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Common fixed-point results for nonlinear contractions in ordered partial metric spaces

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Abstract

In this paper, a new class of a pair of generalized nonlinear contractions on partially ordered partial metric spaces is introduced, and some coincidence and common fixed-point theorems for these contractions are proved. Presented theorems are twofold generalizations of very recent fixed-point theorems of Altun and Erduran (Fixed Point Theory Appl 2011(Article ID 508730):10, 2011), Altun et al. (Topol Appl 157(18):2778-2785, 2010), Matthews (Proceedings of the 8th summer conference on general topology and applications, New York Academy of Sciences, New York, pp. 183-197, 1994) and many other known corresponding theorems.
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1 Introduction

It is well known that the Banach contraction principle is a very useful, simple and classical tool in nonlinear analysis. There exist a vast literature concerning its various generalizations and extensions (see [1-45]). In [22], Matthews extended the Banach contraction mapping theorem to the partial metric context for applications in program verification. After that, fixed-point results in partial metric spaces have been studied [4,8,28,31,34,45]. The existence of several connections between partial metrics and topological aspects of domain theory has been pointed by many authors (see [8,9,16,23,31,33,36-38,41,42,46,47]).

First, we recall some definitions of partial metric spaces and some their properties.

Definition 1.1 *A partial metric on a set X is a function $p : X \times X \to \mathbb{R}^+$ such that for all $x, y, z \in X$:*

(p1) $x = y \Leftrightarrow p(x, x) = p(x, y) = p(y, y)$,

(p2) $p(x, x) \le p(x, y)$,

(p3) $p(x, y) = p(y, x)$,

(p4) $p(x, y) \le p(x, z) + p(z, y) - p(z, z)$.

Note that the self-distance of any point need not be zero, hence the idea of generalizing metrics so that a metric on a non-empty set X is precisely a partial metric p on X such that for any $x \in X$, $p(x, x) = 0$.

Similar to the case of metric space, a partial metric space is a pair (X, p) consisting of a non-empty set X and a partial metric p on X.

Example 1.1 *Let a function* $p : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ *be defined by* $p(x, y) = \max\{x, y\}$ *for any* $x, y \in \mathbb{R}^+$. *Then,* $(\mathbb{R}^+, p)$ *is a partial metric space where the self-distance for any point* $x \in \mathbb{R}^+$ *is its value itself.*

Example 1.2 *Consider a function* $p : \mathbb{R}^- \times \mathbb{R}^- \to \mathbb{R}^+$ *defined by* $p(x, y) = -\min(x, y)$ *for any* $x, y \in \mathbb{R}^-$. *The pair* $(\mathbb{R}^-, p)$ *is a partial metric space for which p is called the usual partial metric on* $\mathbb{R}^-$ *and where the self-distance for any point* $x \in \mathbb{R}^-$ *is its absolute value.*

Example 1.3 *If* $X := \{[a, b] \mid a, b \in \mathbb{R}, a \leq b\}$, *then* $p : X \times X \to \mathbb{R}^+$ *defined by* $p([a, b], [c, d]) = \max\{b, d\} - \min\{a, b\}$ *defines a partial metric on X.*

Each partial metric p on X generates a T_0 topology τ_p on X, which has as a base the family of open p-balls $\{B_p(x, \varepsilon), x \in X, \varepsilon > 0\}$, where

$$B_p(x, \varepsilon) = \{y \in X | p(x, y) < p(x, x) + \varepsilon\} \text{ for all } x \in X \text{ and } \varepsilon > 0.$$

If p is a partial metric on X, then the function $p^s : X \times X \to \mathbb{R}^+$ defined by

$$p^s(x, y) = 2p(x, y) - p(x, x) - p(y, y)$$

is a metric on X.

Definition 1.2 *Let* (X, p) *be a partial metric space and* $\{x_n\}$ *be a sequence in X. Then,*

(*i*) $\{x_n\}$ *converges to a point* $x \in X$ *if and only if* $p(x, x) = \lim_{n\to+\infty} p(x, x_n)$,

(*ii*) $\{x_n\}$ *is a Cauchy sequence if there exists (and is finite)* $\lim_{n,m\to+\infty} p(x_n, x_m)$.

Definition 1.3 *A partial metric space* (X, p) *is said to be complete if every Cauchy sequence* $\{x_n\}$ *in X converges, with respect to* τ_p, *to a point* $x \in X$, *such that* $p(x, x) = \lim_{n,m\to+\infty} p(x_n, x_m)$.

Remark 1.1 *It is easy to see that every closed subset of a complete partial metric space is complete.*

Lemma 1.1 ([22,28]) *Let* (X, p) *be a partial metric space. Then*

(*a*) $\{x_n\}$ *is a Cauchy sequence in* (X, P) *if and only if it is a Cauchy sequence in the metric space* (X, P^s),

(*b*) (X, p) *is complete if and only if the metric space* (X, p^s) *is complete. Furthermore,* $\lim_{n\to+\infty} p^s(x_n, x) = 0$ *if and only if*

$$p(x, x) = \lim_{n\to+\infty} p(x_n, x) = \lim_{n,m\to+\infty} p(x_n, x_m).$$

Matthews [22] obtained the following Banach fixed-point theorem on complete partial metric spaces.

Theorem 1.1 (Matthews [22]) *Let f be a mapping of a complete partial metric space* (X, p) *into itself such that there is a constant* $c \in [0,1)$ *satisfying for all* $x, y \in X$:

$$p(fx, fy) \leq cp(x, y).$$

Then, f has a unique fixed point.

