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Asymptotic pointwise contractive type in modular function spaces

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Abstract
In this paper, we introduce asymptotic pointwise contractive type conditions in modular function spaces and present fixed point results for mappings under such conditions.
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1 Introduction

The notion of asymptotic pointwise contraction was introduced by Kirk [1]: Let (M, d) be a metric space. A mapping $T : M \to M$ is called an asymptotic pointwise contraction if there exists a function $\alpha : M \to [0, 1)$ such that for each integer $n \geq 1$,

$$d(T^n x, T^n y) \leq \alpha_n(x) d(x, y) \quad \text{for each } x, y \in M,$$

where $\alpha_n \to \alpha$ pointwise on M. Moreover, Kirk and Xu [2] proved that if C is a weakly compact convex subset of a Banach space E and $T : C \to C$ an asymptotic pointwise contraction, then T has a unique fixed point $v \in C$, and for each $x \in C$, the sequence of Picard iterates $\{T^n x\}$ converges in norm to v.

Very recently, Saeidi [3] introduced the concept of (weak) asymptotic pointwise contraction type: Let (M, d) be a metric space. A mapping $T : M \to M$ is said to be of asymptotic pointwise contraction type (resp. of weak asymptotic pointwise contraction type) if T^N is continuous for some integer $N \geq 1$ and there exists a function $\alpha : M \to [0, 1)$ such that for each x in M,

$$\limsup_{n\to\infty} \sup_{y \in M} \{ d(T^n x, T^n y) - \alpha_n(x) d(x, y) \} \leq 0, \tag{1.1}$$

$$\Big(\text{resp. } \liminf_{n\to\infty} \sup_{y \in M} \{ d(T^n x, T^n y) - \alpha_n(x) d(x, y) \} \leq 0\Big), \tag{1.2}$$

where $\alpha_n \to \alpha$ pointwise on M.

It is easy to see that an asymptotic pointwise contraction is of asymptotic pointwise contraction type, but the converse is not true [3]. The following result was proved in [3].

Theorem 1.1 [3] *Let C be a nonempty weakly compact subset of a Banach space E, and let $T : C \to C$ be a mapping of weak asymptotic pointwise contraction type. Then T has a* *unique fixed point $v \in C$ and, for each $x \in C$, the sequence of Picard iterates $\{T^n x\}$ converges in norm to v.*

On the other hand, Khamsi and Kozlowski [4] studied the concept of asymptotic pointwise contractions in modular function spaces.

In this paper, motivated by Khamsi and Kozlowski [4, 5] and Saeidi [3], we study the notion of asymptotic pointwise contraction type in a modular function space. Moreover, we present fixed results which extend the earlier results in [3, 4].

2 Preliminaries

Let Ω be a nonempty set, and let Σ be a nontrivial σ-algebra of subsets of Ω. Let $\mathcal{P}$ be a δ-ring of subsets of Ω such that $E \cap A \in \mathcal{P}$ for any $E \in \mathcal{P}$ and $A \in \Sigma$. Let us assume that there exists an increasing sequence of sets $K_n \in \mathcal{P}$ such that $\Omega = \bigcup K_n$. By ξ we denote the linear space of all simple functions with supports from $\mathcal{P}$. By $\mathcal{M}_\infty$ we denote the space of all extended measurable function, *i.e.*, all function $f : \Omega \to [-\infty, +\infty]$ such that there exists a sequence $\{g_n\} \in \xi$, $|g_n| \le |f|$ and $g_n(\omega) \to f(\omega)$ for all $\omega \in \Omega$.

By 1_A we denote the characteristic function of the set A.

