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Fixed point results for contractions involving generalized altering distances in ordered metric spaces

Hemant Kumar Nashine[1], Bessem Samet[2] and Jong Kyu Kim[3*]

* Correspondence: jongkyuk@kyungnam.ac.kr
[3]Department of Mathematics, Kyungnam University, Masan, Kyungnam 631-701, Korea
Full list of author information is available at the end of the article

Abstract
In this article, we establish coincidence point and common fixed point theorems for mappings satisfying a contractive inequality which involves two generalized altering distance functions in ordered complete metric spaces. As application, we study the existence of a common solution to a system of integral equations.
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Introduction and Preliminaries

There are a lot of generalizations of the Banach contraction-mapping principle in the literature (see [1-31] and others).

A new category of contractive fixed point problems was addressed by Khan et al. [1]. In this study, they introduced the notion of an altering distance function which is a control function that alters distance between two points in a metric space.

Definition 1.1. [1] A function ϕ: $[0, +\infty) \to [0, +\infty)$ is called an altering distance function if the following conditions are satisfied.

(i) ϕ is continuous.
(ii) ϕ is non-decreasing.
(iii) $\phi(t) = 0 \Leftrightarrow t = 0$.

Khan et al. [1] proved the following result:

Theorem 1.2. [1]*Let* (X, d) *be a complete metric space,* ϕ: $[0, +\infty) \to [0, +\infty)$ *be an altering distance function, and* $T : X \to X$ *be a self-mapping which satisfies the following inequality:*

$$\varphi(d(Tx, Ty)) \leq c\varphi(d(x, y)) \tag{1.1}$$

for all $x, y \in X$ *and for some* $0 < c < 1$. *Then,* T *has a unique fixed point.*

Letting $\phi(t) = t$ in Theorem 1.2, we retrieve immediately the Banach contraction principle.

In 1997, Alber and Guerre-Delabriere [2] introduced the concept of weak contractions in Hilbert spaces. This concept was extended to metric spaces in [3].

Definition 1.3. Let (X, d) be a metric space. A mapping $T : X \to X$ is said to be weakly contractive if

$$d(Tx, Ty) \leq d(x, y) - \varphi(d(x, y)), \quad \forall x, y \in X,$$

where ϕ: $[0, +\infty) \to [0, +\infty)$ is an altering distance function.

Theorem 1.4. [3]*Let* (X, d) *be a complete metric space and* $T : X \to X$ *be a weakly contractive map. Then,* T *has a unique fixed point.*

Weak inequalities of the above type have been used to establish fixed point results in a number of subsequent studies, some of which are noted in [4-7]. In [5], Choudhury introduced the concept of a generalized altering distance function.

Definition 1.5. [5] A function ϕ: $[0, +\infty) \times [0, +\infty) \times [0, +\infty) \to [0, +\infty)$ is said to be a generalized altering distance function if the following conditions are satisfied:

(i) ϕ is continuous.
(ii) ϕ is non-decreasing in all the three variables.
(iii) $\phi(x, y, z) = 0 \Leftrightarrow x = y = z = 0$.

In [5], Choudhury proved the following common fixed point theorem:

Theorem 1.6. [5]*Let* (X, d) *be a complete metric space and* $S, T : X \to X$ *be two self-mappings such that the following inequality is satisfied:*

$$\Phi_1(d(Sx, Ty)) \leq \psi_1(d(x, y), d(x, Sx), d(y, Ty)) - \psi_2(d(x, y), d(x, Sx), d(y, Ty)) \quad (1.2)$$

for all $x, y \in X$, *where* ψ_1 *and* ψ_2 *are generalised altering distance functions, and* $\Phi_1(x) = \psi_1(x, x, x)$. *Then,* S *and* T *have a common fixed point.*

Recently, there have been so many exciting developments in the field of existence of fixed point in partially ordered sets (see [8-27] and the references cited therein). The first result in this direction was given by Turinici [27], where he extended the Banach contraction principle in partially ordered sets. Ran and Reurings [24] presented some applications of Turinici's theorem to matrix equations. The obtained result by Turinici was further extended and refined in [20-23].

