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# Continuity of solutions for parametric generalized quasi-variational relation problems

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**Abstract**

In this article, we establish sufficient conditions for the solution sets of parametric generalized quasi-variational relation problems with the stability properties such as the upper semicontinuity, lower semi-continuity, the Hausdorff lower semicontinuity, continuity, Hausdorff continuity, and closedness. Our results improve recent existing ones in the literature.

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## Introduction and preliminaries

Let $X$, $Y$ be Hausdorff topological vector spaces and $\Lambda$, $\Gamma$, $M$ be topological spaces. Let $A \subseteq X$ and $B \subseteq Y$ be nonempty sets. Let $K_1$: $A \times \Lambda \to 2^A$, $K_2$: $A \times \Lambda \to 2^A$, $T : A \times A \times \Gamma \to 2^B$ be multifunctions and $R(x, t, y, \mu)$ be a relation linking $x \in A$, $t \in B$, $y \in A$ and $\mu \in M$.

For the sake of simplicity, we adopt the following notations (see [1,2]). Letters w, m, and s are used for weak, middle, and strong, respectively, kinds of considered problems. For subsets $U$ and $V$ under consideration we adopt the notations

$$\begin{aligned}
(u,v)\,\mathrm{w}\,U \times V &\quad \text{means} \quad \forall u \in U, \exists v \in V,\\
(u,v)\,\mathrm{m}\,U \times V &\quad \text{means} \quad \exists v \in V, \forall u \in U,\\
(u,v)\,\mathrm{s}U \times V &\quad \text{means} \quad \forall u \in U, \forall v \in V,\\
\rho_1(U,V) &\quad \text{means} \quad U \subseteq V,\\
\rho_2(U,V) &\quad \text{means} \quad U \cap V \neq \emptyset,\\
(u,v)\,\bar{w}\,U \times V &\quad \text{means} \quad \exists u \in U, \forall v \in V \text{ and similarly for } \bar{m}, \bar{s},\\
\bar{\rho}_1(U,V) &\quad \text{means} \quad U \not\subseteq V \text{ and similarly for } \bar{\rho}_2.
\end{aligned}$$

Let $\alpha \in \{\mathrm{w, m, s}\}$, $\bar{\alpha} \in \{\bar{w}, \bar{m}, \bar{s}\}$, $\rho \in \{\rho_1, \rho_2\}$, and $\bar{\rho} \in \{\bar{\rho}_1, \bar{\rho}_2\}$. We consider the following for parametric generalized quasi-variational relation problem (in short, $(\mathrm{QVR}_\alpha)$):
**(QVR$_\alpha$)**: Find $\bar{x} \in K_1(\bar{x}, \lambda)$ such that $(y, t)\alpha K_2(\bar{x}, \lambda) \times T(\bar{x}, y, \gamma)$ satisfying

$$R(\bar{x}, t, y, \mu) \quad \text{holds.}$$

For each $\lambda \in \Lambda$, $\gamma \in \Gamma$, $\mu \in M$, we let $E(\lambda) := \{x \in A | x \in K_1(x, \lambda)\}$ and let $S_\alpha : \Lambda \times \Gamma \times M \to 2^A$ be a set-valued mapping such that $S_\alpha(\lambda, \gamma, \mu)$ is the solution set of $(\mathrm{QVR}_\alpha)$.

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Throughout the article, we assume that $S_\alpha(\lambda, \gamma, \mu) \neq \emptyset$ for each $(\lambda, \gamma, \mu)$ in the neighborhoods $(\lambda_0, \gamma_0, \mu_0) \in \Lambda \times \Gamma \times M$.

The parametric generalized quasi-variational relation problems are more general than many following problems.

(a) The parametric variational relation problem (VR):

Let $A$, $B$, $X$, $Y$, $M = \Gamma = \Lambda$, $K_1$, $K_2$, $T$, $\alpha = s$ as in ($\mathrm{QVR}_\alpha$). Then, ($\mathrm{QVR}_\alpha$) becomes (VR) is studied in [3]:

Find $\bar{x} \in K_1(\bar{x}, \lambda)$ such that

$$R(\bar{x}, t, y, \lambda) \quad \text{holds}, \forall t \in T(\bar{x}, y, \lambda), \forall y \in K_2(\bar{x}, \lambda).$$

(b) The parametric generalized quasi-variational inclusion problem ($\mathrm{QGVIP}_\alpha$):

Let $A$, $B$, $X$, $Y$, $M$, $\Gamma$, $\Lambda$, $K_1$, $K_2$, $T$ as in ($\mathrm{QVR}_\alpha$) and let $Z$ be a Hausdorff topological vector space. Given a mapping $F : A \times B \times A \times M \to 2^Z$ , the relation $R$ is defined as follows

$$R(x, t, y, \mu) \quad \text{holds iff } 0 \in F(x, t, y, \mu).$$

Then, ($\mathrm{QVR}_\alpha$) becomes ($\mathrm{QGVIP}_\alpha$)

Find $\bar{x} \in K_1(\bar{x}, \lambda)$ such that $(y, t)\alpha K_2(\bar{x}, \lambda) \times T(\bar{x}, y, \gamma)$ satisfying

$$0 \in F(\bar{x}, t, y, \mu).$$

(c) The parametric quasi-variational inclusion problem ($\mathrm{P}_{\alpha\rho}$):

Let $A$, $B$, $X$, $Y$, $M$, $\Gamma$, $\Lambda$, $K_1$, $K_2$, $T$, $R$ as in ($\mathrm{QVR}_\alpha$) and let $Z$ be a Hausdorff topological vector space. Let $F : A \times B \times A \times M \to 2^Z$ and $G : A \times B \times A \times M \to 2^Z$ be multivalued mappings. The relation $R$ is defined as follows

$$R(x, t, y, \mu) \quad \text{holds iff } \rho(F(x, t, y, \mu), \mathrm{G}(x, t, x, \mu)).$$