Definition 2.1 [6] Let $\rho : \mathcal{M}_\infty \to [0, \infty]$ be a nontrivial, convex and even function. We say that ρ is a regular convex function pseudomodular if:

(a) $\rho(0) = 0$;
(b) ρ is monotone, *i.e.*, $|f(\omega)| \le |g(\omega)|$ for all $\omega \in \Omega$ implies $\rho(f) \le \rho(g)$, where $f, g \in \mathcal{M}_\infty$;
(c) ρ is orthogonally subadditive, *i.e.*, $\rho(f 1_{A\cup B}) \le \rho(f 1_A) + \rho(f 1_B)$ for any $A, B \in \Sigma$ such that $A \cap B \neq \emptyset, f \in \mathcal{M}_\infty$;
(d) ρ has the Fatou property, *i.e.*, $|f_n(\omega)| \uparrow |f(\omega)|$ for all $\omega \in \Omega$ implies $\rho(f_n) \uparrow \rho(f)$, where $f \in \mathcal{M}_\infty$;
(e) ρ is order continuous in ξ, *i.e.*, $g_n \in \xi$ and $|g_n(\omega)| \downarrow 0$ implies $\rho(g_n) \downarrow 0$.

Similarly as in the case of measure spaces, we say that a set $A \in \Sigma$ is ρ-null if $\rho(g 1_A) = 0$ for every $g \in \xi$. We say that a property holds ρ-almost everywhere if the exceptional set is ρ-null. As usual we identify any pair of measurable sets whose symmetric difference is ρ-null as well as any pair of measurable functions differing only on a ρ-null set. With this in mind, we define

$$\mathcal{M}(\Omega, \Sigma, \mathcal{P}, \rho) = \{f \in \mathcal{M}_\infty : |f(\omega)| < \infty \ \rho\text{-a.e.}\},$$

where each $f \in \mathcal{M}(\Omega, \Sigma, \mathcal{P}, \rho)$ is actually an equivalence class of functions equal ρ-a.e. rather than an individual function. Where no confusion exists, we write $\mathcal{M}$ instead of $\mathcal{M}(\Omega, \Sigma, \mathcal{P}, \rho)$.

Definition 2.2 [4, 5] Let ρ be a regular function pseudomodular;

(a) we say that ρ is a regular convex function semimodular if $\rho(\alpha f) = 0$ for every $\alpha > 0$ implies $f = 0$ ρ-a.e.;
(b) we say that ρ is a regular convex function modular if $\rho(f) = 0$ implies $f = 0$ ρ-a.e.

The class of all nonzero regular convex function modulars on Ω is denoted by $\mathfrak{R}$.

Definition 2.3 [7–9] Let ρ be a convex function modular.

(a) A modular function space is the vector space $L_\rho(\Omega, \Sigma)$, or briefly L_ρ, defined by

$$L_\rho = \{f \in \mathcal{M} : \rho(\lambda f) \to 0 \text{ as } \lambda \to 0\}.$$

(b) The following formula defines a norm in L_ρ (frequently called Luxemburg norm):

$$\|f\|_\rho = \inf\{\alpha > 0; \rho(f/\alpha) \leq 1\}.$$

In the following theorem, we recall some of the properties of modular function spaces that will be used later on in this paper.

Lemma 2.4 [7–9] *Let $\rho \in \mathfrak{R}$. Defining $L_\rho^0 = \{f \in L_\rho; \rho(f, \cdot)$ is order continuous$\}$ and $E_\rho = \{f \in L_\rho; \lambda f \in L_\rho^0$ for every $\lambda > 0\}$, we have*

(i) $L_\rho \supset L_\rho^0 \supset E_\rho$;

(ii) *E_ρ has the Lebesgue property, i.e., $\rho(\alpha f, D_k) \to 0$, for $\alpha > 0, f \in E_\rho$ and $D_k \downarrow \varnothing$;*

(iii) *E_ρ is the closure of ξ (in the sense of $\|\cdot\|_\rho$).*

Definition 2.5 [4, 5] Let $\rho \in \mathfrak{R}$.

(a) We say that $\{f_n\}$ is ρ-convergent to f and write $f_n \to f(\rho)$ if and only if $\rho(f_n - f) \to 0$.