Recently, Altun et al. [4] obtained the following nice result, which generalizes Theorem 1.1 of Matthews.

Theorem 1.2 (Altun et al. [4]) *Let* (X, p) *be a complete partial metric space and let* $T : X \to X$ *be a map such that*

$$p(Tx, Ty) \leq \varphi\left(\max\left\{p(x, y), p(x, Tx), p(y, Ty), \frac{1}{2}[p(x, Ty) + p(y, Tx)]\right\}\right)$$

for all $x, y \in X$, where $\phi : [0, +\infty) \to [0, +\infty)$ satisfies the following conditions:

(i) ϕ is continuous and non-decreasing,

(ii) $\sum_{n \geq 1} \varphi^n(t)$is convergent for each $t > 0$.

Then, T has a unique fixed point.

On the other hand, existence of fixed points in partially ordered sets has been considered recently in [32], and some generalizations of the result of [32] are given in [1-3,5-7,11,12,14,15,17,19,24-27,29,30,39,40,43] in partial ordered metric spaces. Also, in [32], some applications to matrix equations are presented, and in [15] and [26], some applications to ordinary differential equations are given. In [29], O'Regan and Petruşel established some fixed-point results for self-generalized contractions in ordered metric spaces. Jachymski [19] established a geometric lemma [19, Lemma 1], giving a list of equivalent conditions for some subsets of the plane. Using this lemma, he proved that some very recent fixed-point theorems for generalized contractions on ordered metric spaces obtained by Harjani and Sadarangani [15] and Amini-Harandi and Emami [5] do follow from an earlier result of O'Regan and Petruşel [29, Theorem 3.6].

Very recently, Altun and Erduran [3] generalized Theorem 1.2 to partially ordered complete partial metric spaces and established the following new fixed-point theorems, involving a function $\phi : [0, +\infty) \to [0, +\infty)$ satisfying the conditions (i)-(ii) in Theorem 1.2.

Theorem 1.3 (Altun and Erduran [3]). *Let $(X, \preccurlyeq)$ be a partially ordered set and suppose that there is a partial metric p on X such that (X, p) is a complete partial metric space. Suppose $F : X \to X$ is a continuous and non-decreasing mapping (with respect to $\preccurlyeq$) such that*

$$p(Fx, Fy) \leq \varphi\left(\max\left\{p(x, y), p(x, Fx), p(y, Fy), \frac{1}{2}[p(x, Fy) + p(y, Fx)]\right\}\right)$$

for all $x, y \in X$ with $y \preccurlyeq x$, where $\phi : [0, +\infty) \to [0, +\infty)$ satisfies conditions (i)-(ii) in Theorem 1.2. *If there exists $x_0 \in X$ such that $x_0 \preccurlyeq Fx_0$, then there exists $x \in X$ such that $Fx = x$. Moreover, $p(x, x) = 0$.*

Theorem 1.4 (Altun and Erduran [3]) *Let $(X, \preccurlyeq)$ be a partially ordered set and suppose that there is a partial metric p on X such that (X, p) is a complete partial metric space. Suppose $F : X \to X$ is a non-decreasing mapping such that*

$$p(Fx, Fy) \leq \varphi\left(\max\left\{p(x, y), p(x, Fx), p(y, Fy), \frac{1}{2}[p(x, Fy) + p(y, Fx)]\right\}\right)$$

for all $x, y \in X$ with $y \prec x$ ($y \preccurlyeq x$ and $y \neq x$), where $\phi : [0, +\infty) \to [0, +\infty)$ satisfies conditions (i)-(ii) in Theorem 1.2. *Suppose also that the condition*

$$\begin{cases} \text{if } \{x_n\} \subset X \text{ is a increasing sequence} \\ \text{with } x_n \to x \in X, \text{ then } x_n \prec x \text{ for all } n \end{cases}$$

holds. If there exists $x_0 \in X$ such that $x_0 \preccurlyeq Fx_0$, then there exists $x \in X$ such that $Fx = x$. Moreover, $p(x, x) = 0$.

Theorem 1.5 (Altun and Erduran [3]) *Let $(X, \preccurlyeq)$ be a partially ordered set and suppose that there is a partial metric p on X such that (X, p) is a complete partial metric space. Suppose $F : X \to X$ is a continuous and non-decreasing mapping such that*

$$p(Fx, Fy) \leq \varphi\left(\max\left\{p(x, y), \frac{1}{2}[p(x, Fx) + p(y, Fy)], \frac{1}{2}[p(x, Fy) + p(y, Fx)]\right\}\right)$$

for all $x, y \in X$ with $y \preccurlyeq x$, where $\phi : [0, +\infty) \to [0, +\infty)$ satisfies conditions (i)-(ii) in Theorem 1.2. *If there exists $x_0 \in X$ such that $x_0 \preccurlyeq Fx_0$, then there exists $x \in X$ such that $Fx = x$. Moreover, $p(x, x) = 0$. If we suppose that for all $x, y \in X$ there exists $z \in X$, which is comparable to x and y, we obtain uniqueness of the fixed point of F.*

Altun et al. [4], Altun and Erduran [3] and many authors have obtained fixed-point theorems for contractions under the assumption that a comparison function $\phi : [0, +\infty) \to [0, +\infty)$ is non-decreasing and such that $\sum_{n=1}^{\infty} \varphi^n(t) < \infty$ for each $t > 0$ (see, e. g., [13] and the references in [11,18]-Added in proof). However, the latter condition is strong and rather hard to verify in practice, though some examples and general criteria for this convergence are known (see, e.g., [3,44]). So a natural question arises whether this strong condition can be omitted in partial metric fixed-point theory.