Then, ($\mathrm{QVR}_\alpha$) becomes ($\mathrm{P}_{\alpha\rho}$) is studied in [1,2]:

Find $\bar{x} \in K_1(\bar{x}, \lambda)$ such that $(y, t)\alpha K_2(\bar{x}, \lambda) \times T(\bar{x}, y, \gamma)$ satisfying

$$\rho(F(\bar{x}, t, y, \mu), \mathrm{G}(\bar{x}, t, \bar{x}, \mu)).$$

(d) The parametric vector quasi-equilibrium problems:

Let $A$, $X$, $M$, $\Gamma$, $\Lambda$, $K_1 \equiv K_2 \equiv K$, $T$ as in ($\mathrm{QVR}_\alpha$) and let $Y$ be a Hausdorff topological vector space. Given a mapping $F : A \times A \times M \to 2^Y$ and $C \subseteq Y$ be a closed subset with nonempty interior, the relation $R$ is defined as follows

$$R(x, t, y, \mu) \quad \text{holds iff } \rho(F(t, y, \mu), (Y \backslash -\mathrm{intC})).$$

Then, ($\mathrm{QVR}_\alpha$) becomes the parametric vector quasi-equilibrium problems is studied in [4]. Find $\bar{x} \in \mathrm{clK}(\bar{\mathrm{x}}, \lambda)$ such that $(y, t)\alpha K(\bar{x}, \lambda) \times T(\bar{x}, y, \gamma)$ satisfying

$$\rho(F(t, y, \mu), (Y \backslash -\mathrm{intC})).$$

(e) The parametric multivalued vector quasi-equilibrium problems:

Let $A = B$, $X = Y$, $M = \Gamma$, $\Lambda$, $K_1 = \mathrm{clK}$, $\mathrm{K}_2 = \mathrm{K}$, $\mathrm{T} = \{\mathrm{t}\}$ as in ($\mathrm{QVR}_\alpha$) and let $Z$ be a Hausdorff topological vector space. Given a mapping $F : A \times A \times M \to 2^Z$ and $C \subseteq Z$ be a closed subset with nonempty interior, the relation $R$ is defined as follows

$$R(x, t, y, \mu) \quad \text{holds iff } \rho(F(x, y, \mu), (Z \backslash -\mathrm{intC})).$$

Then, ($QVR_\alpha$) becomes the parametric multivalued vector quasi-equilibrium problems is studied in [5]. Find $\bar{x} \in \mathrm{clK}(\bar{x}, \lambda)$ such that

$$\rho(F(\bar{x}, y, \mu), (Z\setminus -\mathrm{intC})), \forall y \in \mathrm{K}(\bar{x}, \lambda).$$

(f) The parametric generalized vector quasi-equilibrium problems ($QEP_{\alpha\rho}$):

Let $A$, $B$, $X$, $Y$, $M$, $\Gamma$, $\Lambda$, $K_1$, $K_2$, $T$ as in ($QVR_\alpha$) and let $Z$ be a Hausdorff topological vector space. Given a mapping $F : A \times B \times A \times M \to 2^Z$ and $C \subseteq Z$ be a closed subset with nonempty interior, the relation $R$ is defined as follows

$$R(x, t, y, \mu) \quad \text{holds iff}\ \rho(F(x, t, y, \mu), C).$$

Then, ($QVR_\alpha$) becomes ($QEP_{\alpha\rho}$)

Find $\bar{x} \in K_1(\bar{x}, \lambda)$ such that $(y, t)\alpha K_2(\bar{x}, \lambda) \times T(\bar{x}, y, y)$ satisfying

$$\rho(F(\bar{x}, t, y, \mu), C).$$

Stability properties of solution sets for parametric generalized quasi-variational relation problem is an important topic in optimization theory and applications. Recently, the continuity, especially the upper semicontinuity, the lower semicontinuity and the Hausdorff lower semicontinuity of the solution sets have been investigated in models as equilibrium problems [1,2,4-13], variational inequality problems [14-19], and the references therein.

The structure of this article is as follows. In the remaining part of this section, we recall definitions for later uses. Section "Main results" is devoted to the upper semicontinuity, the lower semicontinuity, and the Hausdorff lower semicontinuity of solutions for problem ($QVR_\alpha$). Applications to the parametric vector quasi-equilibrium problem are presented in Section "Applications".

Now we recall some notions see [5,6,20,21]. Let $X$ and $Y$ be as above and $G : X \to 2^Y$ be a multifunction. $G$ is said to be lower semicontinuous (lsc) at $x_0$ if $G(x_0) \cap U \neq \emptyset$ for some open set $U \subseteq Y$ implies the existence of a neighborhood $N$ of $x_0$ such that, for all $x \in N$, $G(x) \cap U \neq \emptyset$. An equivalent formulation is that: $G$ is lsc at $x_0$ if $\forall x_\alpha \to x_0, \forall z_0 \in G(x_0), \exists z_\alpha \in G(x_\alpha), z_\alpha \to z_0$. $G$ is called upper semicontinuous (usc) at $x_0$ if for each open set $U \supseteq G(x_0)$, there is a neighborhood $N$ of $x_0$ such that $U \supseteq G(x)$, for all $x \in N$. $G$ is said to be Hausdorff upper semicontinuous (H-usc in short; Hausdorff lower semicontinuous, H-lsc, respectively) at $x_0$ if for each neighborhood $B$ of the origin in $Y$, there exists a neighborhood $N$ of $x_0$ such that, $G(x) \subseteq G(x_0) + B$, $\forall x \in N$ ($G(x_0) \subseteq G(x) + B$, $\forall x \in N$). $G$ is said to be continuous at $x_0$ if it is both lsc and usc at $x_0$ and to be H-continuous at $x_0$ if it is both H-lsc and H-usc at $x_0$. $G$ is called closed at $x_0$ if for each net $\{(x_\alpha, z_\alpha)\}$ graph$G := \{(x, z) \mid z \in G(x)\}$, $(x_\alpha, z_\alpha) \to (x_0, z_0)$, $z_0$ must belong to $G(x_0)$. We say that $G$ satisfies a certain property in a subset $A \subseteq X$ if $G$ satisfies it at every points of $A$. If $A = X$ we omit "in $X$" in the statement.