(b) A sequence $\{f_n\}$ where $f_n \in L_\rho$ is called ρ-Cauchy if $\rho(f_n - f_m) \to 0$ as $m, n \to \infty$.

(c) A set $C \subset L_\rho$ is called ρ-closed if for any sequence $\{f_n\}$ in C, the convergence $f_n \to f(\rho)$ implies that f belongs to C.

(d) A set $C \subset L_\rho$ is called ρ-bounded if $\sup\{\rho(f - g); f \in C, g \in C\} < \infty$.

(e) For a set $C \subset L_\rho$, the mapping $T : C \to C$ is called ρ-continuous if $f_n \to f(\rho)$, then $T(f_n) \to T(f)(\rho)$.

(f) A set $C \subset L_\rho$ is called ρ-a.e. closed if for any sequence $\{f_n\}$ in C which ρ-a.e. converges to some f, then we must have $f \in C$.

(g) A set $C \subset L_\rho$ is called ρ-a.e. compact if for any sequence $\{f_n\}$ in C, there exists a subsequence $\{f_{n_k}\}$ which ρ-a.e. converges to some $f \in C$.

(h) Let $f \in L_\rho$ and $C \subset L_\rho$. The ρ-distance between f and C is defined as

$$d_\rho(f, C) = \inf\{\rho(f - g); g \in C\}.$$

Let us recall that ρ-convergence does not necessarily imply ρ-Cauchy condition. Also, $f_n \to f$ does not imply in general $\lambda f_n \to \lambda f$, $\lambda > 1$.

Definition 2.6 [4] We say that L_ρ has the property (R) if and only if every nonincreasing sequence $\{C_n\}$ of nonempty, ρ-bounded, ρ-closed, convex subsets of L_ρ has nonempty intersection.

Definition 2.7 [4] We say that the function modular ρ is uniformly continuous if for every $\epsilon > 0$ and $L > 0$, there exists $\delta > 0$ such that

$$\left|\rho(g) - \rho(h + g)\right| \leq \epsilon \quad \text{if } \rho(h) \leq \delta \text{ and } \rho(g) \leq L.$$

Definition 2.8 [4] A function $\lambda : C \to [0,\infty]$, where $C \subset L_\rho$ is nonempty and ρ-closed, is called ρ-lower semicontinuous if for any $\alpha > 0$, the set $C_\alpha = \{f \in C; \lambda(f) \leq \alpha\}$ is ρ-closed.

It can be proved that ρ-lower semicontinuity is equivalent to the condition

$$\lambda(f) \leq \liminf_{n\to\infty} \lambda(f_n) \quad \text{provided } f, f_n \in C \text{ and } \rho(f - f_n) \to 0.$$

The following result plays an important role in the proof of the main results.

Lemma 2.9 [4] *Assume that $\rho \in \Re$ has the property (R). Let $C \subset L_\rho$ be nonempty, convex, ρ-closed and ρ-bounded. If $\varphi : C \to [0,\infty)$ is a ρ-lower semicontinuous convex function, then there exists $x_0 \in C$ such that*

$$\varphi(x_0) = \inf\bigl\{\varphi(x); x \in C\bigr\}.$$

Let us recall the notion of ρ-type.

Definition 2.10 [4] Let $C \subset L_\rho$ be convex and ρ-bounded. A function $\tau : C \to [0,\infty)$ is called a (ρ)-type (or shortly a type) if there exists a sequence $\{y_m\}$ of elements of C such that for any $z \in C$, the following holds:

$$\tau(z) = \limsup_{m\to\infty} \rho(y_m - z).$$

Lemma 2.11 [4] *Let $\rho \in \Re$ be uniformly continuous. Let $C \subset L_\rho$ be nonempty, convex, ρ-closed and ρ-bounded. Then any ρ-type $\tau : C \to [0,\infty)$ is ρ-lower semicontinuous in C.*

