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A convergence result on random products of mappings in metric spaces

Mohamed Amine Khamsi^{1,2*} and Issam Louhichi²

* Correspondence:

mohamed@utep.edu

¹Department of Mathematical Sciences, The University of Texas At El Paso, El Paso, TX 79968, USA
Full list of author information is available at the end of the article

Abstract

Let X be a metric space and $\{T_1, \dots, T_N\}$ be a finite family of mappings defined on $D \subset X$. Let $r : \mathbb{N} \rightarrow \{1, \dots, N\}$ be a map that assumes every value infinitely often. The purpose of this article is to establish the convergence of the sequence (x_n) defined by

$$x_0 \in D; \text{ and } x_{n+1} = T_{r(n)}(x_n), \text{ for all } n \geq 0.$$

In particular, we extend the study of Bauschke [1] from the linear case of Hilbert spaces to metric spaces. Similarly we show that the examples of convergence hold in the absence of compactness. These type of methods have been used in areas like computerized tomography and signal processing.

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1. Introduction

Many problems in mathematics [2] and physical sciences [3-5] uses a technique known as finding common fixed point. Let X be a real Hilbert space and suppose T_1, \dots, T_N are pairwise distinct self-mappings of some nonempty and closed subset D of X . Suppose further that the fixed point set, $\text{Fix}(T_i) = \{x \in D : T_i(x) = x\}$, of each mapping T_i is nonempty and that $C = \text{Fix}(T_1) \cap \dots \cap \text{Fix}(T_N) \neq \emptyset$. The aim is to find a common fixed point of these mappings. One frequently employed approach is the following:

Let r be a random mapping for $\{1, \dots, N\}$, i.e., a surjective mapping from \mathbb{N} onto $\{1, \dots, N\}$ that takes each value in $\{1, \dots, N\}$ infinitely often. Then generate a random sequence $(x_n)_n$ by taken $x_0 \in D$ arbitrary, and

$$x_{n+1} = T_{r(n)}(x_n), \text{ for all } n > 0,$$

and hope that this sequence converges to a point in C . We call it a random or unrestricted product (resp. iteration). For products generated by using control

sequence, there are many results: for instance, cyclic control arises when $r(n) = n + 1 \pmod N$ (see, for example, [6]).

In general, this random product fails to have well convergence behavior. The first positive results were done in the case, when $D = X$ and each mapping T_i is the projection onto nonempty, closed, and convex subset C_i of X ; hence $\text{Fix}(T_i) = C_i$, $i = 1, \dots, N$. The problem of finding a common fixed point is then the well-known *Convex Feasibility Problem* (see, for example, [6]). In [7], there are several interesting applications of this problem. Some of the early known results in this case are:

1. Amemiya and Ando [8]: If each set C_i is a closed subspace, then the random product converges weakly to the projection onto C .
2. Bruck [9]: If some set C_i is compact, then the random product converges in norm to a point in C . If $N = 3$ and each set C_i is symmetric, then the random product converges weakly to some point in C .
3. Dye [10]: If the sets C_i are finite-dimensional subspaces, then the random product converges in norm to a point in C .
4. Dye and Reich [11]: If the sets C_i have a common *weak internal point* or if $N = 3$, then the random product converges weakly to a point in C .
5. Youla [12]: If the sets C_i have a common “inner point”, then the random product converges weakly to a point in C .
6. Aharoni and Censor [13], Flam and Zowe [14], Tseng [15], and Eisner et al. [16]: If X is finite dimensional, then the random product converges in norm to a point in C .

For more recent references, one can consult [17] and the references therein.

Dye et al. [18] were successful in their extension of Amemiya and Ando’s results from Hilbert spaces to Banach spaces. In this study, we try to do the same by extending Bauschke’s results [1] from linear Hilbert spaces to nonlinear metric spaces.

2. Basic definitions and results

Definition 2.1. Let (X, d) be a metric space and D be a nonempty subset of X . The mapping $T : D \rightarrow D$ is said to be projective w.r.t. $c_0 \in \text{Fix}(T)$ whenever

- (i) $d(T(x), c) \leq d(x, c)$, for any $x \in D$, and any $c \in \text{Fix}(T)$;
- (ii) for any bounded sequence $(x_n)_n \subset D$, we have

$$\lim_{n \rightarrow \infty} [d(x_n, c_0) - d(T(x_n), c_0)] = 0 \Rightarrow \lim_{n \rightarrow \infty} d(x_n, \text{Fix}(T)) = 0.$$

Moreover T is called a projective mapping if and only if T is projective w.r.t. any point in $\text{Fix}(T)$.