Let $A$ and $Y$ be as above and $G : A \to 2^Y$ be a multifunction.

(i) If $G$ is usc at $x_0$, then $G$ is $H$-usc at $x_0$. Conversely if $G$ is $H$-usc at $x_0$ and if $G(x_0)$ is compact, then $G$ is usc at $x_0$;

(ii) If $G$ is H-lsc at $x_0$, then $G$ is lsc at $x_0$. The converse is true if $G(x_0)$ is compact;

(iii) If $Y$ is compact and $G$ is closed at $x_0$, then $G$ is usc at $x_0$;

(iv) If $G$ is usc at $x_0$ and $G(x_0)$ is closed, then $G$ is closed at $x_0$;

(v) If $G$ has compact values, then $G$ is usc at $x_0$ if and only if, for each net $\{x_\alpha\} \subseteq A$ which converges to $x_0$ and for each net $\{y_\alpha\} \subseteq G(x_\alpha)$, there are $y_0 \in G(x_0)$ and a subnet $\{y_\beta\}$ of $\{y_\alpha\}$ such that $y_\beta \to y_0$.

Now we let $A$, $B$, $X$, $Y$, $M$, $\Gamma$, $\Lambda$, $R$ as in (QVR$_\alpha$), we use the following notations for level sets of $R$

$$1\mathrm{ev}_{\mathrm{upper}}R := \{(x, t, y, \mu) | R(x, t, y, \mu) \text{ holds}\}.$$
$$1\mathrm{ev}_{\mathrm{upper}}R(., ., ., \mu_0) := \{(x, t, y) | R(x, t, y, \mu_0) \text{ holds}\}.$$
$$1\mathrm{ev}_{\mathrm{lower}}R := \{(x, t, y, \mu) | R(x, t, y, \mu) \text{ does not hold}\}.$$

## Main results

In this section, we discuss the upper semicontinuity, the lower semicontinuity, the Hausdorff lower semicontinuity, continuity, and H-continuity of solution sets for parametric quasi-variational relation problem (QVR$_\alpha$).

**Theorem 1** *Assume for problem* (QVR$_\alpha$) *that*

*(i) $E$ is usc at $\lambda_0$ and $E(\lambda_0)$ is compact, and $K_2$ is lsc in $K_1(A, \Lambda) \times \{\lambda_0\}$;*

*(ii) in $K_1(A, \Lambda) \times K_2(K_1(A, \Lambda), \Lambda) \times \{\gamma_0\}$, $T$ is usc and compact-valued if $\alpha = w$ (or $\alpha = m$), and lsc if $\alpha = s$;*

*(iii) in $K_1(A, \Lambda) \times T(K_1(A, \Lambda), K_2(K_1(A, \Lambda), \Lambda), \Gamma) \times K_2(K_1(A, \Lambda), \Lambda) \times \{\mu_0\}$, $lev_{upper}R$ is closed.*

*Then $S_\alpha$ is both usc and closed at $(\lambda_0, \gamma_0, \mu_0)$.*

**Proof**. Since $\alpha = \{w, m, s\}$, we have in fact three cases. However, the proof techniques are similar. We consider only the cases $\alpha = w$. We first prove that $S_w$ is upper semicontinuous at $(\lambda_0, \gamma_0, \mu_0)$. Indeed, we suppose to the contrary that $S_w$ is not upper semicontinuous at $(\lambda_0, \gamma_0, \mu_0)$, i.e., there is an open subset $U$ of $S_w(\lambda_0, \gamma_0, \mu_0)$ such that for all nets $\{(\lambda_n, \gamma_n, \mu_n)\}$ convergent to $(\lambda_0, \gamma_0, \mu_0)$, there exists $x_n \in S_w(\lambda_n, \gamma_n, \mu_n)$, $x_n \notin U$, $\forall n$. By the upper semicontinuity of $E$ and the compactness of $E(\lambda_0)$, one can assume that $x_n \to x_0$ for some $x_0 \in E(\lambda_0)$. If $x_0 \notin S_w(\lambda_0, \gamma_0, \mu_0)$, then $\exists y_0 \in K_2(x_0, \lambda_0)$, $\forall t_0 \in T(x_0, y_0, \gamma_0)$ such that

$$R(x_0, t_0, y_0, \mu_0) \text{does not hold.} \tag{1}$$

By the lower semicontinuity of $K_2$ at $(x_0, \lambda_0)$, there exists $y_n \in K_2(x_n, \lambda_n)$ such that $y_n \to y_0$. Since $x_n \in S_w(\lambda_n, \gamma_n, \mu_n)$, $\exists t_n \in T(x_n, y_n, \gamma_n)$ such that

$$R(x_n, t_n, y_n, \mu_n) \quad \text{holds.} \tag{2}$$

Since $T$ is usc at $(x_0, y_0, \gamma_0)$ and $T(x_0, y_0, \gamma_0)$ is compact, there exists $t_0 \in T(x_0, y_0, \gamma_0)$ such that $t_n \to t_0$ (can take a subnet if necessary). By the condition (iii) and (2), we have

$$R(x_0, t_0, y_0, \mu_0) \quad \text{holds,} \tag{3}$$

we see a contradiction between (1) and (3). Thus, $x_0 \in S_w(\lambda_0, \gamma_0, \mu_0) \subseteq U$, this contradicts to the fact $x_n \notin U$, $\forall n$. Hence, $S_w$ is upper semicontinuous at $(\lambda_0, \gamma_0, \mu_0)$.

