contractive conditions on a closed ball of a complete metric space. Recently, many results related to the fixed point theorem in complete metric spaces endowed with a partial ordering $\preceq$ appeared in literature. Ran and Reurings [17] proved an analogue of Banach's fixed point theorem in a metric space endowed with a partial order and gave applications to matrix equations. In this way, they weakened the usual contractive condition. Subsequently, Nieto *et al.* [18] extended this result in [17] for nondecreasing mappings and applied it to obtain a unique solution for a first-order ordinary differential equation with periodic boundary conditions. Thereafter, many works related to fixed point problems have also been considered in partially ordered metric spaces (see [17–23]). Indeed, they all deal with monotone mappings (either order-preserving or order-reversing) such that for some $x_0 \in X$, either $x_0 \preceq fx_0$ or $fx_0 \preceq x_0$, where f is a self-map on a metric space. To obtain a unique solution, they used an additional restriction that each pair of elements has a lower bound and an upper bound. We have not used these conditions in our results. In this paper we introduce a new condition of partial order.

On the other hand, the notion of a partial metric space was introduced by Matthews in [24]. In partial metric spaces, the distance of a point from itself may not be zero. He also proved a partial metric version of the Banach fixed point theorem. Karapınar *et al.* [25] have proved a common fixed point in partial metric spaces. Partial metric spaces have applications in theoretical computer science (see [26]). Altun *et al.* [20], Aydi [27], Samet *et al.* [28] and Paesano *et al.* [29] used the idea of a partial metric space and partial order and gave some fixed point theorems for the contractive condition on ordered partial metric spaces. Further useful results can be seen in [28]. To generalize a partial metric, Hitzler and Seda [30] introduced the concept of dislocated topologies and its corresponding generalized metric, named a dislocated metric, and established a fixed point theorem in complete dislocated metric spaces to generalize the celebrated Banach contraction principle. The notion of dislocated topologies has useful applications in the context of logic programming semantics (see [31]). Further useful results can be seen in [32–35]. The dominated mapping, which satisfies the condition $fx \preceq x$, occurs very naturally in several practical problems. For example, if x denotes the total quantity of food produced over a certain period of time and $f(x)$ gives the quantity of food consumed over the same period in a certain town, then we must have $fx \preceq x$. In this paper, we exploit this concept for contractive mappings [36] to generalize, extend and improve some classical fixed point results for two, three and four mappings in the framework of an ordered complete dislocated metric space X. Our results not only extend some primary theorems to ordered dislocated metric spaces, but also restrict the contractive conditions on a closed ball only. The concept of a dominated mapping has been applied to approximate the unique solution of nonlinear functional equations.

Consistent with [30, 32, 34] and [35], the following definitions and results will be needed in the sequel.

Definition 1.1 Let X be a nonempty set and let $d_l : X \times X \to [0,\infty)$ be a function, called a dislocated metric (or simply d_l-metric), if the following conditions hold for any $x, y, z \in X$:

(i) if $d_l(x,y) = 0$, then $x = y$,

(ii) $d_l(x,y) = d_l(y,x)$,

(iii) $d_l(x,y) \leq d_l(x,z) + d_l(z,y)$.

The pair (X, d_l) is then called a dislocated metric space. It is clear that if $d_l(x,y) = 0$, then from (i), $x = y$. But if $x = y$, $d_l(x,y)$ may not be 0.

Recently Sarma and Kumari [34] proved the results that establish the existence of a topology induced by a dislocated metric and the fact that this topology is metrizable. This topology has as a base the family of sets $\{B(x,\varepsilon) \cup \{x\} : x \in X, \varepsilon > 0\}$, where $B(x,\varepsilon)$ is an open ball and $B(x,\varepsilon) = \{y \in X : d_l(x,y) < \varepsilon\}$ for some $x \in X$ and $\varepsilon > 0$. Also, $\overline{B(x,\varepsilon)} = \{y \in X : d_l(x,y) \leq \varepsilon\}$ is a closed ball.

Also, Harandi [37] defined the concept of a metric-like space which is similar to a dislocated metric space. Each metric-like σ on X generates a topology τ_σ on X whose base is the family of open σ-balls

$$B_\sigma(x,\varepsilon) = \{y \in X : |\sigma(x,y) - \sigma(x,x)| < \varepsilon\}.$$

Definition 1.2 Let $p : X \times X \to R^+$, where X is a nonempty set. p is said to be a partial metric on X if for any $x,y,z \in X$:

(P_1) $p(x,x) = p(y,y) = p(x,y)$ if and only if $x = y$,

(P_2) $p(x,x) \leq p(x,y)$,

(P_3) $p(x,y) = p(y,x)$,

(P_4) $p(x,z) \leq p(x,y) + p(y,z) - p(y,y)$.

The pair (X,p) is then called a partial metric space.

Each partial metric p on X induces a T_0 topology p on X which has as a base the family of open balls $\{B_p(x,\varepsilon) : x \in X, \varepsilon > 0\}$, where $B_p(x,\varepsilon) = \{y \in X : p(x,y) < p(x,x) + \varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$.

It is clear that any partial metric is a d_l-metric. A basic example of a partial metric space is the pair (R^+, p), where $p(x,y) = \max\{x,y\}$ for all $x,y \in R^+$. It is also a d_l-metric. An example of a d_l-metric space which is not a partial metric is given below.