In this article, we obtain coincidence point and common fixed point theorems in complete ordered metric spaces for mappings, satisfying a contractive condition which involves two generalized altering distance functions. Presented theorems are the extensions of Theorem 1.6 of Choudhury [5]. In addition, as an application, we study the existence of a common solution for a system of integral equations.

Main Results

At first, we introduce some notations and definitions that will be used later. The following definition was introduced by Jungck [28].

Definition 2.1. [28] Let (X, d) be a metric space and $f, g : X \to X$. If $w = fx = gx$, for some $x \in X$, then x is called a coincidence point of f and g, and w is called a point of coincidence of f and g. The pair $\{f, g\}$ is said to be compatible if and only if $\lim_{n \to +\infty} d(fgx_n, gfx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \to +\infty} fx_n = \lim_{n \to +\infty} gx_n = t$ for some $t \in X$.

Let X be a nonempty set and $R : X \to X$ be a given mapping. For every $x \in X$, we denote by $R^{-1}(x)$ the subset of X defined by

$$R^{-1}(x) := \{u \in X | Ru = x\}.$$

In [19], Nashine and Samet introduced the following concept:

Definition 2.2. [19] Let $(X, \preceq)$ be a partially ordered set, and $T, S, R : X \to X$ are given mappings, such that $TX \subseteq RX$ and $SX \subseteq RX$. We say that S and T are weakly increasing with respect to R if for all $x \in X$, we have

$$Tx \preccurlyeq Sy, \forall y \in R^{-1}(Tx)$$

and

$$Sx \preccurlyeq Ty, \forall y \in R^{-1}(Sx).$$

Remark 2.3. If $R : X \to X$ is the identity mapping ($Rx = x$ for all $x \in X$), then S and T are weakly increasing with respect to R implies that S and T are weakly increasing mappings. It is noted that the notion of weakly increasing mappings was introduced in [9] (also see [16,29]).

Example 2.4. Let $X = [0, +\infty)$ endowed with the usual order $\leq$. Define the mappings $T, S, R : X \to X$ by

$$Tx = \begin{cases} x \text{ if } 0 \leq x < 1 \\ 0 \text{ if } 1 \leq x \end{cases}, \quad Sx = \begin{cases} \sqrt{x} \text{ if } 0 \leq x < 1 \\ 0 \quad \text{if } 1 \leq x \end{cases}$$

and

$$Rx = \begin{cases} x^2 \text{ if } 0 \leq x < 1 \\ 0 \text{ if } 1 \leq x. \end{cases}$$

Then, we will show that the mappings S and T are weakly increasing with respect to R.

Let $x \in X$. We distinguish the following two cases.

- First case: $x = 0$ or $x \geq 1$.

(i) Let $y \in R^{-1}(Tx)$, that is, $Ry = Tx$. By the definition of T, we have $Tx = 0$ and then $Ry = 0$. By the definition of R, we have $y = 0$ or $y \geq 1$. By the definition of S, in both cases, we have $Sy = 0$. Then, $Tx = 0 = Sy$.

(ii) Let $y \in R^{-1}(Sx)$, that is, $Ry = Sx$. By the definition of S, we have $Sx = 0$, and then $Ry = 0$. By the definition of R, we have $y = 0$ or $y \geq 1$. By the definition of T, in both cases, we have $Ty = 0$. Then, $Sx = 0 = Ty$.

- Second case: $0 < x < 1$.

(i) Let $y \in R^{-1}(Tx)$, that is, $Ry = Tx$. By the definition of T, we have $Tx = x$ and then $Ry = x$. By the definition of R, we have $Ry = y^2$, and then $y = \sqrt{x}$. We have

$$Tx = x \leq Sy = S\sqrt{x} = x^{1/4}.$$

(ii) Let $y \in R^{-1}(Sx)$, that is, $Ry = Sx$. By the definition of S, we have $Sx = \sqrt{x}$, and then $Ry = \sqrt{x}$. By the definition of R, we have $Ry = y^2$, and then $y = x^{1/4}$. We have

$$Sx = \sqrt{x} \le Ty = Tx^{1/4} = x^{1/4}.$$

Thus, we proved that S and T are weakly increasing with respect to R.