Now we prove that $S_w$ is closed at $(\lambda_0, \gamma_0, \mu_0)$. Indeed, we supposed that $S_w$ is not closed at $(\lambda_0, \gamma_0, \mu_0)$, i.e., there is a net $\{(x_n, \lambda_n, \gamma_n, \mu_n)\} \to (x_0, \lambda_0, \gamma_0, \mu_0)$ with $x_n \in S_w(\lambda_n, \gamma_n, \mu_n)$ but $x_0 \notin S_w(\lambda_0, \gamma_0, \mu_0)$. The further argument is the same as above. And so we have $S_w$ is closed at $(\lambda_0, \gamma_0, \mu_0)$. □

The following example shows that the upper semicontinuity and the compactness of $E$ are essential.

**Example 2** Let $A = B = X = Y = \mathbb{R}$, $\Lambda = \Gamma = M = [0, 1]$, $\lambda_0 = 0$, $F(x, t, y, \lambda) = 2^{\lambda+1}$, $K_1(x, \lambda) = (-\lambda - 1, \lambda]$, $K_2(x, \lambda) = \{-1\}$ and $T(x, y, \lambda) = [0, e^{2^{\lambda}+\cos\lambda}]$. We let relation $R$ be defined by $R(x, t, y, \lambda)$ holds iff $F(x, t, y, \lambda) \subseteq \mathbb{R}_+$. Then, we have $E(0) = (-1, 0]$ and $E(\lambda) = (-\lambda - 1, \lambda]$, $\forall\lambda \in (0, 1]$. We show that $K_2$ is lsc and assumptions (ii) and (iii) of Theorem 1 are fulfilled. But $S_\alpha$ is neither usc nor closed at (0, 0, 0). The reason is that $E$ is not usc at 0 and $E(0)$ is not compact. In fact, $S_\alpha(0, 0, 0) = (-1, 0]$ and $S_\alpha(\lambda, \gamma, \mu) = (-\lambda -1, \lambda]$, $\forall\lambda \in (0, 1]$.

The following example shows that the lower semicontinuity of $K_2$ is essential.

**Example 3** Let $X, Y, \Lambda, \Gamma, M, \lambda_0$ as in Example 2 and let $A = B = [-3, 3]$, $F(x, t, y, \lambda) = x + y + \lambda$, $K_1(x, \lambda) = [0, 3]$, $T(x, y, \lambda) = \{t\}$. Let relation $R$ be defined by $R(x, t, y, \lambda)$ holds iff $F(x, t, y, \lambda) \subseteq \mathbb{R}_+$ and

$$K_2(x, \lambda) = \begin{cases} \{-3, 0, 3\} & \text{if } \lambda = 0, \\ \{0, 3\} & \text{otherwise.} \end{cases}$$

We have $E(\lambda) = [0, 3]$, $\forall\lambda \in [0, 1]$. Hence $E$ is usc at 0 and $E(0)$ is compact and the conditions (ii) and (iii) of Theorem 1 are easily seen to be fulfilled. But $S_\alpha$ is not upper semicontinuous at (0, 0, 0). The reason is that $K_2$ is not lower semicontinuous. In fact,

$$S_\alpha(\lambda, \gamma, \mu) = \begin{cases} \{3\} & \text{if } \lambda = 0, \\ [0, 3] & \text{if } \lambda \in (0, 1]. \end{cases}$$

The following example shows that the condition (iii) of Theorem 1 is essential.

**Example 4** Let $\Lambda, \Gamma, M, T, \lambda_0$ as in Example 3 and let $X = Y = A = B = [0, 1]$. Let relation $R$ be defined by $R(x, t, y, \lambda)$ holds iff $F(x, t, y, \lambda) \subseteq \mathbb{R}_+$, $K_1(x, \lambda) = K_2(x, \lambda) = [0, 1]$ and $F(x, t, y, 0) = \frac{x}{2} - \frac{y}{2}$, $F(x, t, y, \lambda) = \frac{y}{2} - \frac{x}{3}$, $\forall\lambda \in (0, 1]$. We show that the assumptions (i) and (ii) of Theorem 1 are easily seen to be fulfilled and

$$S_\alpha(\lambda, \gamma, \mu) = \begin{cases} \{0\} & \text{if } \lambda \in (0, 1], \\ \{1\} & \text{if } \lambda = 0. \end{cases}$$

But $S_\alpha$ is not usc at (0, 0, 0). The reason is that assumption (iii) is violated. Indeed, taking $x_n = 0$, $t_n = 0$, $y_n = \frac{1}{2}$, $\lambda_n = \frac{1}{n} \to 0$ as $n \to \infty$, then $\{(x_n, t_n, y_n, \lambda_n)\} \to (0, 0, \frac{1}{2}, 0)$ and $F(x_n, t_n, y_n, \lambda_n) = F(0, 0, \frac{1}{2}, \frac{1}{n}) = \frac{1}{4} > 0$, but $F(0, 0, \frac{1}{2}, 0) = -\frac{1}{4} < 0$.

The following example shows that all assumptions of Theorem 1 are fulfilled. But Theorem 3.2 in [5] cannot be applied.