3 Asymptotic pointwise contractive type conditions in modular function spaces

Definition 3.1 [4] Let $\rho \in \Re$ and $C \subset L_\rho$ be non-empty and ρ-closed. A mapping $T : C \to C$ is called an asymptotic pointwise mapping if there exists a sequence of mappings $\alpha_n : C \to [0,1]$ such that

$$\rho\bigl(T^n f - T^n g\bigr) \leq \alpha_n(f)\rho(f - g) \quad \text{for any } f, g \in C.$$

(a) If $\{\alpha_n\}$ converges pointwise to $\alpha : C \to [0,1)$, then T is called asymptotic pointwise ρ-contraction.
(b) If $\limsup_{n\to\infty} \alpha_n(f) \leq 1$ for any $f \in C$, then T is called asymptotic pointwise nonexpansive.
(c) If $\limsup_{n\to\infty} \alpha_n(f) \leq k$ for any $f \in C$ with $0 < k < 1$, then T is called strongly asymptotic pointwise ρ-contraction.

Khamsi and Kozlowski proved the following results in modular function spaces.

Theorem 3.2 [4] *Let $C \subset L_\rho$ be nonempty, ρ-closed and ρ-bounded. Let $T : C \to C$ be an asymptotic pointwise ρ-contraction. Then T has at most one fixed point in C. Moreover, if x_0 is a fixed point of T, then the orbit $\{T^n x\}$ is ρ-convergent to x_0 for any $x \in C$.*

Theorem 3.3 [4] *Let us assume that $\rho \in \Re$ is uniformly continuous and has the property (R). Let $C \subset L_\rho$ be nonempty, convex, ρ-closed and ρ-bounded. Let $T : C \to C$ be an asymptotic pointwise ρ-contraction. Then T has a unique fixed point $x_0 \in C$. Moreover, the orbit $\{T^n x\}$ is ρ-convergent to x_0 for any $x \in C$.*

Below, we introduce the notion of asymptotic pointwise ρ-contraction type in modular function spaces.

Definition 3.4 Let $C \subset L_\rho$ be nonempty, ρ-bounded and ρ-closed. A mapping $T : C \to C$ is said to be of asymptotic pointwise ρ-contraction type (resp. of weak asymptotic pointwise ρ-contraction type) if T^N is ρ-continuous for some integer $N \geq 1$ and there exists a function $\alpha : C \to [0, 1)$ such that, for each x in C,

$$\limsup_{n\to\infty} \sup_{y\in C} \bigl\{\rho\bigl(T^n x - T^n y\bigr) - \alpha_n(x)\rho(x - y)\bigr\} \leq 0, \tag{3.1}$$

$$\Bigl(\text{resp. } \liminf_{n\to\infty} \sup_{y\in C} \bigl\{\rho\bigl(T^n x - T^n y\bigr) - \alpha_n(x)\rho(x - y)\bigr\} \leq 0\Bigr), \tag{3.2}$$

where $\alpha_n \to \alpha$ pointwise on M.

Taking

$$r_n(x) = \sup_{y\in M} \bigl\{\rho\bigl(T^n x - T^n y\bigr) - \alpha_n(x)\rho(x - y)\bigr\} \in \mathbb{R}^+ \cup \{\infty\},$$

it can be easily seen from (3.1) (resp. (3.2)) that

$$\lim_{n\to\infty} r_n(x) = 0, \tag{3.3}$$

$$\Bigl(\text{resp. } \liminf_{n\to\infty} r_n(x) \leq 0\Bigr) \tag{3.4}$$

for all $x \in M$, and

$$\rho\bigl(T^n x - T^n y\bigr) \leq \alpha_n(x)\rho(x - y) + r_n(x). \tag{3.5}$$

We will obtain fixed point results for these mappings in modular function spaces.

First, it is worth mentioning that the ρ-limit of any ρ-convergent sequence in L_ρ is unique. This fact follows from the following reasoning: Assume that $\rho(u_n - u) \to 0$ and $\rho(u_n - v) \to 0$. Then

$$\rho\left(\frac{u - v}{2}\right) \leq \frac{1}{2}\rho(u - u_n) + \frac{1}{2}\rho(v - u_n) \to 0,$$

which implies that $u = v$.