Example 2.5. Let $X = \{1, 2, 3\}$ endowed with the partial order $\le$ given by

$$\preccurlyeq := \{(1,1),(2,2),(3,3),(2,3),(3,1),(2,1)\}.$$

Define the mappings $T, S, R : X \to X$ by

$$T1 = T3 = 1,\ T2 = 3;$$
$$S1 = S2 = S3 = 1;$$
$$R1 = 1,\ R2 = R3 = 2.$$

We will show that the mappings S and T are weakly increasing with respect to R.

Let $x, y \in X$ such that $y \in R^{-1}(Tx)$. By the definition of S, we have $Sy = 1$. On the other hand, $Tx \in \{1, 3\}$ and $(1, 1), (3, 1) \in \le$. Thus, we have $Tx \le Sy$ for all $y \in R^{-1}(Tx)$.

Let $x, y \in X$ such that $y \in R^{-1}(Sx)$. By the definitions of S and R, we have $R^{-1}(Sx) = R^{-1}(1) = \{1\}$. Then, we have $y = 1$. On the other hand, $1 = Sx \le Ty = T\,1 = 1$. Then, $Sx \le Ty$ for all $y \in R^{-1}(Sx)$. Thus, we proved that S and T are weakly increasing with respect to R.

Our first result is as follows.

Theorem 2.6. *Let $(X, \le)$ be a partially ordered set, and suppose that there exists a metric d on X such that (X, d) is a complete metric space. Let $T, S, R : X \to X$ be given mappings, satisfying for every pair $(x, y) \in X \times X$ such that Rx and Ry are comparable:*

$$\begin{aligned}&\Phi_1(d(Sx,Ty))\\ &\le \psi_1(d(Rx,Ry),d(Rx,Sx),d(Ry,Ty)) - \psi_2(d(Rx,Ry),d(Rx,Sx).d(Ry,Ty)),\end{aligned} \tag{2.1}$$

where ψ_1 and ψ_2 are generalized altering distance functions, and $\Phi_1(x) = \psi_1(x, x, x)$. We assume the following hypotheses:

(i) *T, S, and R are continuous.*
(ii) *$TX \subseteq RX$, $SX \subseteq RX$.*
(iii) *T and S are weakly increasing with respect to R.*
(iv) *the pairs $\{T, R\}$ and $\{S, R\}$ are compatible.*

Then, T, S, and R have a coincidence point, that is, there exists $u \in X$ such that $Ru = Tu = Su$.

Proof. Let $x_0 \in X$ be an arbitrary point. Since $TX \subseteq RX$, there exists $x_1 \in X$ such that $Rx_1 = Tx_0$. Since $SX \subseteq RX$, there exists $x_2 \in X$ such that $Rx_2 = Sx_1$.

Continuing this process, we can construct a sequence $\{Rx_n\}$ in X defined by

$$Rx_{2n+1} = Tx_{2n}, \qquad Rx_{2n+2} = Sx_{2n+1}, \qquad \forall n \in \mathbb{N}. \tag{2.2}$$

We claim that

$$Rx_n \preccurlyeq Rx_{n+1}, \quad \forall n \in \mathbb{N}^*. \tag{2.3}$$

To this aim, we will use the increasing property with respect to R for the mappings T and S. From (2.2), we have

$$Rx_1 = Tx_0 \preccurlyeq Sy, \quad \forall y \in R^{-1}(Tx_0).$$

Since $Rx_1 = Tx_0$, $x_1 \in R^{-1}(Tx_0)$, and we get

$$Rx_1 = Tx_0 \preccurlyeq Sx_1 = Rx_2.$$

Again,

$$Rx_2 = Sx_1 \preccurlyeq Ty, \quad \forall y \in R^{-1}(Sx_1).$$

Since $x_2 \in R^{-1}$ ($Sx1$), we get

$$Rx_2 = Sx_1 \preccurlyeq Tx_2 = Rx_3.$$

Hence, by induction, (2.3) holds.