**Example 5** Let $A, B, X, Y, \Lambda, \Gamma, M, \lambda_0$ as in Example 2 and let $K_1(x, \lambda) = K_2(x, \lambda) = [0, 1]$, $T(x, y, \gamma) = [0, 2^{\cos^6 x + \sin^4 x + 2}]$ and

$$F(x, t, y, \lambda) = \begin{cases} \{0\} & \text{if } \lambda = 0, \\ 3^{\sin^4 x + \cos^2 x + 2} & \text{otherwise.} \end{cases}$$

Let relation $R$ be defined by $R(x, t, y, \lambda)$ holds iff $F(x, t, y, \lambda) \subseteq \mathbb{R}_+$. We show that the assumptions (i), (ii), and (iii) of Theorem 1 are easily seen to be fulfilled and

$$S_\alpha(\lambda, \gamma, \mu) = [0, 1], \forall\lambda \in [0, 1]$$

Hence, $S_\alpha$ is usc at (0, 0, 0). But Theorem 3.2 in [5] cannot be applied. The reason is that $F$ is not usc at $(x, t, y, 0)$.

The following example shows that all assumptions of Theorem 1 are fulfilled. But Theorem 3.4 in [5] cannot be applied.

**Example 6** Let $A$, $B$, $X$, $Y$, $\Lambda$, $\Gamma$, $M$, $\lambda_0$, as in, Example, 5 and, let, $K_1(x, \lambda)$, $= K_2(x, \lambda) =$ $[0, 3]$, $T(x, y, \gamma) = \{t\}$ and

$$F(x, t, y, \lambda) = \begin{cases} \{0\} & \text{if} \quad \lambda = 0, \\ e^{\cos^2\lambda+1} & \text{otherwise.} \end{cases}$$

Let relation $R$ be defined by $R(x, t, y, \lambda)$ holds iff $F(x, t, y, \lambda) \subseteq \mathbb{R}_+$. We show that the assumption (i), (ii,) and (iii) of Theorem 1 are easily seen to be fulfilled

Hence, $S_\alpha$ is usc at (0, 0, 0). But Theorem 3.4 in [5] cannot be applied. The reason is that $F$ is not usc (x, t, y, 0).

Assumptions in Theorem 1, we have $K_2$ is lsc in $K_1(A, \Lambda) \times \{\lambda_0\}$ (which is not imposed in this Theorem 4.1 of [10]). The Example 3 shows that the lower semicontinuity of $K_2$ needs to be added to Theorem 4.1 of [10].

**Remark 7** (i) In the special case, if $T(x, y, \gamma) = \{t\}$, $\Lambda = \Gamma = M$, $A = B$, $X = Y$, $K_1 = K_2 = K$ and the variational relation $R$ is defined as follows $R(x, t, y, \lambda)$ holds iff $F(x, y, \lambda) \not\subset -\mathrm{int}C(x, \lambda)$ (or $F(x, y, \lambda) \cap -\mathrm{int}C(x, \lambda) = \emptyset$), where $F : A \times A \times \Lambda \to 2^Y$ and $C : A \times \Lambda \to 2^Y$ be multifunctions, with $C(x, \lambda)$ being a convex cone. Then, (QVR$_\alpha$) becomes (PGQVEP) and (PEQVEP) in [10].

(ii) In the special case as in Remark 7 (i). Then, Theorem 1 reduces to Theorem 4.1 in [10]. However the proof of the Theorem 4.1 in a different way. Its assumptions (i)-(iv) derive (i) Theorem 1, assumptions (v) and (vi) coincide with (iii) of Theorem 1.

The following example shows a case where the assumed compactness in Theorem 4.1 of [10] is violated but the assumptions of Theorem 1 are fulfilled.

**Example 8** Let $X$, $Y$, $\Lambda$, $\Gamma$, $M$, $T$, $\lambda_0$, as in Example 6 and we let $A = B = [0, 3)$, $F(x, y, \lambda) = x - y$ and $K_1(x, \lambda) = K_2(x, \lambda) = [1, 2]$. Let relation $R$ be defined by $R(x, t, y, \lambda)$ holds iff $F(x, t, y, \lambda) \subseteq \mathbb{R}_+$. We show that the assumptions of Theorem 1 are easily seen to be fulfilled and so $S_\alpha$ is usc and closed at (0, 0, 0), although $A$ is not compact. In fact, $S_\alpha(\lambda, \gamma, \mu) = \{2\}, \forall \lambda \in [0, 1]$.

**Theorem 9** *Assume for problem* (QVR$_\alpha$) *that*

*(i) $E$ is lsc at $\lambda_0$, $K_2$ is usc and compact-valued in $K_1(A, \Lambda) \times \{\lambda_0\}$;*

*(ii) in $K_1(A, \Lambda) \times K_2(K_1(A, \Lambda), \Lambda) \times \{\gamma_0\}$, $T$ is usc and compact-valued if $\alpha = s$, and lsc if $\alpha = w$ (or $\alpha = m$);*

*(iii) in $K_1(A, \Lambda) \times T(K_1(A, \Lambda), K_2(K_1(A, \Lambda), \Lambda), \Gamma) \times K_2(K_1(A, \Lambda), \Lambda) \times \{\mu_0\}$, $lev_{lower}R$ is closed.*

*Then $S_\alpha$ is lower semicontinuous at $(\lambda_0, \gamma_0, \mu_0)$.*

**Proof**. Since $\alpha = \{w, m, s\}$, we have in fact three cases. However, the proof techniques are similar. We consider only the cases $\alpha = s$. Suppose to the contrary that $S_s$ is not lsc at $(\lambda_0, \gamma_0, \mu_0)$, i.e., there are $x_0 \in S_s(\lambda_0, \gamma_0, \mu_0)$ and net $\{(\lambda_n, \gamma_n, \mu_n)\}$, $(\lambda_n, \gamma_n, \mu_n) \to (\lambda_0, \gamma_0, \mu_0)$ such that $\forall x_n \in S_s(\lambda_n, \gamma_n, \mu_n)$, $x_n \to x_0$. Since $E$ is lsc at $\lambda_0$, there is $x'_n \in E(\lambda_n)$ with $x'_n \to x_0$. By the above contradiction assumption, there must be a subnet $\{x'_m\}$ of $\{x'_n\}$ such that, $\forall m$, $x'_m \notin S_s(\lambda_m, \gamma_m, \mu_m)$, i.e., $\exists y_m \in K_2(x'_m, \lambda_m)$, $\exists t_m \in T(x'_m, y_m, \gamma_m)$ such that

$$R(x'_m, t_m, y_m, \mu_m) \quad \text{does not hold.} \tag{4}$$

As $K_2$, is usc at $(x_0, \lambda_0)$ and $K_2(x_0, \lambda_0)$ is compact, one has $y_0 \in K_2(x_0, \lambda_0)$ such that $y_m \to y_0$ (taking a subnet if necessary). By the upper semicontinuity of $T$ at $(x_0, y_0, \gamma_0)$, one has $t_0 \in T(x_0, y_0, \gamma_0)$ such that $t_m \to t_0$.