The aims of this paper is to establish coincidence and common fixed-point theorems in ordered partial metric spaces with a function ϕ satisfying the condition $\phi(t) < t$ for all $t > 0$, which is weaker than the condition $\sum_{n=1}^{\infty} \varphi^n(t) < \infty$. Presented theorems generalize and extend to a pair of mappings the results of Altun and Erduran [3], Altun et al. [4], Matthews [22] and many other known corresponding theorems.

2 Main results

We start this section by some preliminaries.

Definition 2.1 (Altun and Erduran [3]) *Let (X, p) be a partial metric space, $F : X \to X$ be a given mapping. We say that F is continuous at $x_0 \in X$, if for every $\varepsilon > 0$, there exists $\delta > 0$ such that $F(B_p(x_0, \delta)) \subseteq B_p(Fx_0, \varepsilon)$.*

The following result is easy to check.

Lemma 2.1 *Let (X, p) be a partial metric space, $F : X \to X$ be a given mapping. Suppose that F is continuous at $x_0 \in X$. Then, for all sequence $\{x_n\} \subset X$, we have*

$$x_n \to x_0 \Rightarrow Fx_n \to Fx_0.$$

Definition 2.2 (Ćirić et al. [11]) *Let $(X, \preccurlyeq)$ be a partially ordered set and $F, g : X \to X$ are mappings of X into itself. One says F is g-non-decreasing if for $x, y \in X$, we have*

$$gx \preccurlyeq gy \Rightarrow Fx \preccurlyeq Fy.$$

We introduce the following definition.

Definition 2.3 *Let (X, p) be a partial metric space and $F, g: X \to X$ are mappings of X into itself. We say that the pair $\{F, g\}$ is partial compatible if the following conditions hold:*

(b1) $p(x, x) = 0 \Rightarrow p(gx, gx) = 0$,

(b2) $\lim_{n\to+\infty} p(Fgx_n, gFx_n) = 0$, *whenever $\{x_n\}$ is a sequence in X such that $Fx_n \to t$ and $gx_n \to t$ for some $t \in X$.*

It is clear that Definition 2.3 extends and generalizes the notion of compatibility introduced by Jungck [21].

Define by φ the set of functions $\phi : [0, +\infty) \to [0, +\infty)$ satisfying the following conditions:

(c1) ϕ is continuous and non-decreasing,

(c2) $\phi(t) < t$ for each $t > 0$.

Now, we are ready to state and prove our first result.

Theorem 2.1 *Let* $(X, \preccurlyeq)$ *be a partially ordered set and suppose that there is a partial metric* p *on* X *such that* (X, p) *is a complete partial metric space. Let* $F, g : X \to X$ *be two continuous self-mappings of* X *such that* $FX \subseteq gX$, F *is a* g*-non-decreasing mapping, the pair* $\{F, g\}$ *is partial compatible, and*

$$p(Fx, Fy) \leq \varphi\left(\max\left\{p(gx, gy), p(gx, Fx), p(gy, Fy), \frac{1}{2}[p(gx, Fy) + p(gy, Fx)]\right\}\right) \tag{1}$$

for all $x, y \in X$ *for which* $gy \preccurlyeq gx$, *where a function* $\phi \in \varphi$. *If there exists* $x_0 \in X$ *with* $gx_0 \preccurlyeq Fx_0$, *then* F *and* g *have a coincidence point, that is, there exists* $x \in X$ *such that* $Fx = gx$. *Moreover, we have* $p(x, x) = p(Fx, Fx) = p(gx, gx) = 0$.

Proof. Let $x_0 \in X$ such that $gx_0 \preccurlyeq Fx_0$. Since $FX \subseteq gX$, we can choose $x_1 \in X$ so that $gx_1 = Fx_0$. Again, from $FX \subseteq gX$, there exists $x_2 \in X$ such that $gx_2 = Fx_1$. Continuing this process, we can choose a sequence $\{x_n\} \subset X$ such that

$$gx_{n+1} = Fx_n, \quad \forall n \geq 0.$$

Since $gx_0 \preccurlyeq Fx_0$ and $Fx_0 = gx_1$, then $gx_0 \preccurlyeq gx_1$. Since F is a g-non-decreasing mapping, we have $Fx_0 \preccurlyeq Fx_1$, that is, $gx_1 \preccurlyeq gx_2$. Again, using that F is a g-non-decreasing mapping, we have $Fx_1 \preccurlyeq Fx_2$, that is, $gx_2 \preccurlyeq gx_3$. Continuing this process, we get

$$gx_1 \preccurlyeq gx_2 \preccurlyeq gx_3 \preccurlyeq \cdots \preccurlyeq gx_n \preccurlyeq gx_{n+1} \preccurlyeq \cdots \tag{2}$$

Suppose that there exists $n \in N$ such that $p(Fx_n, Fx_{n+1}) = 0$. This implies that $Fx_n = Fx_{n+1}$, that is, $gx_{n+1} = Fx_{n+1}$. Then, x_{n+1} is a coincidence point of F and g, and so we have finished the proof. Thus, we can assume that

$$p(Fx_n, Fx_{n+1}) > 0, \quad \forall n \in \mathbb{N}. \tag{3}$$

We will show that

$$p(Fx_n, Fx_{n+1}) \leq \varphi(p(Fx_{n-1}, Fx_n)) \text{ for all } n \geq 1. \tag{4}$$