Without loss of the generality, we can assume that

$$Rx_n \neq Rx_{n+1}, \quad \forall n \in \mathbb{N}^*. \tag{2.4}$$

Now, we will prove our result on three steps.

Step I. We will prove that

$$\lim_{n\to+\infty} d(Rx_{n+1}, Rx_{n+2}) = 0. \tag{2.5}$$

Letting $x = x_{2n+1}$ and $y = x_{2n}$, from (2.3) and the considered contraction, we have

$$\begin{aligned}
&\Phi_1(d(Rx_{2n+2}, Rx_{2n+1}))\\
&= \Phi_1(d(Sx_{2n+1}, Tx_{2n}))\\
&\leq \psi_1(d(Rx_{2n+1}, Rx_{2n}), d(Rx_{2n+1}, Sx_{2n+1}), d(Rx_{2n}, Tx_{2n}))\\
&\quad -\psi_2(d(Rx_{2n+1}, Rx_{2n}), d(Rx_{2n+1}, Sx_{2n+1}), d(Rx_{2n}, Tx_{2n}))\\
&= \psi_1(d(Rx_{2n+1}, Rx_{2n}), d(Rx_{2n+1}, Rx_{2n+2}), d(Rx_{2n}, Rx_{2n+1}))\\
&\quad -\psi_2(d(Rx_{2n+1}, Rx_{2n}), d(Rx_{2n+1}, Rx_{2n+2}), d(Rx_{2n}, Rx_{2n+1})).
\end{aligned} \tag{2.6}$$

Suppose that

$$d(Rx_{2n+1}, Rx_{2n+2}) > d(Rx_{2n}, Rx_{2n+1}). \tag{2.7}$$

Using the property of the generalized altering function, this implies that

$$\begin{aligned}
&\psi_1(d(Rx_{2n+1}, Rx_{2n}), d(Rx_{2n+1}, Rx_{2n+2}), d(Rx_{2n}, Rx_{2n+1}))\\
&\leq \Phi_1(d(Rx_{2n+2}, Rx_{2n+1})).
\end{aligned}$$

Hence, we obtain

$$\begin{aligned}
&\Phi_1(d(Rx_{2n+2}, Rx_{2n+1}))\\
&\leq \Phi_1(d(Rx_{2n+2}, Rx_{2n+1}))\\
&-\psi_2(d(Rx_{2n+1}, Rx_{2n}), d(Rx_{2n+1}, Rx_{2n+2}), d(Rx_{2n}, Rx_{2n+1})).
\end{aligned}$$

This implies that

$$\psi_2(d(Rx_{2n+1}, Rx_{2n}), d(Rx_{2n+1}, Rx_{2n+2}), d(Rx_{2n}, Rx_{2n+1})) = 0$$

and

$$d(Rx_{2n+1}, Rx_{2n}) = 0.$$

Hence, we obtain a contradiction with (2.4). We deduce that

$$d(Rx_{2n}, Rx_{2n+1}) \geq d(Rx_{2n+1}, Rx_{2n+2}), \quad \forall n \in \mathbb{N}^*. \tag{2.8}$$

Similarly, letting $x = x_{2n+1}$ and $y = x_{2n+2}$, from (2.3) and the considered contraction, we have

$$\begin{aligned} &\Phi_1(d(Rx_{2n+2}, Rx_{2n+3})) \\ &\leq \psi_1(d(Rx_{2n+1}, Rx_{2n+2}), d(Rx_{2n+1}, Rx_{2n+2}), d(Rx_{2n+2}, Rx_{2n+3})) \\ &-\psi_2(d(Rx_{2n+1}, Rx_{2n+2}), d(Rx_{2n+1}, Rx_{2n+2}), d(Rx_{2n+2}, Rx_{2n+3})). \end{aligned} \tag{2.9}$$