Since $(x'_m, t_m, y_m, \lambda_m, \gamma_m, \mu_m) \to (x_0, t_0, y_0, \lambda_0, \gamma_0, \mu_0)$ and by the condition (iii) and (4), yields that

$$R(x_0, t_0, y_0, \mu_0) \quad \text{does not hold,}$$

which is impossible since $x_0 \in S_s(\lambda_0, \gamma_0, \mu_0)$. Therefore, $S_s$ is lsc at $(\lambda_0, \gamma_0, \mu_0)$. □

The following example shows that the lower semicontinuity of $E$ is essential

**Example 10** Let $A$, $B$, $X$, $Y$, $\Lambda$, $\Gamma$, $M$, $\lambda_0$ as in Example 2 and let $F(x, t, y, \lambda) = 2^{\lambda}$, $T(x, y, \lambda) = \{t\}$, $K_2(x, \lambda) = [0, 1]$. Let relation $R$ be defined by $R(x, t, y, \lambda)$ holds iff $F(x, t, y, \lambda) \subseteq (0, +\infty)$ and

$$K_1(x, \lambda) = \begin{cases} [-1, 1] & \text{if} \quad \lambda = 0, \\ [-\lambda - 1, 0] & \text{otherwise.} \end{cases}$$

We have $E(0) = [-1, 1]$, $E(\lambda) = [-\lambda - 1, 0]$, $\forall \lambda \in (0, 1]$. Hence $K_2$ is usc and the conditions (ii) and (iii) of Theorem 9 are easily seen to be fulfilled. But $S$ is not lower semicontinuous at (0, 0, 0). The reason is that $E$ is not lower semicontinuous at 0. In fact, $S_\alpha(0, 0, 0) = [-1, 1]$ and $S_\alpha(\lambda, \gamma, \mu) = [-\lambda - 1, 0]$, $\forall \lambda \in (0, 1]$.

The following example shows that all assumptions of Theorem 9 are fulfilled. But Theorems 2.1 and 2.3 in [5] and Theorem 2.2 in [4] are not fulfilled.

**Example 11** Let $A$, $B$; $X$, $Y$, $T$, $\Lambda$, $\Gamma$, $M$, $\lambda_0$ as in Example 10, let $K_1(x, \lambda) = K_2(x, \lambda) = [0, \frac{1}{2}]$ and

$$F(x, y, \lambda) = \begin{cases} [\frac{1}{2}, 1] & \text{if} \quad \lambda = 0, \\ [2, 3^{\lambda+2}] & \text{otherwise.} \end{cases}$$

and we let relation $R$ be defined by $R(x, t, y, \mu)$ holds iff $F(x, y, \lambda) \subseteq (0, +\infty)$. We show that the assumptions (i), (ii) and (iii) of Theorem 9 are satisfied and $\forall \lambda \in [0, 1]$, $\forall \lambda \in [0, 1]$. Theorems 2.1 and 2.3 in [5] and Theorem 2.2 in [4] are not fulfilled. The reason is that $F$ is neither usc nor lsc at $(x, y, 0)$.

**Theorem 12** *Impose the assumption of Theorem 9 and the following additional conditions:*

*(iv)* $K_2(., \lambda_0)$ *is lsc in* $K_1(A, \Lambda)$ *and* $E(\lambda_0)$ *is compact*;

*(v) in* $K_1(A, \Lambda) \times K_2(K_1(A, \Lambda), \Lambda)$, $T(., ., \gamma_0)$ *is usc and compact-valued if* $\alpha = w$ *(or* $\alpha = m$*), and lsc if* $\alpha = s$;

*(vi) in* $K_1(A, \Lambda) \times T(K_1(A, \Lambda), K_2(K_1(A, \Lambda), \Lambda), \Gamma) \times K_2(K_1(A, \Lambda), \Lambda)$, $lev_{upper}R(., ., \mu_0)$ *is closed* ;

*Then* $S_\alpha$ *is Hausdorff lower semicontinuous at* $(\lambda_0, \gamma_0, \mu_0)$.

**Proof**. We consider only for the cases $\alpha = s$. We first prove that $S_s(\lambda_0, \gamma_0, \mu_0)$ is closed. Indeed, we let $x_n \in S_s(\lambda_0, \gamma_0, \mu_0)$ such that $x_n \to x_0$. If $x_0 \notin S_s(\lambda_0, \gamma_0, \mu_0)$, then $\exists y_0 \in K_2(x_0, \lambda_0)$, $\exists t_0 \in T(x_0, y_0, \gamma_0)$ such that

$$R(x_0, t_0, y_0, \mu_0) \quad \text{does not hold.} \tag{5}$$

By the lower semicontinuity of $K_2(., \lambda_0)$ at $x_0$, one has $y_n \in K_2(x_n, \lambda_0)$ such that $y_n \to y_0$. By the lower semicontinuity of $T(., ., \gamma_0)$ at $(x_0, y_0)$, one has $t_n \in T(x_n, y_n, \gamma_0)$ such that $t_n \to t_0$. Since $x_n \in S_s(\lambda_0, \gamma_0, \mu_0)$, we have

$$R(x_n, t_n, y_n, \mu_0) \quad \text{holds.} \tag{6}$$

Since $(x_n, t_n, y_n) \to (x_0, t_0, y_0)$ and by the condition (vi) and (6) yields that

$$R(x_0, t_0, y_0, \mu_0) \quad \text{holds,} \tag{7}$$

we see a contradiction between (5) and (7). Therefore, $S_s(\lambda_0, \gamma_0, \mu_0)$ is closed.