Using (2) and applying the considered contraction (1) with $x = x_n$ and $y = x_{n+1}$, we get

$$\begin{aligned}
&p(Fx_n, Fx_{n+1}) \leq \\
&\varphi\left(\max\left\{p(gx_n, gx_{n+1}), p(Fx_n, gx_n), p(Fx_{n+1}, gx_{n+1}), \frac{1}{2}[p(gx_n, Fx_{n+1}) + p(Fx_n, gx_{n+1})]\right\}\right) \\
&= \varphi\left(\max\left\{p(Fx_{n-1}, Fx_n), p(Fx_{n+1}, Fx_n), \frac{1}{2}[p(Fx_{n-1}, Fx_{n+1}) + p(Fx_n, Fx_n)]\right\}\right) \\
&\leq \varphi\left(\max\left\{p(Fx_{n-1}, Fx_n), p(Fx_{n+1}, Fx_n), \frac{1}{2}[p(Fx_{n-1}, Fx_n) + p(Fx_n, Fx_{n+1})]\right\}\right).
\end{aligned}$$

Hence, as

$$p(Fx_n, Fx_n) + p(Fx_{n-1}, Fx_{n+1}) \leq p(Fx_{n-1}, Fx_n) + p(Fx_n, Fx_{n+1})$$

and ϕ is non-decreasing, we have

$$p(Fx_n, Fx_{n+1}) \leq \varphi\left(\max\left\{p(Fx_{n-1}, Fx_n), p(Fx_{n+1}, Fx_n)\right\}\right). \tag{5}$$

If we suppose that $\max\{p(Fx_{n-1}, Fx_n), p(Fx_{n+1}, Fx_n)\} = p(Fx_{n+1}, Fx_n)$, then from (5),

$$p(Fx_n, Fx_{n+1}) \le \varphi(p(Fx_{n+1}, Fx_n)).$$

Using (3) and the fact that $\phi(t) < t$ for all $t > 0$, we have

$$p(Fx_n, Fx_{n+1}) \le \varphi(p(Fx_{n+1}, Fx_n)) < p(Fx_{n+1}, Fx_n),$$

a contradiction. Therefore,

$$\max\{p(Fx_{n-1}, Fx_n), p(Fx_{n+1}, Fx_n)\} = p(Fx_{n-1}, Fx_n),$$

and so from (5),

$$p(Fx_n, Fx_{n+1}) \le \varphi(p(Fx_{n-1}, Fx_n)).$$

Thus, we proved (4).

Since ϕ is non-decreasing, repeating the inequality (4) n times, we get

$$p(Fx_n, Fx_{n+1}) \le \varphi^n(p(Fx_0, Fx_1)), \quad \forall n \in \mathbb{N}. \tag{6}$$

Letting $n \to +\infty$ in the inequality (6) and using the fact that $\phi^n(t) \to 0$ as $n \to +\infty$ for all $t > 0$, we obtain

$$\lim_{n\to+\infty} p(Fx_n, Fx_{n+1}) = 0. \tag{7}$$

On the other hand, we have

$$\begin{aligned} p^s(Fx_n, Fx_{n+1}) &= 2p(Fx_n, Fx_{n+1}) - p(Fx_n, Fx_n) - p(Fx_{n+1}, Fx_{n+1}) \\ &\le 2p(Fx_n, Fx_{n+1}). \end{aligned}$$

Letting $n \to +\infty$ in this inequality, by (7), we get

$$\lim_{n\to+\infty} p^s(Fx_n, Fx_{n+1}) = 0. \tag{8}$$

Now, we shall prove that $\{Fx_n\}$ is a Cauchy sequence in the metric space (X, p^s). Suppose, to the contrary, that $\{Fx_n\}$ is not a Cauchy sequence in (X, p^s). Then, there exists $\varepsilon > 0$ such that for each positive integer k, there exist two sequences of positive integers $\{m(k)\}$ and $\{n(k)\}$ such that

$$n(k) > m(k) > k \quad \text{and} \quad p^s(Fx_{m(k)}, Fx_{n(k)}) \ge \varepsilon. \tag{9}$$

Since $p^s(x, y) \le 2p(x, y)$ for all $x, y \in X$, from (9), for all positive integer k, we have

$$n(k) > m(k) > k \quad \text{and} \quad p(Fx_{m(k)}, Fx_{n(k)}) \ge \frac{\varepsilon}{2}.$$

Without loss of generality, we can suppose that also

$$n(k) > m(k) > k, \quad p(Fx_{m(k)}, Fx_{n(k)}) \ge \frac{\varepsilon}{2}, \quad p(Fx_{m(k)}, Fx_{n(k)-1}) < \frac{\varepsilon}{2}. \tag{10}$$

From (10) and the triangular inequality (that holds for a partial metric), we have

$$\begin{aligned} \frac{\varepsilon}{2} &\le p(Fx_{m(k)}, Fx_{n(k)}) \\ &\le p(Fx_{m(k)}, Fx_{n(k)-1}) + p(Fx_{n(k)-1}, Fx_{n(k)}) - p(Fx_{n(k)-1}, Fx_{n(k)-1}) \\ &< \frac{\varepsilon}{2} + p(Fx_{n(k)-1}, Fx_{n(k)}). \end{aligned}$$