Example 1.3 If $X = R^+ \cup \{0\}$, then $d_l(x,y) = x + y$ defines a dislocated metric d_l on X. Note that this metric is not a partial metric as (P_2) is not satisfied.

From the examples and definitions, it is clear that any partial metric is a d_l-metric, whereas a d_l-metric may not be a partial metric. We also remark that for those d_l-metrics which are also partial metrics, we have $B_{d_l}(x,\varepsilon) \subseteq B_p(x,\varepsilon)$. Also, for any d_l-metric, $B_{d_l}(x,\varepsilon) \subseteq B_\sigma(x,\varepsilon)$. Thus it is better to find a fixed point on a closed ball defined by Hitzler in a d_l-metric because we restrict ourselves to applying the contractive condition on the smallest closed ball. In this way, we also weaken the contractive condition.

Definition 1.4 [30] A sequence $\{x_n\}$ in a d_l-metric space (X, d_l) is called a Cauchy sequence if given $\varepsilon > 0$, there corresponds $n_0 \in N$ such that for all $n,m \geq n_0$, we have $d_l(x_m,x_n) < \varepsilon$ or $\lim_{n,m\to\infty} d_l(x_n,x_m) = 0$.

Definition 1.5 [30] A sequence $\{x_n\}$ in a d_l-metric space converges with respect to d_l if there exists $x \in X$ such that $d_l(x_n, x) \to 0$ as $n \to \infty$. In this case, x is called the limit of $\{x_n\}$, and we write $x_n \to x$.

Definition 1.6 [30] A d_l-metric space (X, d_l) is called complete if every Cauchy sequence in X converges to a point in X.

In Harandi's sense, a sequence $\{x_n\}$ in the metric-like space (X, σ) converges to a point $x \in X$ if and only if $\lim_{n\to\infty} \sigma(x_n, x) = \sigma(x, x)$. The sequence $\{x_n\}_{n=0}^{\infty}$ of elements of X is called σ-Cauchy if the limit $\lim_{n,m\to\infty} \sigma(x_n, x_m)$ exists and is finite. The metric-like space (X, σ) is called complete if for each σ-Cauchy sequence $\{x_n\}_{n=0}^{\infty}$, there is some $x \in X$ such that

$$\lim_{n\to\infty} \sigma(x_n, x) = \sigma(x, x) = \lim_{n,m\to\infty} \sigma(x_n, x_m).$$

Romaguera [38] has given the idea of a 0-Cauchy sequence and a 0-complete partial metric space. Using his idea, we can observe the following:

(a) Every Cauchy sequence with respect to Hitzler is a Cauchy sequence with respect to Harandi.
(b) Every complete metric space with respect to Harandi is complete with respect to Hitzler. The following example shows that the converse assertions of (a) and (b) do not hold.

Example 1.7 Let $X = Q^{+} \cup \{0\}$ and let $d_l : X \times X \to X$ be defined by $d_l(x, y) = x + y$. Note that $\{x_n\} = (1 + \frac{1}{n})^n$ is a Cauchy sequence with respect to Harandi, but it is not a Cauchy sequence with respect to Hitzler. Also, every Cauchy sequence (with respect to Hitzler) in X converges to a point '0' in X. Hence X is complete with respect to Hitzler, but X is not complete with respect to Harandi as $\lim_{n\to\infty}(1 + \frac{1}{n})^n = e \notin X$.

Definition 1.8 Let X be a nonempty set. Then $(X, \preceq, d_l)$ is called an ordered dislocated metric space if (i) d_l is a dislocated metric on X and (ii) $\preceq$ is a partial order on X.

Definition 1.9 Let $(X, \preceq)$ be a partial ordered set. Then $x, y \in X$ are called comparable if $x \preceq y$ or $y \preceq x$ holds.

Definition 1.10 [39] Let $(X, \preceq)$ be a partially ordered set. A self-mapping f on X is called dominated if $fx \preceq x$ for each x in X.

Example 1.11 [39] Let $X = [0, 1]$ be endowed with the usual ordering and $f : X \to X$ be defined by $fx = x^n$ for some $n \in \mathbb{N}$. Since $fx = x^n \leq x$ for all $x \in X$, therefore f is a dominated map.

Definition 1.12 Let X be a nonempty set and $T, f : X \to X$. A point $y \in X$ is called a point of coincidence of T and f if there exists a point $x \in X$ such that $y = Tx = fx$. The mappings T, f are said to be weakly compatible if they commute at their coincidence point (*i.e.*, $Tfx = fTx$ whenever $Tx = fx$).

For $A \subset X$, we denote by $D(A)$ the set of all limit points of A and $\overline{A}$ closure of A in X. We state without proof the following simple facts due to [34].

Lemma 1.13 *A subset of A of a dislocated metric space is closed if and only if $\overline{A} = A$.*

Lemma 1.14 *The topology induced by a dislocated metric is a Hausdorff topology.*

Lemma 1.15 *Every closed ball in a complete dislocated metric space is complete.*

We also need the following results for subsequent use.