Suppose that

$$d(Rx_{2n+2}, Rx_{2n+3}) > d(Rx_{2n+1}, Rx_{2n+2}). \tag{2.10}$$

Then, from (2.9) and (2.10), we obtain

$$\begin{aligned} &\Phi_1(d(Rx_{2n+2}, Rx_{2n+3})) \\ &\leq \Phi_1(d(Rx_{2n+2}, Rx_{2n+3})) \\ &-\psi_2(d(Rx_{2n+1}, Rx_{2n+2}), d(Rx_{2n+1}, Rx_{2n+2}), d(Rx_{2n+2}, Rx_{2n+3})). \end{aligned}$$

This implies that

$$\psi_2(d(Rx_{2n+1}, Rx_{2n+2}), d(Rx_{2n+1}, Rx_{2n+2}), d(Rx_{2n+2}, Rx_{2n+3})) = 0$$

and

$$d(Rx_{2n+1}, Rx_{2n+2}) = 0.$$

Hence, we obtain a contradiction with (2.4). We deduce that

$$d(Rx_{2n+1}, Rx_{2n+2}) \geq d(Rx_{2n+2}, Rx_{2n+3}), \quad \forall n \in \mathbb{N}. \tag{2.11}$$

Combining (2.8) and (2.11), we obtain

$$d(Rx_{n+1}, Rx_{n+2}) \geq d(Rx_{n+2}, Rx_{n+3}), \quad \forall n \in \mathbb{N}. \tag{2.12}$$

Hence, $\{d(Rx_{n+1}, Rx_{n+2})\}$ is a decreasing sequence of positive real numbers. This implies that there exists $r \geq 0$ such that

$$\lim_{n\to+\infty} d(Rx_{n+1}, Rx_{n+2}) = r. \tag{2.13}$$

Define the function Φ_2: $[0, +\infty) \to [0, +\infty)$ by

$$\Phi_2(x) = \psi_2(x, x, x), \quad \forall x \geq 0.$$

From (2.6) and (2.12), we obtain

$$\Phi_1(d(Rx_{2n+2}, Rx_{2n+1})) \leq \Phi_1(d(Rx_{2n+1}, Rx_{2n})) - \Phi_2(d(Rx_{2n+2}, Rx_{2n+1})),$$

which implies that

$$\Phi_2(d(Rx_{2n+2}, Rx_{2n+1})) \leq \Phi_1(d(Rx_{2n+1}, Rx_{2n})) - \Phi_1(d(Rx_{2n+2}, Rx_{2n+1})). \tag{2.14}$$

Similarly, from (2.9) and (2.12), we obtain

$$\Phi_1(d(Rx_{2n+2}, Rx_{2n+3})) \leq \Phi_1(d(Rx_{2n+1}, Rx_{2n+2})) - \Phi_2(d(Rx_{2n+2}, Rx_{2n+3})),$$

which implies that

$$\Phi_2(d(Rx_{2n+2}, Rx_{2n+3})) \leq \Phi_1(d(Rx_{2n+1}, Rx_{2n+2})) - \Phi_1(d(Rx_{2n+2}, Rx_{2n+3})). \tag{2.15}$$

Now, combining (2.14) and (2.15), we obtain

$$\Phi_2(d(Rx_{k+2}, Rx_{k+1})) \leq \Phi_1(d(Rx_{k+1}, Rx_k)) - \Phi_1(d(Rx_{k+2}, Rx_{k+1})), \quad \forall k \in \mathbb{N}^*.$$

This implies that for all $n \in \mathbb{N}^*$, we have

$$\begin{aligned}\sum_{k=1}^{n} \Phi_2(d(Rx_{k+2}, Rx_{k+1})) &\leq \sum_{k=1}^{n} [\Phi_1(d(Rx_{k+1}, Rx_k)) - \Phi_1(d(Rx_{k+2}, Rx_{k+1}))] \\ &= \Phi_1(d(Rx_2, Rx_1)) - \Phi_1(d(Rx_{n+2}, Rx_{n+1})) \\ &\leq \Phi_1(d(Rx_2, Rx_1)).\end{aligned}$$

This implies that

$$\sum_{n=1}^{+\infty} \Phi_2(d(Rx_{k+2}, Rx_{k+1})) < \infty.$$

Hence,

$$\lim_{n\to+\infty} \Phi_2(d(Rx_{n+2}, Rx_{n+1})) = 0. \tag{2.16}$$

Now, using (2.13), (2.16), and the continuity of Φ_2, we obtain

$$\psi_2(r, r, r) = \Phi_2(r) = 0,$$

which implies that $r = 0$. Hence, (2.5) is proved.