It is worth to mention that in [1] it is assumed that T is nonexpansive to be projective. In particular it is not immediate that T is continuous under our definition.

Example 2.1. Let X be a Hilbert space and Y be a closed subspace of X . Let $P : X \rightarrow Y$ be the orthogonal projection. Then $\text{Fix}(P) = Y$. For any $x \in X$ and $c \in Y$, we have

$$\|x - c\|^2 = \|P(x) - c\|^2 + \|x - P(x)\|^2.$$

Let $(x_n)_n$ be a bounded sequence such that $\lim_{n \rightarrow \infty} [\|x_n - c\| - \|P(x_n) - c\|] = 0$, for some $c \in Y$. Then we have

$$\lim_{n \rightarrow \infty} \|x_n - P(x_n)\| = 0.$$

Since $P(x) \in \text{Fix}(P)$, for any $x \in X$, we conclude that $\lim_{n \rightarrow \infty} d(x_n, \text{Fix}(P)) = 0$. In other words, P is a projective mapping. This example explains why the word projective is used in the above definition.

More examples of projective mappings may be found in [1]. We have the following result about projective mappings.

Proposition 2.1. *Let (X, d) be a complete metric space and D be a nonempty and closed subset of X . Let $T : D \rightarrow D$ be a projective mapping. Then for any $x \in D$, the iterates $(T^n(x))_n$ converge. Set $P(x) = \lim_{n \rightarrow \infty} T^n(x)$. Then $P : D \rightarrow \text{Fix}(T)$ is a retract which satisfies*

- (i) $P \circ T = T \circ P = P$;
- (ii) $d(P(x), c) \leq d(x, c)$, for any $x \in D$, and $c \in \text{Fix}(T)$.

Moreover P is nonexpansive whenever T is nonexpansive.

Proof. Let $x \in D$ and consider the iterates $(T^n(x))_n$. Then from the assumptions on T , we can see that $(d(T^n(x), c))_n$ is decreasing. So this sequence must be convergent. In particular we must have $\lim_{n \rightarrow \infty} [d(T^n(x), c) - d(T^{n+1}(x), c)] = 0$. Hence $\lim_{n \rightarrow \infty} d(T^n(x), \text{Fix}(T)) = 0$. Let us show that $(T^n(x))_n$ is a Cauchy sequence. Indeed, for any $n < m$ in \mathbb{N} , we have

$$d(T^n(x), T^m(x)) \leq d(T^n(x), c) + d(c, T^m(x)) \leq 2d(T^n(x), c),$$

for any $c \in \text{Fix}(T)$, which implies

$$d(T^n(x), T^m(x)) \leq d(T^n(x), c) + d(c, T^m(x)) \leq 2d(T^n(x), \text{Fix}(T)).$$

Since $\lim_{n \rightarrow \infty} d(T^n(x), \text{Fix}(T)) = 0$, we conclude that $(T^n(x))_n$ is indeed a Cauchy sequence. The completeness of X will imply that $(T^n(x))_n$ is convergent. Denote its limit by $P(x)$. It is easy to check that, since T is projective, $\text{Fix}(T)$ is closed. Moreover for each $n \geq 1$, there exists $c_n \in \text{Fix}(T)$ such that

$$d(T^n(x), c_n) \leq d(T^n(x), \text{Fix}(T)) + \frac{1}{n}.$$

Therefore $(c_n)_n$ also converges to $P(x)$. Since $\text{Fix}(T)$ is closed, we must have $P(x) \in \text{Fix}(T)$ which implies $T \circ P = P$. The fact that $P \circ T = P$ is obvious from the properties of the iterates. The last part (ii) follows from the fact

$$\lim_{n \rightarrow \infty} d(T^n(x), c) = \inf_{n \geq 0} d(T^n(x), c) = d(P(x), c),$$

for any $x \in D$ and $c \in \text{Fix}(T)$.

For a family of mappings, we have the following extension.