The following theorem is our main result.

Theorem 3.5 *Let $\rho \in \Re$ be uniformly continuous and have the property (R). Let $C \subset L_\rho$ be nonempty, convex, ρ-closed and ρ-bounded. Let $T : C \to C$ be a mapping of weak asymptotic pointwise ρ-contraction type. Then T has a unique fixed point $v \in C$ and, for each $x \in C$, the sequence of Picard iterates $\{T^n x\}$ is ρ-convergent to v.*

Proof Fix an $x \in C$ and define a function τ by

$$\tau(u) = \limsup_{n\to\infty} \rho\bigl(T^n x - u\bigr), \quad u \in C.$$

By Lemma 2.11, τ is ρ-lower semicontinuous in C. By Lemma 2.9, then there exists $x_0 \in C$ such that

$$\tau(x_0) = \inf\bigl\{\tau(x); x \in C\bigr\}.$$

Let us prove that $\tau(x_0) = 0$. Indeed, for any $n, m \geq 1$, we have

$$\begin{aligned}\tau\bigl(T^m x_0\bigr) &= \limsup_{n\to\infty} \rho\bigl(T^n x - T^m x_0\bigr)\\ &= \limsup_{n\to\infty} \rho\bigl(T^{m+n} x - T^m x_0\bigr)\\ &= \limsup_{n\to\infty} \rho\bigl(T^m\bigl(T^n x\bigr) - T^m x_0\bigr)\\ &\leq \limsup_{n\to\infty} \alpha_m(x_0)\rho\bigl(T^n x - x_0\bigr) + r_m(x_0)\\ &= \alpha_m(x_0)\tau(x_0) + r_m(x_0),\end{aligned}$$

which implies

$$\tau(x_0) = \inf\bigl\{\tau(x); x \in C\bigr\} \leq \tau\bigl(T^m x_0\bigr) \leq \alpha_m(x_0)\tau(x_0) + r_m(x_0). \tag{3.6}$$

Since T is of weak asymptotic pointwise ρ-contraction type, by (3.4) we have $\liminf_{n\to\infty} r_m(x_0) \leq 0$. Thus, for a subsequence $\{r_{m_k}(x_0)\}$ of $\{r_m(x_0)\}$, we have

$$\lim_{k\to\infty} r_{m_k}(x_0) \leq 0. \tag{3.7}$$

Now, by (3.6) and (3.7), we obtain

$$\tau(x_0) \leq \liminf_{k\to\infty}\bigl[\alpha_{m_k}(x_0)\tau(x_0) + r_{m_k}(x_0)\bigr] = \alpha(x_0)\tau(x_0),$$

which forces $\tau(x_0) = 0$ as $\alpha(x_0) < 1$. Hence $\rho(T^n x - x_0) \to 0$ as $n \to \infty$. From this and the continuity of T^N, for some $N \geq 1$, it follows that $\rho(T^{N+n} x - T^N x_0) \to 0$ as $n \to \infty$. Since the ρ-limit of any ρ-convergent sequence is unique, we must have $T^N x_0 = x_0$, namely, x_0 is a fixed point of T^N. Now, repeating the above proof for x_0 instead of x, we deduce that $T^n x_0$ is ρ-convergent to a member ν of C; *i.e.*, $\rho(T^n x_0 - \nu) \to 0$. But $T^{kN} x_0 = x_0$ for all $k \geq 1$. Hence, $\nu = x_0$ and then $T^n x_0 \to x_0(\rho)$.

We show that $Tx_0 = x_0$; for this purpose, consider an arbitrary $\epsilon > 0$. Then there exists a $k_0 > 0$ such that $\rho(T^n x_0 - x_0) < \epsilon$ for all $n > k_0$. So, by choosing a natural number $k > k_0/N$, we obtain

$$\rho(Tx_0 - x_0) = \rho\bigl(T\bigl(T^{kN} x_0\bigr) - x_0\bigr) = \rho\bigl(T^{kN+1} x_0 - x_0\bigr) < \epsilon.$$

Since the choice of $\epsilon > 0$ is arbitrary and $\rho \in \Re$, we get $Tx_0 = x_0$.