On the other hand, since $S_s(\lambda_0, \gamma_0, \mu_0) \subseteq E(\lambda_0)$ is compact by $E(\lambda_0)$ compact. Since $S_s$ is lower semicontinuous at $(\lambda_0, \gamma_0, \mu_0)$ and $S_s(\lambda_0, \gamma_0, \mu_0)$ compact. Hence $S_s$ is Hausdorff lower, semicontinuous at $(\lambda_0, \gamma_0, \mu_0)$. And so we complete the proof. □

The following example shows that the assumed compactness in (iv) is essential

**Example 13** Let $X = A = B = \mathbb{R}^2$, $Y = \mathbb{R}$, $\Lambda = \Gamma = M = [0, 1]$, $\lambda_0 = 0$, and $x = (x_1, x_2) \in \mathbb{R}^2$, $T(x, y, \lambda) = [0, 3^{\sin^4 x + \sin^2 x + 1}]$, $T(x, y, \lambda) = [0, 3^{\sin^4 x + \sin^2 x + 1}]$ and

$$F(x, t, y, \lambda) = \begin{cases} \left\{\frac{1}{2}\right\} & \text{if} \quad \lambda = 0, \\ \left\{\frac{1}{2} + \frac{\lambda}{2^{\lambda+1}}\right\} & \text{otherwise.} \end{cases}$$

Let relation $R$ be defined by $R(x, t, y, \lambda)$ holds iff $F(x, t, y, \lambda) \subseteq (0, +\infty)$. We have $E(0) =$, $\{x \in \mathbb{R}^2 \mid x^2 = 0\}$ and $E(\lambda) = \{x \in \mathbb{R}^2 | x_2 = \lambda x_1^4)\}$, $\forall \lambda \in (0, 1]$. We show that the assumptions, of Theorem 12 are satisfied, but the compactness of $E(0)$ is not satisfied. Hence, $S_\alpha$ is not, Hausdorff lower semicontinuous at (0, 0, 0). In fact, $S_\alpha(0, 0, 0) = \{(x_1, x_2) \in \mathbb{R}^2 | x_2 = 0\}$ and $S_\alpha(\lambda, \gamma, \mu) = \{(x_1, x_2) \in \mathbb{R}^2 | x_2 = \lambda x_1^4)\}$, $\forall \lambda \in (0, 1]$.

**Corollary 14** *Suppose that all conditions in Theorems 1 and 9 are satisfied. Then, we have* $S\alpha$ *is both continuous and closed at* $(\lambda_0, \gamma_0, \mu_0)$.

**Corollary 15** *Suppose that all conditions in Theorems 1 and 12 are satisfied. Then, we have* $S_\alpha$ *is Hausdorff continuous and closed at* $(\lambda_0, \gamma_0, \mu_0)$.

## Applications

Since our generalized quasi-variational relation problems include many rather general problems as particular cases as mentioned in Section "Introduction". The results of Section "Main results" can derive corresponding to results of these special cases. In Section "Applications" we discuss only some corollaries for generalized vector quasi-equilibrium problems as example.

In this section, we discuss the upper semicontinuity, the lower semicontinuity, the Hausdorff lower semicontinuity, continuity, H-continuity of solution sets for generalized parametric vector quasi-equilibrium problems ($\text{QEP}_{\alpha\rho}$).

For each $\lambda \in \Lambda$, $\gamma \in \Gamma$, $\mu \in M$, let $\Psi_{\alpha\rho} : \Lambda \times \Gamma \times M \to 2^A$ e a set-valued mapping such that $\Psi_{\alpha\rho}(\lambda, \gamma, \mu)$ is the solution set of ($\text{QEP}_{\alpha\rho}$).

Throughout the article, we assume that $\Psi_{\alpha\rho}(\lambda, \gamma, \mu) \neq \emptyset$ for each $(\lambda, \gamma, \mu)$ in the neighborhoods $(\lambda_0, \gamma_0, \mu_0) \in \Lambda \times \Gamma \times M$.

**Corollary 16** *Assume for problem* ($\text{QEP}_{\alpha\rho}$) *that*

*(i)* $E$ *is usc at* $\lambda_0$ *and* $E(\lambda_0)$ *is compact, and* $K_2$ *is lsc in* $K_1(A, \Lambda) \times \{\lambda_0\}$*;*

*(ii) in* $K_1(A, \Lambda) \times K_2(K_1(A, \Lambda), \Lambda) \times \{\gamma_0\}$*,* $T$ *is usc and compact-valued if* $\alpha = w$ *(or* $\alpha = m$*), and lsc if* $\alpha = s$*;*

*(iii) in* $K_1(A, \Lambda) \times T(K_1(A, \Lambda), K_2(K_1(A, \Lambda), \Lambda), \Gamma) \times K_2(K_1(A, \Lambda), \Lambda) \times \{\mu_0\}$*, the set* $\{(x, t, y, \mu) \in A \times B \times A \times M | \rho(F(\bar{x}, t, y, \mu); C)\}$ *is closed.*

*Then* $\Psi_{\alpha\rho}$ *is both usc and closed at* $(\lambda_0, \gamma_0, \mu_0)$.

