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Fixed point theorems for contraction mappings in modular metric spaces, Fixed Point Theory Appl. 2011, 2011:93

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Abstract
This article is written due to a small gap in our published paper. In this erratum, we point out and fix the problem to set our existed results at the best of their perfection.

1. On the results in [1]

In [1], the authors have studied and introduced some fixed point theorems in the frame-work of a modular metric space. We shall first state their results and then discuss some small gap herewith.

Theorem 1.1 (Theorem 3.2 in Mongkolkeha et al. [1]). *Let X_ω be a complete modular metric space and f be a self-mapping on X satisfying the inequality*

$$\omega_\lambda(fx, fy) \le k\omega_\lambda(x, y),$$

for all $x, y \in X_\omega$, where $k \in [0, 1)$. Then, f has a unique fixed point in $x_* \in X_\omega$ and the sequence $\{f^n x\}$ converges to x_*.

Theorem 1.2 (Theorem 3.6 in Mongkolkeha et al. [1]). *Let X_ω be a complete modular metric space and f be a self mapping on X satisfying the inequality*

$$\omega_\lambda(fx, fy) \le k[\omega_{2\lambda}(x, fx) + \omega_{2\lambda}(y, fy)],$$

for all $x, y \in X_\omega$, where$k \in \left[0, \frac{1}{2}\right)$. Then, f has a unique fixed point in $x_* \in X_\omega$ and the sequence $\{f^n x\}$ converges to x_*.

We now claim that the conditions in the above theorems are not sufficient to guarantee the existence and uniqueness of the fixed points. We state a counterexample to Theorem 1.1 in the following:

Example 1.3. Let $X := \{0, 1\}$ and ω be given by

$$\omega_\lambda(x, y) = \begin{cases} \infty, & \text{if } 0 < \lambda < 1 \text{ and } x \ne y, \\ 0, & \text{if } \lambda \ge 1 \text{ or } x = y. \end{cases}$$

Thus, the modular metric space $X_\omega = X$. Now let f be a self-mapping on X defined by

$$\begin{cases} f(0) = 1, \\ f(1) = 0. \end{cases}$$

Then, f is satisfies the inequality (1.1) with any $k \in [0, 1)$ but it possesses no fixed point after all.

Notice that this gap flaws the theorems only when ∞ is involved.

2. Revised theorems

In this section, we shall now give the corrections to our theorems in [1].

Theorem 2.1. *Let X_ω be a complete modular metric space and f be a self mapping on X satisfying the inequality*

$$\omega_\lambda(fx, fy) \le k\omega_\lambda(x, y),$$

for all $x, y \in X_\omega$, where $k \in [0, 1)$. Suppose that there exists $x_0 \in X$ such that $\omega_\lambda(x_0, fx_0) < \infty$ for all $\lambda > 0$. Then, f has a unique fixed point in $x_* \in X_\omega$ and the sequence $\{f^n x_0\}$ converges to x_*.

Theorem 2.2. *Let X_ω be a complete modular metric space and f be a self-mapping on X satisfying the inequality*

$$\omega_\lambda(fx, fy) \le k[\omega_{2\lambda}(x, fx) + \omega_{2\lambda}(y, fy)],$$

for all $x, y \in X_\omega$ .where $k \in \left[0, \frac{1}{2}\right)$ .Suppose that there exists $x_0 \in X$ such that $\omega_\lambda(x_0, fx_0) < \infty$ for all $\lambda > 0$. Then, f has a unique fixed point in $x_* \in X_\omega$ and the sequence $\{f^n x\}$ converges to x_*.

Proof (of Theorem 2.1). Let $\lambda > 0$ and observe that

$\omega_\lambda(f^n x_0, f^{n+1} x_0) \le k\omega_\lambda(f^{n-1} x_0, f^n x_0) \le \cdots \le k^n \omega_\lambda(x_0, fx_0) < \infty$, for all $n \in \mathbb{N}$

Assume $m > n$ be two positive integers. Observe that

$$\begin{aligned}
\omega_\lambda(f^m x_0, f^n x_0) &\le \omega_\lambda(f^n x_0, f^{n+1} x_0) + (f^{n+1} x_0, f^{n+2} x_0) + \cdots + \omega_\lambda(f^{m-1} x_0, f^m x_0) \\
&\le (k^n + k^{n+1} + \cdots + k^{m-1})\omega_\lambda(x_0, fx_0) \\
&\le (k^n + k^{n+1} + \cdots)\omega_\lambda(x_0, fx_0) \\
&= \frac{k^n}{1-k}\omega_\lambda(x_0, fx_0).
\end{aligned}$$

Since $\omega_\lambda(x_0, fx_0) < \infty$, we deduce that for any given $\varepsilon > 0$, $\omega\lambda(f^m x_0, f^n x_0) < \varepsilon$ for $m > n > N$ with $N \in \mathbb{N}$ big enough. Thus, $\{f^n x_0\}$ is Cauchy and hence it converges to some $x_* \in X_\omega$ in essence of the completeness of X_ω. Observe further that

$$\omega_\lambda(x_*, fx_*) \le \omega_\lambda(x_*, f^n x_0) + k\omega_\lambda(f^{n-1} x_0, x_*).$$

Letting $n \to \infty$ to obtain that $\omega_\lambda(x_*, fx_*) = 0$ for all $\lambda > 0$. Therefore, x^* is a fixed point of f. Suppose also that $y_* = fy_*$. Note that

$$\omega_\lambda(x_*, y_*) = \omega_\lambda(fx_*, fy_*) \le k\omega_\lambda(x_*, y_*),$$

which implies that $\omega_\lambda(x_*, fx_*) = 0$ for all $\lambda > 0$. Therefore, the theorem is proved. □

For the proofs of the remaining theorem, take the idea of the above correction and combine with the proof aforementioned in [1] to obtain the expected results.

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