Letting $k \to +\infty$ and using (7), we get

$$\lim_{k\to+\infty} p(Fx_{m(k)}, Fx_{n(k)}) = \frac{\varepsilon}{2}. \tag{11}$$

Again, using the triangular inequality, we obtain

$$\begin{aligned}\frac{\varepsilon}{2} &\le p(Fx_{m(k)}, Fx_{n(k)}) \le p(Fx_{m(k)}, Fx_{m(k)-1}) + p(Fx_{m(k)-1}, Fx_{n(k)})\\ &\le p(Fx_{m(k)}, Fx_{m(k)-1}) + p(Fx_{n(k)}, Fx_{m(k)}) + p(Fx_{m(k)-1}, Fx_{m(k)}).\end{aligned}$$

Letting $k \to +\infty$ in this inequality, and using (11) and (7), we get

$$\frac{\varepsilon}{2} \le \lim_{k\to+\infty} p(Fx_{n(k)}, Fx_{m(k)-1}) \le \frac{\varepsilon}{2}.$$

Hence,

$$\lim_{k\to+\infty} p(Fx_{n(k)}, Fx_{m(k)-1}) = \frac{\varepsilon}{2}. \tag{12}$$

On the other hand, we have

$$p(Fx_{n(k)}, Fx_{m(k)}) \;\le\; p(Fx_{n(k)}, Fx_{n(k)+1}) + p(Fx_{n(k)+1}, Fx_{m(k)}). \tag{13}$$

From (1) with $x = x_n$ and $y = x_{n+1}$, we get

$$\begin{aligned}&p(Fx_{n(k)+1}, Fx_{m(k)}) \le\\ &\varphi\left(\max\left\{p(Fx_{n(k)}, Fx_{m(k)-1}), p(Fx_{n(k)+1}, Fx_{n(k)}), p(Fx_{m(k)}, Fx_{m(k)-1}),\right.\right.\\ &\left.\left.\frac{1}{2}[p(Fx_{n(k)}, Fx_{m(k)}) + p(Fx_{n(k)+1}, Fx_{m(k)-1})]\right\}\right)\\ &\le \varphi\left(\max\left\{p(Fx_{n(k)}, Fx_{m(k)-1}), p(Fx_{n(k)+1}, Fx_{n(k)}), p(Fx_{m(k)}, Fx_{m(k)-1}),\right.\right.\\ &\left.\left.\frac{1}{2}[p(Fx_{n(k)}, Fx_{m(k)}) + p(Fx_{n(k)+1}, Fx_{n(k)}) + p(Fx_{n(k)}, Fx_{m(k)-1})]\right\}\right)\\ &:= \varphi(\xi(k)).\end{aligned}$$

Therefore, from (13) and since ϕ is a non-decreasing function, we get

$$p(Fx_{n(k)}, Fx_{m(k)}) \le p(Fx_{n(k)}, F_{n(k)+1}) + \varphi(\xi(k)).$$

Letting $k \to +\infty$ in the above inequality, using (7), (11), (12) and the continuity of ϕ, we have

$$\frac{\varepsilon}{2} \le \varphi\left(\frac{\varepsilon}{2}\right) < \frac{\varepsilon}{2},$$

a contradiction. Thus, our supposition that $\{Fx_n\}$ is not a Cauchy sequence was wrong. Therefore, $\{Fx_n\}$ is a Cauchy sequence in the metric space (X, p^s), and so we have

$$\lim_{m,n\to+\infty} p^s(Fx_n, Fx_m) = 0. \tag{14}$$

Now, since (X, p) is complete, then from Lemma 1.1, (X, p^s) is a complete metric space. Therefore, the sequence $\{Fx_n\}$ converges to some $x \in X$, that is,

$$\lim_{n\to+\infty} p^s(Fx_n, x) = \lim_{n\to+\infty} p^s(gx_{n+1}, x) = 0.$$

From the property (b) in Lemma 1.1, we have

$$p(x,x)=\lim_{n\to+\infty}p(Fx_n,x)=\lim_{n\to+\infty}p(gx_{n+1},x)=\lim_{m,n\to+\infty}p(Fx_n,Fx_m). \tag{15}$$

On the other hand, from property (p2) of a partial metric, we have

$$p(Fx_n,Fx_n)\le p(Fx_n,Fx_{n+1}) \text{ for all } n\in\mathbb{N}.$$

Letting $n\to+\infty$ in the above inequality and using (7), we obtain

$$\lim_{n\to+\infty}p(Fx_n,Fx_n)=0.$$

Therefore, from the definition of p^s and using (14), we get $\lim_{m,n\to+\infty} p(Fx_n, Fx_m) = 0$. Thus, from (15), we have

$$p(x,x)=\lim_{n\to+\infty}p(Fx_n,x)=\lim_{m,n\to+\infty}p(Fx_n,Fx_m)=0. \tag{16}$$

Now, since F is continuous, from (16) and using Lemma 2.1, we get

$$\lim_{n\to+\infty}p(F(Fx_n),Fx)=p(Fx,Fx). \tag{17}$$

Using the triangular inequality, we obtain

$$p(Fx,gx)\le p(Fx,F(Fx_n))+p(F(gx_{n+1}),g(Fx_{n+1}))+p(g(Fx_{n+1}),gx). \tag{18}$$

Letting $n\to+\infty$ in the above inequality, using (17), (15), (16), the partial compatibility of $\{F, g\}$, the continuity of g and Lemma 2.1, we have

$$p(Fx,gx)\le p(Fx,Fx)+p(gx,gx)=p(Fx,Fx). \tag{19}$$

Now, suppose that $p(Fx, gx) > 0$. Then, from (1) with $x = y$, we get

$$p(Fx,Fx)\le\varphi\left(\max\{p(gx,gx),p(Fx,gx)\}\right)=\varphi\left(p(Fx,gx)\right)<p(Fx,gx).$$

Therefore, from (19), we have

$$p(Fx,gx)<p(Fx,gx),$$

a contradiction. Thus, we have $p(Fx, gx) = 0$, which implies that $Fx = gx$, that is, x is a coincidence point of F and g. Moreover, from (16) and since the pair $\{F, g\}$ is partial compatible, we have $p(x, x) = 0 = p(gx, gx) = p(Fx, Fx)$. This completes the proof. ■

An immediate consequence of Theorem 2.1 is the following result.