**Proof**. Since $\alpha = \{w, m, s\}$, $\rho = \{\rho_1, \rho_2\}$, we have in fact six cases. However, the proof techniques are similar. We consider only the cases $\alpha = w$, $\rho = \rho_1$. Let relation $R$ be defined by $R(x, t, y, \mu)$ holds iff $F(\bar{x}, t, y, \mu) \subseteq C$. To apply Theorem 1, we need to check only that in $K_1(A, \Lambda) \times T(K_1(A, \Lambda), K_2(K_1(A, \Lambda), \Lambda), \Gamma) \times K_2(K_1(A, \Lambda), \Lambda) \times \{\mu_0\}$, the set $\{(x, t, y, \mu) \in A \times B \times A \times M | F(\bar{x}, t, y, \mu) \subseteq C\}$ is closed.

Indeed, for all nets $\{(x_n, t_n, y_n, _n)\} \rightarrow (x_0, t_0, y_0, \mu_0)$ such that

$$R(x_n, t_n, y_n, \mu_n) \quad \text{holds.}$$

By assumption (iii), we have

$$F(x_0, t_0, y_0, \mu_0) \subseteq C.$$

□

**Corollary 17** *Assume for problem* (QEP$_{\alpha\rho}$) *that*

*(i) E is lsc at $\lambda_0$, $K_2$ is usc and compact-valued in $K_1(A, \Lambda) \times \{\lambda_0\}$;*

*(ii) in $K_1(A, \Lambda) \times K_2(K_1(A, \Lambda), \Lambda) \times \{\gamma_0\}$, T is usc and compact-valued if $\alpha = s$, and lsc if $\alpha = w$ (or $\alpha = m$);*

*(iii) in $K_1(A, \Lambda) \times T(K_1(A, \Lambda), K_2(K_1(A, \Lambda), \Lambda), \Gamma) \times K_2(K_1(A, \Lambda), \Lambda) \times \{\mu_0\}$, the set $\{(x, t, y, \mu) \in A \times B \times A \times M | \bar{\rho}(F(x, t, y, \mu); C)\}$ is closed.*

*Then $\Psi_{\alpha\rho}$ is lower semicontinuous at $(\lambda_0, \gamma_0, \mu_0)$.*

**Proof**. Since $\alpha = \{w, m, s\}$, $\rho = \{\rho_1, \rho_2\}$, we have in fact six cases. However, the proof techniques are similar. We consider only the cases $\alpha = s$, $\rho = \rho_1$. Let relation $R$ be defined by $R(x, t, y, \mu)$ holds iff $F(x, t, y, \mu) \subseteq C$. To apply Theorem 9, we need to check only that in $K_1(A, \Lambda) \times T\ (K_1(A, \Lambda), K_2(K_1(A, \Lambda), \Lambda), \Gamma) \times K_2(K_1(A, \Lambda), \Lambda) \times \{\mu_0\}$, the set $\{(x, t, y, \mu) \in A \times B \times A \times M \mid F(x, t, y, \mu) \subseteq C)\}$ is closed.

Indeed, for all nets $\{(x_n, t_n, y_n, \mu_n)\} \rightarrow (x_0, t_0, y_0, \mu_0)$ such that

$$R(x_n, t_n, y_n, \mu_n) \quad \text{does not hold.}$$

By assumption (iii), we have

$$F(x_0, t_0, y_0, \mu_0) \not\subset C.$$

□

**Corollary 18** *Impose the assumption of Corollary 17 and the following additional conditions:*

*(iv) $K_2(., \lambda_0)$ is lsc in $K_1(A, \Lambda)$ and $E(\lambda_0)$ is compact;*

*(v) in $K_1(A, \Lambda) \times K_2(K_1(A, \Lambda), \Lambda)$, $T\ (., ., \gamma_0)$ is usc and compact-valued if $\alpha = w$ (or $\alpha = m$), and lsc if $\alpha = s$;*

*(vi) in $K_1(A, \Lambda) \times T(K_1(A, \Lambda), K_2(K_1(A, \Lambda), \Lambda), \Gamma) \times K_2(K_1(A, \Lambda), \Lambda)$, the set $\{(x, t, y) \in A \times B \times A \mid \rho(F(x, t, y, \mu_0); C)\}$ is closed.*

*Then $\Psi_{\alpha\rho}$ is Hausdorff lower semicontinuous at $(\lambda_0, \gamma_0, \mu_0)$.*

**Proof**. Since $\alpha = \{w, m, s\}$, $\rho = \{\rho_1, \rho_2\}$, we have in fact six cases. However, the proof techniques are similar. We consider only the cases $\alpha = s$, $\rho = \rho_1$. Let relation $R$ be defined by $R(x, t, y, \mu)$ holds iff $F(x, t, y, \mu) \subseteq C$. To apply Theorem 12, we need to check only that in $K_1(A, \Lambda) \times T\ (K_1(A, \Lambda), K_2(K_1(A, \Lambda), \Lambda), \Gamma) \times K_2(K_1(A, \Lambda), \Lambda)$, the set $\{(x, t, y) \in A \times B \times A \mid F(x, t, y, \mu_0) \subseteq C)\}$ is closed. Indeed, for all nets $\{(x_n, t_n, y_n)\} \rightarrow (x_0, t_0, y_0)$ such that $R(x_n, t_n, y_n, \mu_0)$ holds. By assumption (vi), we have $F(x_0, t_0, y_0, \mu_0) \subseteq C$. □

**Remark 19** (i) Suppose that all conditions in Corollaries 16 and 17 are satisfied. Then, we have $\Psi_\alpha$ is both continuous and closed at $(\lambda_0, \gamma_0, \mu_0)$.

(ii) Suppose that all conditions in Corollaries 16 and 18 are satisfied. Then, we have $\Psi_{\alpha\rho}$ is Hausdorff continuous and closed at $(\lambda_0, \gamma_0, \mu_0)$.

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The author declares that they have no competing interests.

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