Lemma 1.16 [40] *Let X be a nonempty set and let $f : X \to X$ be a function. Then there exists a subset $E \subset X$ such that $fE = fX$ and $f : E \to X$ is one-to-one.*

Lemma 1.17 [1] *Let X be a nonempty set and let the mappings $S, T, f : X \to X$ have a unique point of coincidence v in X. If (S,f) and (T,f) are weakly compatible, then S, T, f have a unique common fixed point.*

Theorem 1.18 [36, p.303] *Let (X, d) be a complete metric space, let $S : X \to X$ be a mapping, let $r > 0$ and x_0 be an arbitrary point in X. Suppose that there exists $k \in [0,1)$ with*

$$d(Sx, Sy) \leq kd(x, y) \quad \text{for all } x, y \in Y = \overline{B(x_0, r)}$$

and $d(x_0, Sx_0) < (1-k)r$. Then there exists a unique point x^* in $\overline{B(x_0, r)}$ such that $x^* = Sx^*$.

2 Fixed points of contractive mappings

Theorem 2.1 *Let $(X, \preceq, d_l)$ be an ordered complete dislocated metric space, let $S, T : X \to X$ be dominated maps and let x_0 be an arbitrary point in X. Suppose that for $k \in [0,1)$ and for $S \neq T$, we have*

$$d_l(Sx, Ty) \leq kd_l(x, y) \quad \text{for all comparable elements } x, y \text{ in } \overline{B(x_0, r)} \tag{2.1}$$

and

$$d_l(x_0, Sx_0) \leq (1-k)r. \tag{2.2}$$

If for a non-increasing sequence $\{x_n\}$ in $\overline{B(x_0, r)}$, $\{x_n\} \to u$ implies that $u \preceq x_n$, then there exists $x^* \in \overline{B(x_0, r)}$ such that $d_l(x^*, x^*) = 0$ and $x^* = Sx^* = Tx^*$. Also if, for any two points x, y in $\overline{B(x_0, r)}$, there exists a point $z \in \overline{B(x_0, r)}$ such that $z \preceq x$ and $z \preceq y$, that is, every pair of elements has a lower bound, then x^* is a unique common fixed point in $\overline{B(x_0, r)}$.

Proof Choose a point x_1 in X such that $x_1 = Sx_0$. As $Sx_0 \preceq x_0$, so $x_1 \preceq x_0$ and let $x_2 = Tx_1$. Now $Tx_1 \preceq x_1$ gives $x_2 \preceq x_1$. Continuing this process, we construct a sequence x_n of points in X such that

$$x_{2i+1} = Sx_{2i}, \qquad x_{2i+2} = Tx_{2i+1} \quad \text{and} \quad x_{2i+1} = Sx_{2i} \preceq x_{2i}, \quad \text{where } i = 0, 1, 2, \ldots.$$

First we show that $x_n \in \overline{B(x_0, r)}$ for all $n \in N$. Using inequality (2.2), we have

$$d_l(x_0, x_1) \leq (1-k)r \leq r.$$

It follows that

$$x_1 \in \overline{B(x_0, r)}.$$

Let $x_2, \ldots, x_j \in \overline{B(x_0, r)}$ for some $j \in N$. If $j = 2i + 1$, then $x_{2i+1} \preceq x_{2i}$, where $i = 0, 1, 2, \ldots, \frac{j-1}{2}$. So, using inequality (2.1), we obtain

$$\begin{aligned} d_l(x_{2i+1}, x_{2i+2}) &= d_l(Sx_{2i}, Tx_{2i+1}) \leq k\big[d_l(x_{2i}, x_{2i+1})\big] \\ &\leq k^2\big[d_l(x_{2i-1}, x_{2i})\big] \leq \cdots \leq k^{2i+1} d_l(x_0, x_1). \end{aligned} \tag{2.3}$$

If $j = 2i + 2$, then as $x_1, x_2, \ldots, x_j \in \overline{B(x_0, r)}$ and $x_{2i+2} \preceq x_{2i+1}$ $(i = 0, 1, 2, \ldots, \frac{j-2}{2})$. We obtain

$$d_l(x_{2i+2}, x_{2i+3}) \leq k^{2(i+1)} d_l(x_0, x_1). \tag{2.4}$$

Thus from inequalities (2.3) and (2.4), we have

$$d_l(x_j, x_{j+1}) \leq k^j d_l(x_0, x_1). \tag{2.5}$$

Now

$$\begin{aligned} d_l(x_0, x_{j+1}) &\leq d_l(x_0, x_1) + \cdots + d_l(x_j, x_{j+1}) \\ &\leq d_l(x_0, x_1) + \cdots + k^j d_l(x_0, x_1) \quad \text{(by (2.5))} \\ &\leq d_l(x_0, x_1)\big[1 + \cdots + k^{j-1} + k^j\big] \\ &\leq \frac{(1 - k^{j+1})}{1 - k} d_l(x_0, x_1) \\ &\leq \frac{(1 - k^{j+1})}{1 - k} (1 - k)r \quad \text{(by (2.2))} \\ &\leq \big(1 - k^{j+1}\big)r \leq r. \end{aligned}$$

Thus $x_{j+1} \in \overline{B(x_0, r)}$. Hence $x_n \in \overline{B(x_0, r)}$ for all $n \in N$. It implies that

$$d_l(x_n, x_{n+1}) \leq k^n d_l(x_0, x_1) \quad \text{for all } n \in N. \tag{2.6}$$