Definition 2.2. *Let (X, d) be a metric space and D be a nonempty subset of X . Let \mathcal{T} be a family of mappings defined from D into D . The fixed point set of \mathcal{T} is the set*

. We will say that \mathcal{T} is projective w.r.t. $c \in \text{Fix}(\mathcal{T})$ if any $T \in \mathcal{T}$ is projective w.r.t. c . Similarly, we will say that \mathcal{T} is projective if and only if any $T \in \mathcal{T}$ is a projective mapping w.r.t. any point in $\text{Fix}(\mathcal{T})$.

In the proof of the previous proposition, we used some interesting properties of the iterates. These properties are satisfied by what is known as Fejér sequences.

Definition 2.3. Let (X, d) be a metric space and D be a nonempty subset of X . A sequence $(x_n)_n \subset X$ is said to be Fejér monotone w.r.t. D if

$$d(x_{n+1}, c) \leq d(x_n, c); \text{ for all } c \in D \text{ and } n \geq 0.$$

Note that if $(x_n)_n$ is Fejér monotone then it is bounded, and $(d(x_n, D))_n$ is monotone decreasing.

Definition 2.4. Let (X, d) be a metric space. Let $\{C_1, \dots, C_N\}$ be a finite family of nonempty subsets of X . We will say that $\{C_1, \dots, C_N\}$ is boundedly regular, if for any bounded sequence $(x_n)_n \subset X$ we have

$$\lim_{n \rightarrow \infty} \max_{i=1, \dots, N} d(x_n, c_i) = 0 \Rightarrow \lim_{n \rightarrow \infty} d\left(x_n, \bigcap_{i=1}^N C_i\right) = 0.$$

We will say that $\{C_1, \dots, C_N\}$ is innately boundedly regular if $(C_i)_{i \in J}$ is boundedly regular for any nonempty subset J of $\{1, \dots, N\}$.

For examples on boundedly regular sets, the reader may consult [1]. In fact this property is the one that weakens the compactness assumption used in many results.

Next, we define the concept of quasi-projection as introduced by Baillon and Bruck [19].

Definition 2.5. Let (X, d) be a metric space and C be a nonempty subset of X . Let $x_0 \in X$.

The quasi-projection of x_0 onto C , denoted by $\mathcal{Q}_C(x_0)$, is defined by

$$\mathcal{Q}_C(x_0) = \{x \in C : d(x, c) \leq d(x_0, c), \text{ for any } c \in C\}.$$

Under the above assumptions, the definition of the projection is given by

$$P_C(x_0) = \{c \in C : d(x_0, c) = d(x_0, C)\}.$$

Clearly we have $P_C(x_0) \subset \mathcal{Q}_C(x_0)$. And if $x_0 \in C$, we have

$$P_C(x_0) = \mathcal{Q}_C(x_0) = \{x_0\}.$$

3. Main results

Definition 3.1. Let (X, d) be a metric space and D be a nonempty subset of X . Let $\{T_1, \dots, T_N\}$ be a finite family of mappings defined from D into D . A mapping $T : D \rightarrow D$ is called a full word if T can be written as a finite product of the mappings $\{T_1, \dots, T_N\}$, where each T_i will occur at least once. The set of all full word mappings will be denoted by \mathcal{F} . A mapping $T : D \rightarrow D$ is called a M -word, where $M \in \{1, \dots, N\}$, if T is a finite product of the mappings $\{T_1, \dots, T_N\}$, where at most M different $\{T_{i_1}, \dots, T_{i_M}\}$ occur in this product for some $\{i_1, \dots, i_M\} \subset \{1, \dots, N\}$. The set of all M -words will be denoted by \mathcal{W}_M .

Note that the identity map belongs to \mathcal{W}_M for any $M \in \{1, \dots, N\}$, and $\mathcal{F} \subset \mathcal{W}_N$.

Proposition 3.1. *Let (X, d) be a complete metric space. Let $\{T_1, \dots, T_N\}$ be a finite family of mappings defined from D into D , where D is a nonempty and closed subset of X . Assume that $\{T_1, \dots, T_N\}$ are projective w.r.t. a common fixed point $c \in D$. Also assume that $\{\text{Fix}(T_1), \dots, \text{Fix}(T_N)\}$ is innately boundedly regular. Let $(x_n)_n$ be a bounded sequence in D and $(W_n)_n \subset \mathcal{W}_N$ such that $\lim_{n \rightarrow \infty} [d(x_n, c) - d(W_n(x_n), c)] = 0$. Then*

$$(*) \quad \lim_{n \rightarrow \infty} d(x_n, W_n(x_n)) = 0.$$

Moreover if $(W_n)_n \subset \mathcal{F}$, then $\lim_{n \rightarrow \infty} d\left(x_n, \bigcap_{i=1}^N \text{Fix}(T_i)\right) = 0$.