Step II. We claim that $\{Rx_n\}$ is a Cauchy sequence.

From (2.5), it will be sufficient to prove that $\{Rx_{2n}\}$ is a Cauchy sequence. We proceed by negation, and suppose that $\{Rx_{2n}\}$ is not a Cauchy sequence. Then, there exists $\varepsilon > 0$ for which we can find two sequences of positive integers $\{m(i)\}$ and $\{n(i)\}$ such that for all positive integer i,

$$n(i) > m(i) > i, \; d(Rx_{2m(i)}, Rx_{2n(i)}) \geq \varepsilon, \; d(Rx_{2m(i)}, Rx_{2n(i)-2}) < \varepsilon. \tag{2.17}$$

From (2.17) and using the triangular inequality, we get

$$\begin{aligned}\varepsilon &\leq d(Rx_{2m(i)}, Rx_{2n(i)}) \\ &\leq d(Rx_{2m(i)}, Rx_{2n(i)-2}) + d(Rx_{2n(i)-2}, Rx_{2n(i)-1}) \\ &\quad + d(Rx_{2n(i)-1}, Rx_{2n(i)}) \\ &< \varepsilon + d(Rx_{2n(i)-2}, Rx_{2n(i)-1}) + d(Rx_{2n(i)-1}, Rx_{2n(i)}).\end{aligned}$$

Letting $i \to +\infty$ in the above inequality, and using (2.5), we obtain

$$\lim_{i\to+\infty} d(Rx_{2m(i)}, Rx_{2n(i)}) = \varepsilon. \tag{2.18}$$

Again, the triangular inequality gives us

$$\left|d(Rx_{2n(i)}, Rx_{2m(i)-1}) - d(Rx_{2n(i)}, Rx_{2m(i)})\right| \leq d(Rx_{2m(i)-1}, Rx_{2m(i)}).$$

Letting $i \to +\infty$ in the above inequality, and using (2.5) and (2.18), we get

$$\lim_{i\to+\infty} d(Rx_{2n(i)}, Rx_{2m(i)-1}) = \varepsilon. \tag{2.19}$$

On the other hand, we have

$$\begin{aligned} d(Rx_{2n(i)}, Rx_{2m(i)}) &\le d(Rx_{2n(i)}, Rx_{2n(i)+1}) + d(Rx_{2n(i)+1}, Rx_{2m(i)}) \\ &= d(Rx_{2n(i)}, Rx_{2n(i)+1}) + d(Tx_{2n(i)}, Sx_{2m(i)-1}). \end{aligned}$$

Then, from (2.5), (2.18), and the continuity of Φ_1, and letting $i \to +\infty$ in the above inequality, we have

$$\Phi_1(\varepsilon) \le \lim_{i\to+\infty} \Phi_1(d(Sx_{2m(i)-1}, Tx_{2n(i)})). \tag{2.20}$$

Now, using the considered contractive condition for $x = x_{2m(i)-1}$ and $y = x_{2n(i)}$, we have

$$\begin{aligned} &\Phi_1(d(Sx_{2m(i)-1}, Tx_{2n(i)})) \\ &\le \psi_1(d(Rx_{2m(i)-1}, Rx_{2n(i)}), d(Rx_{2m(i)-1}, Rx_{2m(i)}), d(Rx_{2n(i)}, Rx_{2n(i)+1})) \\ &-\psi_2(d(Rx_{2m(i)-1}, Rx_{2n(i)}), d(Rx_{2m(i)-1}, Rx_{2m(i)}), d(Rx_{2n(i)}, Rx_{2n(i)+1})). \end{aligned}$$