It implies that

$$\begin{aligned} d_l(x_n, x_{n+i}) &\leq d_l(x_n, x_{n+1}) + \cdots + d_l(x_{n+i-1}, x_{n+i}) \\ &\leq k^n d_l(x_0, x_1) + \cdots + k^{n+i-1} d_l(x_0, x_1) \quad \text{(by (2.6))} \\ &\leq k^n d_l(x_0, x_1)\big[1 + \cdots + k^{i-2} + k^{i-1}\big] \\ &\leq \frac{k^n(1 - k^i)}{1 - k} d_l(x_0, x_1) \longrightarrow 0 \quad \text{as } n \to \infty. \end{aligned}$$

Notice that the sequence $\{x_n\}$ is a Cauchy sequence in $(\overline{B(x_0, r)}, d_l)$. Therefore there exists a point $x^* \in \overline{B(x_0, r)}$ with $\lim_{n\to\infty} x_n = x^*$. Also,

$$\lim_{n\to\infty} d_l\big(x_n, x^*\big) = 0. \tag{2.7}$$

Now,

$$d_l(x^*, Sx^*) \leq d_l(x^*, x_{2n+2}) + d_l(x_{2n+2}, Sx^*).$$

On taking limit as $n \to \infty$ and using the fact that $x^* \preceq x_n$ when $x_n \to x^*$, we have

$$d_l(x^*, Sx^*) \leq \lim_{n\to\infty} \left[d_l(x^*, x_{2n+2}) + kd_l(x_{2n+1}, x^*)\right].$$

By equation (2.7), we obtain

$$d_l(x^*, Sx^*) \leq 0,$$

and hence $x^* = Sx^*$. Similarly, by using

$$d_l(x^*, Tx^*) \leq d_l(x^*, x_{2n+1}) + d_l(x_{2n+1}, Tx^*),$$

we can show that $x^* = Tx^*$. Hence S and T have a common fixed point in $\overline{B(x_0, r)}$. Now,

$$d_l(x^*, x^*) = d_l(Sx^*, Tx^*) \leq kd_l(x^*, x^*).$$

This implies that

$$d_l(x^*, x^*) = 0.$$

For uniqueness, assume that y is another fixed point of T and S in $\overline{B(x_0, r)}$. If x^* and y are comparable, then

$$\begin{aligned} d_l(x^*, y) &= d_l(Sx^*, Ty) \\ &\leq kd_l(x^*, y). \end{aligned}$$

This shows that $x^* = y$. Now if x^* and y are not comparable, then there exists a point $z_0 \in \overline{B(x_0, r)}$ such that $z_0 \preceq x^*$ and $z_0 \preceq y$. Choose a point z_1 in X such that $z_1 = Tz_0$. As $Tz_0 \preceq z_0$, so $z_1 \preceq z_0$and let $z_2 = Sz_1$. Now $Sz_1 \preceq z_1$ gives $z_2 \preceq z_1$. Continuing this process and having chosen z_n in X such that

$$z_{2i+1} = Tz_{2i}, \qquad z_{2i+2} = Sz_{2i+1} \quad \text{and} \quad z_{2i+1} = Tz_{2i} \preceq z_{2i}, \quad \text{where } i = 0, 1, 2, \ldots,$$

we obtain that $z_{n+1} \preceq z_n \preceq \cdots \preceq z_0 \preceq x^*$. As $z_0 \preceq x^*$ and $z_0 \preceq y$, it follows that $z_n \preceq Tx^*$ and $z_n \preceq Ty$ for all $n \in N$. We will prove that $z_n \in \overline{B(x_0, r)}$ for all $n \in N$ by using mathematical induction. For $n = 1$,

$$\begin{aligned} d_l(x_0, z_1) &\leq d_l(x_0, x_1) + d_l(x_1, z_1) \\ &\leq (1 - k)r + kd_l(x_0, z_0) \\ &\leq (1 - k)r + kr = r. \end{aligned}$$

It follows that $z_1 \in \overline{B(x_0, r)}$. Let $z_2, z_3, \dots, z_j \in \overline{B(x_0, r)}$ for some $j \in N$. Note that if j is odd, then

$$d_l(x_{j+1}, z_{j+1}) = d_l(Tx_j, Sz_j) \preceq kd_l(x_j, z_j) \preceq \cdots \preceq k^{j+1} d_l(x_0, z_0),$$

and if j is even, then

$$d_l(x_{j+1}, z_{j+1}) = d_l(Sx_j, Tz_j) \preceq kd_l(x_j, z_j) \preceq \cdots \preceq k^{j+1} d_l(x_0, z_0).$$

Now

$$\begin{aligned}
d_l(x_0, z_{j+1}) &\le d_l(x_0, x_1) + d_l(x_1, x_2) + \cdots + d_l(x_{j+1}, z_{j+1}) \\
&\le d_l(x_0, x_1) + kd_l(x_0, x_1) + \cdots + k^{j+1} d_l(x_0, z_0) \\
&\le d_l(x_0, x_1)\bigl[1 + k + \cdots + k^j\bigr] + k^{j+1} r \\
&\le (1-k) r \frac{(1 - k^{j+1})}{1-k} + k^{j+1} r, \\
d_l(x_0, z_{j+1}) &\le r,
\end{aligned}$$

which implies that

$$d_l(x_{j+1}, z_{j+1}) = d_l(Sx_j, Tz_j) \preceq kd_l(x_j, z_j) \preceq \cdots \preceq k^{j+1} d_l(x_0, z_0).$$