Theorem 2.2 *Let* $(X, \preccurlyeq)$ *be a partially ordered set and suppose that there is a partial metric p on X such that* (X, p) *is a complete partial metric space. Suppose* $F : X \to X$ *is a continuous and non-decreasing mapping (with respect to* $\preccurlyeq$*) such that*

$$p(Fx,Fy)\le\varphi\left(\max\left\{p(x,y),p(x,Fx),p(y,Fy),\frac{1}{2}[p(x,Fy)+p(y,Fx)]\right\}\right) \tag{20}$$

for all $x, y \in X$ *with* $y \preccurlyeq x$, *where* $\phi : [0, +\infty) \to [0, +\infty)$ *is continuous non-decreasing and* $\phi(t) < t$ *for all* $t > 0$. *If there exists* $x_0 \in X$ *such that* $x_0 \preccurlyeq Fx_0$, *then there exists* $x \in X$ *such that* $Fx = x$. *Moreover,* $p(x, x) = 0$.

Proof. Putting $gx = Ix = x$ in Theorem 2.1, we obtain Theorem 2.2. ■

Now we shall present an example in which $F: X \to X$ and $\phi : [0, +\infty) \to [0, +\infty)$ satisfy all hypotheses of our Theorem 2.2, but not the hypotheses of Theorems of

Altun et al. [4], Altun and Erduran [3] with ϕ given in an illustrative example in [3], Matthews [22] and of many other known corresponding theorems.

Before giving our example, we need the following result.

Lemma 2.2 *Consider $X = [0, +\infty)$ endowed with the partial metric $p : X \times X \to [0, +\infty)$ defined by $p(x, y) = \max\{x, y\}$ for all $x, y \geq 0$. Let $F : X \to X$ be a non-decreasing function. If F is continuous with respect to the standard metric $d(x, y) = |x - y|$ for all $x, y \geq 0$, then F is continuous with respect to the partial metric p.*

Proof. Let $\{x_n\}$ be a sequence in X such that $\lim_{n\to+\infty} p(x_n, x) = p(x, x)$ for some $x \in X$, that is, $\lim_{n\to+\infty} \max\{x_n, x\} = x$. Using Lemma 2.1, we have to prove that $\lim_{n\to+\infty} p(Fx_n, Fx) = p(Fx, Fx)$, that is, $\lim_{n\to+\infty} \max\{Fx_n, Fx\} = Fx$.

Since F is a non-decreasing mapping, we have

$$\max\{Fx_n, Fx\} = F(\max\{x_n, x\}). \tag{21}$$

Now, using that F is continuous with respect to the standard metric, we have

$$\lim_{n\to+\infty} \max\{x_n, x\} = x \Rightarrow \lim_{n\to+\infty} F(\max\{x_n, x\}) = Fx.$$

Therefore, from (21), it follows that

$$\lim_{n\to+\infty} \max\{Fx_n, Fx\} = Fx.$$

This makes end to the proof. ■

Example 2.1 *Let $X = [0, +\infty)$ and (X, p) be a complete partial metric space, where $p : X \times X \to \mathbb{R}^+$ is defined by $p(x, y) = \max\{x, y\}$. Let us define a partial order $\preccurlyeq$ on X as follows:*

$$x \preccurlyeq y \Leftrightarrow x = y \text{ or } (x, y \in [0, 1) \text{ with } x \leq y).$$

Define $F : X \to X$ by

$$F(x) = \begin{cases} \dfrac{x}{1+x} & \text{if } x \in [0, 1), \\ \dfrac{\sqrt{x}}{2} & \text{if } x \geq 1, \end{cases}$$

and let $\phi : [0, +\infty) \to [0, +\infty)$ be defined by

$$\varphi(t) = \begin{cases} \dfrac{t}{1+t} & \text{if } t \in (0, 1], \\ \dfrac{t}{2} & \text{if } t > 1. \end{cases}$$

Clearly the function $\phi \in \varphi$, that is, ϕ is continuous non-decreasing and $\phi(t) < t$ for each $t > 0$. On the other hand, using Lemma 2.2, since F is non-decreasing (with respect to the usual order) and continuous in X with respect to the standard metric, then it is continuous with respect to the partial metric p. The function F is also non-decreasing with respect to the partial order $\preccurlyeq$.

We now show that F satisfies the nonlinear contractive condition (20) for all $x, y \in X$ with $y \preccurlyeq x$. By definition of F, we have

$$\begin{aligned} p(Fx, Fy) &= \max\left\{\frac{x}{1+x}, \frac{y}{1+y}\right\} \\ &= \frac{x}{1+x} \\ &= \varphi(\max\{x, y\}) \\ &= \varphi(p(x, y)). \end{aligned}$$

Thus,

$$p(Fx, Fy) \le \varphi\left(\max\left\{p(x, y), p(Fx, x), p(Fy, y), \frac{1}{2}[p(x, Fy) + p(Fx, y)]\right\}\right).$$

Therefore, the contractive condition (20) *is satisfied for all* $x, y \in X$ *for which* $y \preccurlyeq x$.