Then, from (2.5), (2.19), and the continuity of ψ_1 and ψ_2, and letting $i \to +\infty$ in the above inequality, we have

$$\lim_{i\to+\infty} \Phi_1(d(Sx_{2m(i)-1}, Tx_{2n(i)})) \le \psi_1(\varepsilon, 0, 0) - \psi_2(\varepsilon, 0, 0) \le \Phi_1(\varepsilon) - \psi_2(\varepsilon, 0, 0).$$

Now, combining (2.20) with the above inequality, we get

$$\Phi_1(\varepsilon) \le \Phi_1(\varepsilon) - \psi_2(\varepsilon, 0, 0),$$

which implies that $\psi_2(\varepsilon, 0, 0) = 0$, that is a contradiction since $\varepsilon > 0$. We deduce that $\{Rx_n\}$ is a Cauchy sequence.

Step III. Existence of a coincidence point.

Since $\{Rx_n\}$ is a Cauchy sequence in the complete metric space (X, d), there exists $u \in X$ such that

$$\lim_{n\to+\infty} Rx_n = u. \tag{2.21}$$

From (2.21) and the continuity of R, we get

$$\lim_{n\to+\infty} R(Rx_n) = Ru. \tag{2.22}$$

By the triangular inequality, we have

$$d(Ru, Tu) \le d(Ru, R(Rx_{2n+1})) + d(R(Tx_{2n}), T(Rx_{2n})) + d(T(Rx_{2n}), Tu). \tag{2.23}$$

On the other hand, we have

$$Rx_{2n} \to u, \quad Tx_{2n} \to u \quad \text{as } n \to +\infty.$$

Since R and T are compatible mappings, this implies that

$$\lim_{n\to+\infty} d(R(Tx_{2n}), T(Rx_{2n})) = 0. \tag{2.24}$$

Now, from the continuity of T and (2.21), we have

$$\lim_{n\to+\infty} d(T(Rx_{2n}), Tu) = 0. \tag{2.25}$$

Combining (2.22), (2.24), and (2.25), and letting $n \to +\infty$ in (2.23), we obtain

$$d(Ru, Tu) \leq 0,$$

that is,

$$Ru = Tu. \tag{2.26}$$

Again, by the triangular inequality, we have

$$d(Ru, Su) \leq d(Ru, R(Rx_{2n+2})) + d(R(Sx_{2n+1}), S(Rx_{2n+1})) + d(S(Rx_{2n+1}), Su). \tag{2.27}$$

On the other hand, we have

$$Rx_{2n+1} \to u, \quad Sx_{2n+1} \to u \quad \text{as } n \to +\infty.$$

Since R and S are compatible mappings, this implies that

$$\lim_{n\to+\infty} d(R(Sx_{2n+1}), S(Rx_{2n+1})) = 0. \tag{2.28}$$

Now, from the continuity of S and (2.21), we have

$$\lim_{n\to+\infty} d(S(Rx_{2n+1}), Su) = 0. \tag{2.29}$$

Combining (2.22), (2.28), and (2.29), and letting $n \to +\infty$ in (2.27), we obtain

$$d(Ru, Su) \leq 0,$$

that is,

$$Ru = Su. \tag{2.30}$$

Finally, from (2.26) and (2.30), we have

$$Tu = Ru = Su,$$

that is, u is a coincidence point of T, S, and R. This completes the proof.

In the next theorem, we omit the continuity hypotheses on T, S, and R.

Definition 2.7. Let $(X,\leq, d)$ be a partially ordered metric space. We say that X is regular if the following hypothesis holds: if $\{z_n\}$ is a non-decreasing sequence in X with respect to $\leq$ such that $z_n \to z \in X$ as $n \to +\infty$, then $z_n \leq z$ for all $n \in \mathbb{N}$.

Now, our second result is the following.