Thus $z_{j+1} \in \overline{B(x_0, r)}$. Hence $z_n \in \overline{B(x_0, r)}$ for all $n \in N$. As $z_0 \preceq x^*$ and $z_0 \preceq y$, it follows that $z_n \preceq T^n x^*$, $z_n \preceq S^n x^*$, $z_n \preceq S^n y$ and $z_n \preceq T^n y$ for all $n \in N$ as $S^n x^* = T^n x^* = x^*$ and $S^n y = T^n y = y$ for all $n \in N$. If n is odd, then

$$\begin{aligned}
d_l\bigl(x^*, y\bigr) &= d_l\bigl(T^n x^*, T^n y\bigr) \\
&\le d_l\bigl(T^n x^*, Sz_n\bigr) + d_l\bigl(Sz_n, T^n y\bigr) \\
&\le kd_l\bigl(T^{n-1} x^*, z_n\bigr) + kd_l\bigl(z_n, T^{n-1} y\bigr) \\
&= kd_l\bigl(S^{n-1} x^*, Tz_{n-1}\bigr) + kd_l\bigl(Tz_{n-1}, S^{n-1} y\bigr) \\
&\le k^2 d_l\bigl(S^{n-2} x^*, z_{n-1}\bigr) + k^2 d_l\bigl(z_{n-1}, S^{n-2} y\bigr) \\
&\;\;\vdots \\
&\le k^{n+1} d_l\bigl(x^*, z_0\bigr) + k^{n+1} d_l(z_0, y) \longrightarrow 0 \quad \text{as } n \to \infty.
\end{aligned}$$

So, $x^* = y$. Similarly, we can show that $x^* = y$ if n is even. Hence x^* is a unique common fixed point of T and S in $\overline{B(x_0, r)}$. □

Theorem 2.1 extends Theorem 1.18 to ordered complete dislocated metric spaces.

Example 2.2 Let $X = Q^+ \cup \{0\}$ be endowed with the order $(x_1, y_1) \preceq (x_2, y_2)$ if $x_1 \le x_2$, $y_1 \le y_2$. Let $S, T : X^2 \to X^2$ be defined by

$$S(x, y) = \begin{cases} (\frac{x}{7}, \frac{3y}{11}) & \text{if } x + y \le 1, \\ (x - \frac{1}{3}, y - \frac{3}{8}) & \text{if } x + y > 1 \end{cases}$$

and

$$T(x,y) = \begin{cases} (\frac{4x}{15}, \frac{2y}{7}) & \text{if } x+y \leq 1, \\ (x-\frac{1}{4}, y-\frac{1}{5}) & \text{if } x+y > 1. \end{cases}$$

Clearly, S and T are dominated mappings. Let $d_l : X^2 \times X^2 \to X$ be defined by $d_l((x_1,y_1),(x_2,y_2)) = x_1 + y_1 + x_2 + y_2$. Then it is easy to prove that (X^2, d_l) is a complete dislocated metric space. Let $(x_0,y_0) = (\frac{3}{7}, \frac{4}{7})$, $r = 2$, then

$$\overline{B\big((x_0,y_0),r\big)} = \big\{(x,y) \in X : x+y \leq 1\big\}$$

with $k = \frac{3}{10} \in [0,1)$,

$$(1-k)r = \left(1-\frac{3}{10}\right)2 = \frac{7}{5},$$

$$d_l\big((x_0,y_0), S(x_0,y_0)\big) = \frac{656}{539} < \frac{7}{5}.$$

Also, for all comparable elements $(x_1,y_1),(x_2,y_2) \in X^2$ such that $x_1+y_1 > 1$ and $x_2+y_2 > 1$, we have

$$\begin{aligned} d_l\big(S(x_1,y_1), T(x_2,y_2)\big) &= x_1 - \frac{1}{3} + y_1 - \frac{3}{8} + x_2 - \frac{1}{4} + y_2 - \frac{1}{5} \\ &\geq \frac{3}{10}\{x_1+y_1+x_2+y_2\}, \\ d_l(Sx,Ty) &\geq kd_l\big[(x_1,y_1),(x_2,y_2)\big]. \end{aligned}$$

So, the contractive condition does not hold on X^2. Now if $(x_1,y_1),(x_2,y_2) \in \overline{B((x_0,y_0),r)}$, then

$$\begin{aligned} d_l\big(S(x_1,y_1), T(x_2,y_2)\big) &= \frac{x_1}{7} + \frac{3y_1}{11} + \frac{4x_2}{15} + \frac{2y_2}{7} \\ &\leq \frac{3}{10}\{x_1+y_1+x_2+y_2\} = kd_l\big[(x_1,y_1),(x_2,y_2)\big]. \end{aligned}$$

Therefore, all the conditions of Theorem 2.1 are satisfied. Moreover, $(0,0)$ is the common fixed point of S and T. Also, note that for any metric d on X^2, the respective condition does not hold on $\overline{B((x_0,y_0),r)}$ since

$$\begin{aligned} d\left(S\left(\frac{2}{5},\frac{3}{5}\right), T\left(\frac{2}{5},\frac{3}{5}\right)\right) &= d\left(\left(\frac{2}{35},\frac{9}{55}\right),\left(\frac{8}{75},\frac{6}{35}\right)\right) \\ &> kd\left(\left(\frac{2}{5},\frac{3}{5}\right),\left(\frac{2}{5},\frac{3}{5}\right)\right) = 0 \quad \text{for any } k \in [0,1). \end{aligned}$$

Moreover, X^2 is not complete for any metric d on X^2.