Proof Consider the following statement $\mathcal{P}(M)$, where $M \in \{1, \dots, N\}$:

For any bounded sequence $(x_n)_n \subset D$ and any sequence of words $(W_n)_n \subset \mathcal{W}_N$ such that $\lim_{n \rightarrow \infty} [d(x_n, c) - d(W_n(x_n), c)] = 0$ we must have

$$\lim_{n \rightarrow \infty} d(x_n, W_n(x_n)) = 0.$$

The proof will be done via an induction argument on M . First we prove $\mathcal{P}(M)$, when $M = 1$. Assume it is not true. Then there exists a bounded sequence $(x_n)_n \subset D$, an $i \in \{1, \dots, N\}$ and a sequence of words $(W_n)_n$, where $W_n = T_i^{l_n}$ such that

$$\lim_{n \rightarrow \infty} [d(x_n, c) - d(W_n(x_n), c)] = 0$$

and $\lim_{n \rightarrow \infty} d(x_n, W_n(x_n)) \neq 0$. Note that we have

$$d(W_n(x_n), c) = d(T_i^{l_n}(x_n), c) \leq d(T_i(x_n), c) \leq d(x_n, c), \text{ for any } n \geq 0,$$

where the last inequality holds, since T_i is projective w.r.t. c . Thus, we have $\lim_{n \rightarrow \infty} [d(x_n, c) - d(T_i(x_n), c)] = 0$, which implies that $\lim_{n \rightarrow \infty} d(x_n, \text{Fix}(T_i)) = 0$. For any $z \in \text{Fix}(T_i)$, we have

$$d(x_n, W_n(x_n)) = d(x_n, T_i^{l_n}(x_n)) \leq d(x_n, z) + d(T_i^{l_n}(x_n), z) \leq 2d(x_n, z).$$

Since $z \in \text{Fix}(T_i)$ is arbitrary, we obtain $d(x_n, W_n(x_n)) \leq 2d(x_n, \text{Fix}(T_i))$. This is a contradiction. Therefore, $\mathcal{P}(M)$ is true for $M = 1$. Assume $\mathcal{P}(M - 1)$, for $M \in \{2, \dots, N\}$, is true and let us prove that $\mathcal{P}(M)$ is also true. Let $(x_n)_n \subset D$ be a bounded sequence and $(W_n)_n \subset \mathcal{W}_N$ be any sequence of words such that $\lim_{n \rightarrow \infty} [d(x_n, c) - d(W_n(x_n), c)] = 0$. From the definition of \mathcal{W}_M , there exists $\{i_1, \dots, i_M\} \subset \{1, \dots, N\}$ such that each W_n is a full word of $\{T_{i_1}, \dots, T_{i_M}\}$ Fix $i_m \in \{i_1, \dots, i_M\}$. Then we have $W_n = L_n \circ T_{i_m} \circ W_n^*$ where L_n is a word generated by $\{T_{i_1}, \dots, T_{i_M}\}$ and W_n^* is a word generated by $\{T_{i_1}, \dots, T_{i_M}\} \setminus \{T_{i_m}\}$ for any $n \geq 0$. Since

$$d(W_n(x_n), c) \leq d(T_{i_m} \circ W_n^*(x_n), c) \leq d(W_n^*(x_n), c) \leq d(x_n, c),$$

for any $n \geq 0$, we obtain

$$\lim_{n \rightarrow \infty} [d(x_n, c) - d(W_n^*(x_n), c)] = 0 \text{ and } \lim_{n \rightarrow \infty} [d(W_n^*(x_n), c) - d(T_{i_m} \circ W_n^*(x_n), c)] = 0.$$

Since $(x_n)_n$ is bounded, we deduce that $(W_n^*(x_n))_n$ is also bounded. Using the fact that $\mathcal{P}(M - 1)$ is true and T_{i_m} is projective w.r.t. c , we conclude that

$$\lim_{n \rightarrow \infty} d(x_n, W_n^*(x_n)) = 0 \text{ and } \lim_{n \rightarrow \infty} d(W_n^*(x_n), \text{Fix}(T_{i_m})) = 0.$$