Also, for $x_0 = 0$, *we have* $x_0 \preccurlyeq Fx_0$.

Therefore, all hypotheses of Theorem 2.2 *are satisfied and F has a fixed point. Note that it is easy to see that the hypothesis* (23) *as well as all other hypotheses in Theorems* 2.3 *and* 2.4 *below is also satisfied.*

Observe that in this example, ϕ *does not satisfy the condition* $\sum_{n=1}^{\infty} \varphi^n(t) < \infty$ *for each* $t > 0$ *of Theorems in* [3,4]. *Indeed, let* $t_0 \in (0, 1]$ *be arbitrary. Then, it is easy to show by induction that* $\phi^n(t_0) = t_0/(1 + nt_0)$. *Thus,*

$$\sum_{n=1}^{\infty} \varphi^n(t_0) = \sum_{n=1}^{\infty} \frac{t_0}{1+nt_0} = +\infty.$$

Note that F does not satisfy the contractive condition (20) *in Theorem* 2.2 *with a function*

$$\varphi(t) = \frac{t^2}{1+t}.$$

This function is given by Altun and Erduran in their illustrative example in [3]. *It is easy to show that for* $y \preccurlyeq x$,

$$\begin{aligned} p(Fx, Fy) &= \max\left\{\frac{x}{1+x}, \frac{y}{1+y}\right\} = \frac{x}{1+x} > \frac{x^2}{1+x} \\ &= \varphi\left(\max\left\{p(x, y), p(x, Fx), p(y, Fy), \frac{1}{2}[p(x, Fy) + p(y, Fx)]\right\}\right) \\ &\ge \varphi\left(\max\left\{p(x, y), p(x, Fx), p(y, Fy), \frac{1}{2}[p(x, Fy) + p(y, Fx)]\right\}\right). \end{aligned}$$

Now, we will prove the following result.

Theorem 2.3 *Let* $(X, \preccurlyeq)$ *be a partially ordered set and suppose that there is a partial metric p on X such that* (X, p) *is a complete partial metric space. Let* $F, g : X \to X$ *be two self-mappings of X such that* $FX \subseteq gX$, *F is a g-non-decreasing mapping and,*

$$p(Fx, Fy) \le \varphi\left(\max\left\{p(gx, gy), p(gx, Fx), p(gy, Fy), \frac{1}{2}[p(gx, Fy) + p(gy, Fx)]\right\}\right) \quad (22)$$

for all $x, y \in X$ *for which* $gx \succ gy$, *where* $\phi \in \varphi$. *Also suppose*

$$\begin{cases} \text{if } \{gx_n\} \subset X \text{ is a increasing sequence} \\ \text{with } gx_n \to gz \in gX, \text{ then } gx_n \prec gz, gz \preccurlyeq g(gz) \text{ for all } n \end{cases} \quad (23)$$

holds. Also suppose gX is closed. If there exists $x_0 \in X$ *with* $gx_0 \preccurlyeq Fx_0$*, then F and g have a coincidence point* $x \in X$ *such that* $p(Fx, Fx) = p(gx, gx) = 0$*. Further, if F and g commute at their coincidence points, then F and g have a common fixed point.*

Proof. Denote

$$M[F,g](x,y) := \max\left\{p(gx,gy), p(gx,Fx), p(gy,Fy), \frac{1}{2}[p(gx,Fy)+p(gy,Fx)]\right\}$$

for all $x, y \in X$.

As in the proof of Theorem 2.1, we can construct a sequence $\{x_n\}$ in X by $gx_{n+1} = Fx_n$ for all $n \geq 0$. Also, we can assume that $Fx_n \neq Fx_{n+1}$ for all $n \geq 0$; otherwise, we are finished. Therefore, we have

$$gx_1 \prec gx_2 \prec \cdots \prec gx_n \prec gx_{n+1} \prec \cdots \tag{24}$$

Again, as in the proof of Theorem 2.1, we can show that $\{Fx_n\}$ is a Cauchy sequence in the complete metric space (X, p^s), and therefore, there exists $y \in X$ such that

$$p(y,y) = \lim_{n\to+\infty} p(Fx_n, y) = \lim_{m,n\to+\infty} p(Fx_n, Fx_m) = 0. \tag{25}$$

Since $\{Fx_n\} \subset gX$ and gX is closed, there exists $x \in X$ such that $y = gx$. From (24) and hypothesis (23), we have

$$gx_n \prec gx \text{ for all } n, \quad gx \preccurlyeq g(gx). \tag{26}$$

Now, we will show that x is a coincidence point of F and g. Using the triangular inequality, we have

$$p(gx, Fx) \leq p(gx, gx_{n+1}) + p(Fx_n, Fx).$$

From (26), using the considered contraction, we have

$$p(Fx, Fx_n) \leq \varphi(M[F,g](x,x_n)).$$

Thus,

$$p(gx, Fx) \leq p(gx, Fx_n) + \varphi(M[F,g](x,x_n)). \tag{27}$$

Now, we have

$$\begin{aligned} &M[F,g](x,x_n) = \\ &\max\left\{p(gx,Fx_{n-1}), p(Fx,gx), p(Fx_n,Fx_{n-1}), \frac{1}{2}[p(gx,Fx_n)+p(Fx,Fx_{n-1})]\right\} \\ &\leq \max\{p(gx,Fx_{n-1}), p(Fx,gx), p(Fx_n,Fx_{n-1}), \\ &\left.\frac{1}{2}[p(gx,Fx_n)+p(Fx,gx)+p(gx,Fx_{n-1})]\right\}. \end{aligned}$$

Since ϕ is a non-decreasing function, using (25), the above inequality and $n \to +\infty$ in (27), we get

$$p(gx, Fx) \leq \varphi(p(gx, Fx)).$$

If $p(gx, Fx) > 0$, we obtain $p(gx, Fx) \leq \phi(p(gx, Fx)) < p(gx, Fx)$: a contradiction. We deduce that $p(gx, Fx) = 0$, which implies that $gx = Fx$, that is, x is a coincidence point of F and g.