Remark 2.3 If we impose a Banach-type contractive condition for a pair of mappings $S,T : X \to X$ on a metric space (X,d), that is,

$$d(Sx,Ty) \leq kd(x,y) \quad \text{for all } x,y \in X,$$

then it follows that $Sx = Tx$ for all $x \in X$ (that is, S and T are equal). Therefore the above condition fails to find common fixed points of S and T. However, the same condition in a dislocated metric space does not assert that $S = T$, which is seen in Example 2.2. Hence Theorem 2.1 cannot be obtained from a metric fixed point theorem.

Theorem 2.4 *Let $(X, \preceq, d_l)$ be an ordered complete dislocated metric space, let $S : X \to X$ be a dominated map and let x_0 be an arbitrary point in X. Suppose that there exists $k \in [0, 1)$ with*

$$d_l(Sx, Sy) \leq kd_l(x, y) \quad \text{for all comparable elements } x, y \text{ in } \overline{B(x_0, r)}$$

and

$$d_l(x_0, Sx_0) \leq (1 - k)r.$$

If, for a non-increasing sequence $\{x_n\}$ in $\overline{B(x_0, r)}$, $\{x_n\} \to u$ implies that $u \preceq x_n$, and also, for any two points x, y in $\overline{B(x_0, r)}$, there exists a point $z \in \overline{B(x_0, r)}$ such that every pair of elements has a lower bound, then there exists a unique fixed point x^* of S in $\overline{B(x_0, r)}$. Further, $d_l(x^*, x^*) = 0$.

Proof By following similar arguments to those we have used to prove Theorem 2.1, one can easily prove the existence of a unique fixed point x^* of S in $\overline{B(x_0, r)}$. □

In Theorem 2.1, condition (2.2) is imposed to restrict condition (2.1) only for x, y in $\overline{B(x_0, r)}$ and Example 2.2 explains the utility of this restriction. However, the following result relaxes condition (2.2) but imposes condition (2.1) for all comparable elements in the whole space X.

Theorem 2.5 *Let $(X, \preceq, d_l)$ be an ordered complete dislocated metric space, let $S, T : X \to X$ be the dominated map and let x_0 be an arbitrary point in X. Suppose that for $k \in [0, 1)$ and for $S \neq T$, we have*

$$d_l(Sx, Ty) \leq kd_l(x, y) \quad \text{for all comparable elements } x, y \text{ in } X.$$

Also, if for a non-increasing sequence $\{x_n\}$ in X, $\{x_n\} \to u$ implies that $u \preceq x_n$, and for any two points x, y in X, there exists a point $z \in X$ such that $z \preceq x$ and $z \preceq y$, then there exists a unique point x^* in X such that $x^* = Sx^* = Tx^*$. Further, $d_l(x^*, x^*) = 0$.

In Theorem 2.1, the condition 'for a non-increasing sequence, $\{x_n\} \to u$ implies that $u \preceq x_n$' and the existence of z or a lower bound is imposed to restrict condition (2.1) only for comparable elements. However, the following result relaxes these restrictions but imposes condition (2.1) for all elements in $\overline{B(x_0, r)}$. In Theorem 2.1, it may happen that S has more fixed points, but these fixed points of S are not the fixed points of T, because a common fixed point of S and T is unique, whereas without order we can obtain a unique fixed point of S and T separately, which is proved in the following theorem.

Theorem 2.6 *Let (X, d_l) be a complete dislocated metric space, let $S, T : X \to X$ be self-maps and let x_0 be an arbitrary point in X. Suppose that for $k \in [0,1)$ and for $S \neq T$, we have*

$$d_l(Sx, Ty) \leq kd_l(x,y) \quad \textit{for all elements } x, y \textit{ in } \overline{B(x_0, r)}$$

and

$$d_l(x_0, Sx_0) \leq (1-k)r.$$

Then there exists a unique $x^* \in \overline{B(x_0, r)}$ such that $d_l(x^*, x^*) = 0$ and $x^* = Sx^* = Tx^*$. Further, S and T have no fixed point other than x^*.

Proof By Theorem 2.1, $x^* = Sx^* = Tx^*$. Let y be another point such that $y = Ty$. Then

$$d_l\bigl(x^*, y\bigr) = d_l\bigl(Sx^*, Ty\bigr) \leq kd_l\bigl(x^*, y\bigr).$$

This shows that $x^* = y$. Thus T has no fixed point other than x^*. Similarly, S has no fixed point other than x^*. □

Now we apply our Theorem 2.1 to obtain a unique common fixed point of three mappings on a closed ball in an ordered complete dislocated metric space.

Theorem 2.7 *Let $(X, \preceq, d_l)$ be an ordered dislocated metric space, let S, T be self-mappings and let f be a dominated mapping on X such that $SX \cup TX \subset fX$, $Tx \preceq fx$, $Sx \preceq fx$, and let x_0 be an arbitrary point in X. Suppose that for $k \in [0,1)$ and for $S \neq T$, we have*

$$d_l(Sx, Ty) \leq kd_l(fx, fy) \tag{2.8}$$

for all comparable elements $fx, fy \in \overline{B(fx_0, r)} \subseteq fX$; and

$$d_l(fx_0, Tx_0) \leq (1-k)r. \tag{2.9}$$

If for a non-increasing sequence, $\{x_n\} \to u$ implies that $u \preceq x_n$, and for any two points z and x in $\overline{B(fx_0, r)}$, there exists a point $y \in \overline{B(fx_0, r)}$ such that $y \preceq z$ and $y \preceq x$, that is, every pair of elements in $\overline{B(fx_0, r)}$ has a lower bound in $\overline{B(fx_0, r)}$; if fX is a complete subspace of X and (S, f) and (T, f) are weakly compatible, then S, T and f have a unique common fixed point fz in $\overline{B(fx_0, r)}$. Also, $d_l(fz, fz) = 0$.