This will easily imply $\lim_{n \rightarrow \infty} d(x_n, \text{Fix}(T_{i_m})) = 0$. Since i_m was arbitrarily chosen in $\{i_1, \dots, i_M\}$, we conclude that

$$\lim_{n \rightarrow \infty} \max_{i_m \in \{i_1, \dots, i_M\}} d(x_n, \text{Fix}(T_{i_m})) = 0.$$

Using the innately boundedness regularity of $\text{Fix}(T_{i_m})$, for $i_m \in \{1, \dots, N\}$, we conclude that

$$\lim_{n \rightarrow \infty} d \left(x_n, \bigcap_{i=1}^{i_M} \text{Fix}(T_i) \right) = 0.$$

Obviously, we have $\bigcap_{i=1}^{i_M} \text{Fix}(T_i) \subset \text{Fix}(W_n)$, for any $n \geq 0$. Hence we have

$$d(x_n, W_n(x_n)) \leq 2d \left(x_n, \bigcap_{i=1}^{i_M} \text{Fix}(T_i) \right),$$

which implies that $\lim_{n \rightarrow \infty} d(x_n, W_n(x_n)) = 0$. This concludes that $\mathcal{P}(M)$ is true for any $M \in \{1, \dots, N\}$. Therefore, we have proven that $\mathcal{P}(N)$ is true as well. This completes the proof of the first part of our proposition. The second part follows easily from the fact that for any $i \in \{1, \dots, N\}$, we have $\lim_{n \rightarrow \infty} d(x_n, \text{Fix}(T_i)) = 0$ and the innately boundedness regularity of $\text{Fix}(T_i)$, for $i \in \{1, \dots, N\}$.

Note that condition (*) first appeared in Dye and Reich [20] as the condition (S).

Next, we give the main result of this article.

Theorem 3.1. *Let (X, d) be a complete metric space. Let $\{T_1, \dots, T_N\}$ be a finite family of mappings defined from D into D , where D is a nonempty and closed subset of X . Assume that $\{T_1, \dots, T_N\}$ are projective w.r.t. a common fixed point $c \in D$. Also Assume that $\{\text{Fix}(T_1), \dots, \text{Fix}(T_N)\}$ is innately boundedly regular. Let $r : \mathbb{N} \rightarrow \{1, \dots, N\}$ be a random mapping which assumes each value infinitely often. Then the random sequence $(x_n)_n$ defined by*

$$x_0 \in D; \text{ and } x_{n+1} = T_{r(n)}(x_n), \text{ for all } n \geq 0,$$

converges to a point in $\mathcal{Q}_C(x_0)$, where $C = \bigcap_{i=1}^N \text{Fix}(T_i)$.

Proof. Since r is a random mapping, there exists a subsequence $(x_{n_k})_k$ of $(x_n)_n$ such that $W_k = T_{r(n_{k+1}-1)} \circ \dots \circ T_{r(n_k)}$ is a full word of $\{T_1, \dots, T_N\}$, for any k . It is easy to check that $(x_n)_n$ is Fejér monotone w.r.t. C . Our assumptions and Proposition 3.1 will then imply $\lim_{n_k \rightarrow \infty} d(x_{n_k}, C) = 0$. Since $(d(x_n, C))_n$ is decreasing, we conclude that $\lim_{n \rightarrow \infty} d(x_n, C) = 0$. As we did before, we can easily show that for any $n < m$, we have $d(x_n, x_m) < 2d(x_n, C)$. Hence $(x_n)_n$ is a Cauchy sequence which converges to a point $\omega \in D$. Since $(x_n)_n$ is Fejér monotone w.r.t. C , we have

$$d(\omega, c) = \lim_{n \rightarrow \infty} d(x_n, c) = \inf_{n \geq 0} d(x_n, c)$$

for any $c \in C$. In particular, we have $d(\omega, c) \leq d(x_0, c)$ for any $c \in C$. Moreover since C is closed and $\lim_{n \rightarrow \infty} d(x_n, C) = 0$, we conclude that $\omega \in C$. Putting everything together we obtain that $\omega \in Q_C(x_0)$. This proves Theorem 3.1.

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Author details

¹Department of Mathematical Sciences, The University of Texas At El Paso, El Paso, TX 79968, USA ²Department of Mathematics and Statistics, King Fahd University of Petroleum & Minerals, Dhahran 31261, Saudi Arabia

Authors' contributions

All authors participated in the design of this work and performed equally. All authors read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests.

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