Suppose now that F and g commute at x. Set $w = Fx = gx$. Then,

$$Fw = F(gx) = g(Fx) = gw. \tag{28}$$

From the hypothesis (23), we have $gx \preccurlyeq g(gx) = gw$. If $gx = gw$, we get $w = gw = Fw$, and the proof is finished. Then, suppose that $gx \prec gw$. Applying the considered contraction, we get

$$p(Fw, Fx) \leq \varphi(M[F, g](w, x)), \tag{29}$$

where

$$\begin{aligned} &M[F, g](w, x) \\ &= \max\left\{p(gw, gx), p(Fw, gw), p(Fx, gx), \frac{1}{2}[p(gw, Fx) + p(Fw, gx)]\right\} \\ &= \max\left\{p(Fw, Fx), p(Fw, Fw), p(Fx, Fx), \frac{1}{2}[p(Fw, Fx) + p(Fw, Fx)]\right\} \\ &= \max\{p(Fw, Fx), p(Fw, Fw)\} \\ &= p(Fw, Fx). \end{aligned}$$

Suppose that $p(Fw, Fx) > 0$, From (29), we get

$$p(Fw, Fx) \leq \varphi(M[F, g](w, x)) = \varphi(p(Fw, Fx)) < p(Fw, Fx),$$

which is a contradiction. Thus, we have $p(Fw, Fx) = 0$, which implies that $Fw = Fx = w$. Therefore, from (28), we have $w = Fw = gw$, and w is a common fixed point of F and g. This completes the proof. ■

Remark 2.1 *The result given by Theorem* 2.3 *is also valid if the contraction condition* (22) *is satisfied for all $x, y \in X$ with $gx \succcurlyeq gy$ and* (23) *is replaced by*

$$\begin{cases} \text{if } \{gx_n\} \subset X \text{ is a increasing sequence} \\ \text{with } gx_n \to gz \in gX, \quad \text{then } gx_n \preccurlyeq gz, gz \preccurlyeq g(gz) \text{ for all } n \end{cases}$$

An immediate consequence of Theorem 2.3 is the following.

Theorem 2.4 *Let $(X, \preccurlyeq)$ be a partially ordered set and suppose that there is a partial metric p on X such that (X, p) is a complete partial metric space. Suppose $F : X \to X$ is a non-decreasing mapping such that*

$$p(Fx, Fy) \leq \varphi\left(\max\left\{p(x, y), p(x, Fx), p(y, Fy), \frac{1}{2}[p(x, Fy) + p(y, Fx)]\right\}\right),$$

for all $x, y \in X$ with $y \prec x$, where $\phi : [0, +\infty) \to [0, +\infty)$ is continuous non-decreasing and $\phi(t) < t$ for all $t > 0$. Suppose also that the condition

$$\begin{cases} \text{if } \{x_n\} \subset X \text{ is a increasing sequence} \\ \text{with } x_n \to x \in X, \text{ then } x_n \prec x \text{ for all } n \end{cases} \tag{30}$$

holds. If there exists $x_0 \in X$ such that $x_0 \preccurlyeq Fx_0$, then there exists $x \in X$ such that $Fx = x$. Moreover, $p(x, x) = 0$.

Now, we give a simple example to show that our result given by Theorem 2.3 is more general than Theorem 3.6 of O'Regan and Petruşel [29].

Example 2.2 *Let $X = [0, +\infty)$ endowed with the partial metric $p(x, y) = \max\{x, y\}$ for all $x, y \in X$. We endow X with the usual order $\leq$. Consider the mappings $F, g : X \to X$*

and $\phi : [0, +\infty) \to [0, +\infty)$ *defined by*

$$F(x) = 2x, \quad g(x) = 4x, \quad \varphi(t) = (3/4)t.$$

Let $y \le x$. *We have*

$$p(F(x), F(y)) = F(x) = 2x < 3 \cdot \frac{1}{4} \cdot 4x = \frac{3}{4} p(g(x), g(y)) = \varphi(p(g(x), g(y)).$$

Then, (22) *is satisfied. It is easy to show that all the other hypotheses of Theorem* 2.3 *are also satisfied. Since F and g commute, we deduce that F and g have a common fixed point* $z = 0$, *that is*, $0 = F(0) = g(0)$.

On the other hand, if we endow X with the standard metric $d(x, y) = |x - y|$ *for all x*, $y \in X$, *we have*

$$d(F(x), F(y)) = |F(x) - F(y)| = 2|x - y| > \varphi(|x - y|)$$

for $x \ne y$ *and for any* $\phi : [0, +\infty) \to [0, +\infty)$ *satisfying* $\phi(t) < t$ *for* $t > 0$. *Therefore, Theorem 3.6 of O'Regan and Petruşel* [29]*is not applicable.*

Note that F also does not satisfy the contractive conditions in the rest theorems of O'Regan and Petruşel [29].

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