Proof By Lemma 1.16, there exists $E \subset X$ such that $fE = fX$ and $f : E \to X$ is one-to-one. Now, since $SX \cup TX \subset fX$, we define two mappings $g, h : fE \to fE$ by $g(fx) = Sx$ and $h(fx) = Tx$, respectively. Since f is one-to-one on E, then g, h are well defined. As $Sx \preceq fx$ implies that $g(fx) \preceq fx$ and $Tx \preceq fx$ implies that $h(fx) \preceq fx$, therefore g and h are dominated maps. Now $fx_0 \in \overline{B(fx_0, r)} \subseteq fX$. Then $fx_0 \in fX$. Let $y_0 = fx_0$, choose a point y_1 in fX such that $y_1 = h(y_0)$. As $h(y_0) \preceq y_0$, so $y_1 \preceq y_0$ and let $y_2 = g(y_1)$. Now $g(y_1) \preceq y_1$ gives $y_2 \preceq y_1$. Continuing this process and having chosen y_n in fX such that

$$y_{2i+1} = h(y_{2i}) \quad \text{and} \quad y_{2i+2} = g(y_{2i+1}), \quad \text{where } i = 0, 1, 2, \ldots,$$

then $y_{n+1} \preceq y_n$ for all $n \in N$. Following similar arguments of Theorem 2.1, $y_n \in \overline{B(fx_0, r)}$. Also, by inequality (2.9),

$$d_l\big(fx_0, h(fx_0)\big) \leq (1-k)r.$$

Note that for $fx, fy \in \overline{B(fx_0, r)}$, where fx, fy are comparable. Then by using inequality (2.8), we have

$$d_l\big(g(fx), h(fy)\big) \leq k d_l(fx, fy).$$

As fX is a complete space, all the conditions of Theorem 2.1 are satisfied, we deduce that there exists a unique common fixed point $fz \in \overline{B(fx_0, r)}$ of g and h. Also, $d_l(fz, fz) = 0$. Now $fz = g(fz) = h(fz)$ or $fz = Sz = Tz = fz$. Thus fz is the point of coincidence of S, T and f. Let $v \in \overline{B(fx_0, r)}$ be another point of coincidence of f, S and T, then there exists $u \in \overline{B(fx_0, r)}$ such that $v = fu = Su = Tu$, which implies that $fu = g(fu) = h(fu)$, a contradiction as $fz \in \overline{B(fx_0, r)}$ is a unique common fixed point of g and h. Hence $v = fz$. Thus S, T and f have a unique point of coincidence $fz \in \overline{B(fx_0, r)}$. Now, since (S, f) and (T, f) are weakly compatible, by Lemma 1.17 fz is a unique common fixed point of S, T and f. □

In a similar way, we can apply our Theorems 2.5 and 2.6 to obtain a unique common fixed point of three mappings in an ordered complete dislocated metric space and a unique common fixed point of three mappings on a closed ball in a complete dislocated metric space, respectively.

In the following theorem, we use Theorem 2.6 to establish the existence of a unique common fixed point of four mappings on a closed ball in a complete dislocated metric space. One cannot prove the following theorem for an ordered dislocated metric space in a way similar to that of Theorem 2.7. In order to prove the unique common fixed point of four mappings on a closed ball in an ordered dislocated metric space, we should prove that S and T have no fixed point other than x^* in Theorem 2.1.

Theorem 2.8 *Let (X, d_l) be a dislocated metric space and let S, T, g and f be self-mappings on X such that $SX, TX \subset fX = gX$. Assume that for x_0, an arbitrary point in X, and for $k \in [0, 1)$ and for $S \neq T$, the following conditions hold*:

$$d_l(Sx, Ty) \leq k d_l(fx, gy) \tag{2.10}$$

for all elements $fx, gy \in \overline{B(fx_0, r)} \subseteq fX$; and

$$d_l(fx_0, Sx_0) \leq (1-k)r. \tag{2.11}$$

If fX is a complete subspace of X, then there exists $fz \in X$ such that $d_l(fz, fz) = 0$. Also, if (S, f) and (T, g) are weakly compatible, then S, T, f and g have a unique common fixed point fz in $\overline{B(fx_0, r)}$.