It is easy to verify that T can have only one fixed point. Indeed, if $u, v \in C$ are fixed points of T, then by (3.5), we have

$$\rho(u - v) = \rho\big(T^n u - T^n v\big) \le \alpha_n(u)\rho(u - v) + r_n(u), \quad \forall n \ge 1.$$

Taking $\liminf$ in the above inequality, we obtain

$$\rho(u - v) \le \alpha(u)\rho(u - v).$$

Since $\alpha(u) < 1$ and $\rho \in \mathfrak{R}$, we immediately get $u = v$. □

Next, using the ρ-a.e. strong Opial property of the function modular, we prove a fixed point theorem which does not assume the uniform continuity of ρ.

Definition 3.6 [4, 10] We say that L_ρ satisfies the ρ-a.e. strong Opial property (or shortly SO-property) if for every $\{f_n\} \in L_\rho$ which is ρ-a.e. convergent to zero such that there exists a $\beta > 1$ for which

$$\sup\big\{\rho(\beta f_n)\big\} < \infty,$$

the following equality holds for any $g \in L_\rho$:

$$\liminf_{n\to\infty}(f_n + g) = \liminf_{n\to\infty}\rho(f_n) + \rho(g).$$

Lemma 3.7 [4] *Let $\rho \in \mathfrak{R}$. Assume that L_ρ has the ρ-a.e. strong Opial property. Let $C \subset E_\rho$ be a nonempty, ρ-a.e. compact subset such that there exists $\beta > 1$ such that $\delta_\rho(\beta C) = \sup\{\rho(\beta(x - y)); x, y \in C\} < \infty$. Let $D \subset C$ be a nonempty ρ-a.e. closed subset. For any $n \ge 1$, let $\lambda_n : D \to [0, \infty)$ be such that for any $y \in D$, there exists a sequence $\{y_n\} \subset C$ such that, for every $n \ge 1$, the following holds*:

$$\lambda_n(y) - \frac{1}{n} \le \rho(y - y_n),$$

and $\rho(x - y_n) \le \lambda_n(x)$ for every $x \in D$ and $n \ge 1$. Let $\lambda(x) = \limsup_{n\to\infty} \lambda_n(x)$ for any $x \in D$. Then there exists $x_0 \in D$ at which λ attains infimum, i.e.,

$$\lambda(x_0) = \inf\big\{\lambda(x); x \in D\big\}.$$

Theorem 3.8 *Let $\rho \in \mathfrak{R}$. Assume that L_ρ has the ρ-a.e. strong Opial property. Let $C \subset E_\rho$ be a nonempty ρ-a.e. compact convex subset such that $\delta_\rho(\beta C) = \sup\{\rho(\beta(x - y)); x, y \in C\} < \infty$ for some $\beta > 1$. Then any $T : C \to C$ of weak asymptotic pointwise ρ-contraction type has a unique fixed point $x_0 \in C$. Moreover, the orbit $\{T^n x\}$ is ρ-convergent to x_0 for any $x \in C$.*

Proof Fix an $x \in C$ and define a function τ by

$$\tau(u) = \limsup_{n\to\infty}\rho\big(T^n x - u\big), \quad u \in C.$$

By Lemma 3.7 applied with $\lambda(u) = \tau(u)$, $D = C$, $\lambda_n(u) = \rho(T^n x - u)$, and with $y_n = T^n x$ chosen for all $u \in C$, there exists $x_0 \in C$ such that

$$\tau(x_0) = \inf\{\tau(x); x \in C\}.$$

The rest of the proof is like the one used for Theorem 3.5. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All the authors contributed equally. All authors read and approved the final manuscript.

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