Proof By Lemma 1.16, there exist $E_1, E_2 \subset X$ such that $fE_1 = fX = gX = gE_2$, $f : E_1 \to X$, $g : E_2 \to X$ are one-to-one. Now define the mappings $A, B : fE_1 \to fE_1$ by $A(fx) = Sx$ and

$B(gx) = Tx$, respectively. Since f, g are one-to-one on E_1 and E_2, respectively, then the mappings A, B are well defined. As fX is a complete space, all the conditions of Theorem 2.6 are satisfied, we deduce that there exists a unique common fixed point $fz \in \overline{B(fx_0, r)}$ of A and B. Further, A and B have no fixed point other than fz. Also, $d_l(fz,fz) = 0$. Now $fz = A(fz) = B(fz)$ or $fz = Sz = fz$. Thus fz is a point of coincidence of f and S. Let $w \in \overline{B(fx_0, r)}$ be another point of coincidence of S and f, then there exists $u \in \overline{B(fx_0, r)}$ such that $w = fu = Su$, which implies that $fu = A(fu)$, a contradiction as $fz \in \overline{B(fx_0, r)}$ is a unique fixed point of A. Hence $w = fz$. Thus S and f have a unique point of coincidence $fz \in \overline{B(fx_0, r)}$. Since (S,f) are weakly compatible, by Lemma 1.17 fz is a unique common fixed point of S and f. As $fX = gX$, then there exists $v \in X$ such that $fz = gv$. Now, as $A(fz) = B(fz) = fz \Rightarrow A(gv) = B(gv) = gv \Rightarrow Tv = gv$, thus gv is the point of coincidence of T and g. Now, if $Tx = gx \Rightarrow B(gx) = gx$, a contradiction. This implies that $gv = gx$. As (T,g) are weakly compatible, we obtain gv, a unique common fixed point for T and g. But $gv = fz$. Thus S, T, g and f have a unique common fixed point $fz \in \overline{B(fx_0, r)}$. □

Corollary 2.9 *Let $(X, \preceq, d_l)$ be an ordered dislocated metric space, let S, T be self-mappings and let f be a dominated mapping on X such that $SX \cup TX \subset fX$, $Tx \preceq fx$, $Sx \preceq fx$, and let x_0 be an arbitrary point in X. Suppose that for $k \in [0,1)$ and for $S \neq T$, we have*

$$d_l(Sx, Ty) \leq kd_l(fx,fy)$$

for all comparable elements $fx,fy \in \overline{B(fx_0, r)} \subseteq fX$; and

$$d_l(fx_0, Tx_0) \leq (1-k)r.$$

If for a non-increasing sequence, $\{x_n\} \to u$ implies that $u \preceq x_n$, and for any two points z and x in $\overline{B(fx_0, r)}$, there exists a point $y \in \overline{B(fx_0, r)}$ such that $y \preceq z$ and $y \preceq x$; if fX is a complete subspace of X, then S, T and f have a unique point of coincidence $fz \in \overline{B(fx_0, r)}$. Also, $d_l(fz,fz) = 0$.

In a similar way, we can obtain a coincidence point result of four mappings as a corollary of Theorem 2.8.

A partial metric version of Theorem 2.1 is given below.

Theorem 2.10 *Let $(X, \preceq, p)$ be an ordered complete partial metric space, let $S, T : X \to X$ be dominated maps and let x_0 be an arbitrary point in X. Suppose that for $k \in [0,1)$ and for $S \neq T$,*

$$p(Sx, Ty) \leq kp(x, y) \quad \textit{for all comparable elements } x, y \textit{ in } \overline{B(x_0, r)}$$

and

$$p(x_0, Sx_0) \leq (1-k)\bigl[r + p(x_0, x_0)\bigr].$$

Then there exists $x^* \in \overline{B(x_0, r)}$ such that $p(x^*, x^*) = 0$. Also, if for a non-increasing sequence $\{x_n\}$ in $\overline{B(x_0, r)}$, $\{x_n\} \to u$ implies that $u \preceq x_n$, and for any two points x, y in $\overline{B(x_0, r)}$, there exists a point $z \in \overline{B(x_0, r)}$ such that $z \preceq x$ and $z \preceq y$, then there exists a unique point x^* in $\overline{B(x_0, r)}$ such that $x^* = Sx^* = Tx^*$.

A partial metric version of Theorem 2.7 is given below.

Theorem 2.11 *Let $(X, \preceq, p)$ be an ordered partial metric space, let S, T be self-mappings and let f be a dominated mapping on X such that $SX \cup TX \subset fX$ and $Tx, Sx \preceq fx$. Assume that for x_0, an arbitrary point in X, and for $k \in [0,1)$ and for $S \neq T$, the following conditions hold*:

$$p(Sx, Ty) \leq kp(fx, fy)$$

for all comparable elements $fx, fy \in \overline{B(fx_0, r)} \subseteq fX$; and

$$p(fx_0, Tx_0) \leq (1-k)\big[r + p(fx_0, fx_0)\big].$$

If for a non-increasing sequence, $\{x_n\} \to u$ implies that $u \preceq x_n$, also for any two points z and x in $\overline{B(fx_0, r)}$, there exists a point $y \in \overline{B(fx_0, r)}$ such that $y \preceq z$ and $y \preceq x$; if fX is complete subspace of X and (S, f) and (T, f) are weakly compatible, then S, T and f have a unique common fixed point fz in $\overline{B(fx_0, r)}$. Also, $p(fz, fz) = 0$.

Remark 2.12 We can obtain a partial metric version as well as a metric version of other theorems in a similar way.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
IB gave the idea. MA and AS wrote the initial draft. IB and MA finalized the manuscript. Correspondence was mainly done by IB. All authors read and approved the final manuscript.

Author details
[1]Department of Mathematics, International Islamic University, Islamabad, 44000, Pakistan. [2]Centre for Mathematics and Statistical Sciences, Lahore School of Economics, Lahore, Pakistan.

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