Theorem 2.8. *Let $(X,\leq)$ be a partially ordered set, and suppose that there exists a metric d on X such that (X, d) is a complete metric space. Let $T, S, R : X \to X$ be given mappings satisfying for every pair $(x, y) \in X \times X$ such that Rx and Ry are comparable,*

$$\begin{aligned}&\Phi_1(d(Sx, Ty))\\ &\leq \psi_1(d(Rx, Ry), d(Rx, Sx), d(Ry, Ty)) - \psi_2(d(Rx, Ry), d(Rx, Sx), d(Ry, Ty)),\end{aligned}$$

where ψ_1 and ψ_2 are generalized altering distance functions and $\Phi_1(x) = \psi_1(x, x, x)$. We assume the following hypotheses:

(i) *X is regular.*

(ii) *T and S are weakly increasing with respect to R.*
(iii) *RX is a closed subset of* (X, d).
(iv) $TX \subseteq RX$, $SX \subseteq R\,X$.

Then, T, S, and R have a coincidence point.

Proof. From the proof of Theorem 2.6, we have that $\{Rx_n\}$ is a Cauchy sequence in (RX, d) which is complete, since RX is a closed subspace of (X, d). Hence, there exists $u = Rv$, $v \in X$ such that

$$\lim_{n\to+\infty} Rx_n = u = Rv. \tag{2.31}$$

Since $\{Rx_n\}$ is a non-decreasing sequence and X is regular, it follows from (2.31) that $Rx_n \le Rv$ for all $n \in \mathbb{N}^*$. Hence, we can apply the considered contractive condition. Then, for $x = v$ and $y = x_{2n}$, we obtain

$$\begin{aligned}\Phi_1(d(Sv, Rx_{2n+1})) &= \Phi_1(d(Sv, Tx_{2n}))\\ &\le \psi_1(d(Rv, Rx_{2n}), d(Rv, Sv), d(Rx_{2n}, Rx_{2n+1}))\\ &\quad - \psi_2(d(Rv, Rx_{2n}), d(Rv, Sv), d(Rx_{2n}, Rx_{2n+1})).\end{aligned}$$

Letting $n \to +\infty$ in the above inequality, and using (2.5), (2.31), and the properties of ψ_1 and ψ_2, then we have

$$\begin{aligned}\Phi_1(d(Sv, Rv)) &\le \psi_1(0, d(Rv, Sv), 0) - \psi_2(0, d(Rv, Sv), 0)\\ &\le \Phi_1(d(Sv, Rv)) - \psi_2(0, d(Rv, Sv), 0).\end{aligned}$$

This implies that $\psi_2(0, d(Rv, Sv), 0) = 0$, which gives us that $d(Rv, Sv) = 0$, i.e.,

$$Rv = Sv. \tag{2.32}$$

Similarly, for $x = x_{2n+1}$ and $y = v$, we obtain

$$\begin{aligned}\Phi_1(d(Rx_{2n+2}, Tv)) &= \Phi_1(d(Sx_{2n+1}, Tv))\\ &\le \psi_1(d(Rx_{2n+2}, Rv), d(Rx_{2n+1}, Rx_{2n+2}), d(Rv, Tv))\\ &\quad - \psi_2(d(Rx_{2n+2}, Rv), d(Rx_{2n+1}, Rx_{2n+2}), d(Rv, Tv)).\end{aligned}$$

Letting $n \to +\infty$ in the above inequality, we get

$$\begin{aligned}\Phi_1(d(Rv, Tv)) &\le \psi_1(0, 0, d(Rv, Tv)) - \psi_2(0, 0, d(Rv, Tv))\\ &\le \Phi_1(d(Rv, Tv)) - \psi_2(0, 0, d(Rv, Tv)).\end{aligned}$$

This implies that $\psi_2(0, 0, d(Rv, Tv)) = 0$ and then,

$$Rv = Tv. \tag{2.33}$$

Now, combining (2.32) and (2.33), we obtain

$$Rv = Tv = Sv.$$

Hence, v is a coincidence point of T, S, and R. This completes the proof.

Now, we present an example to illustrate the obtained result given by the previous theorem. Moreover, in this example, we will show that Theorem 1.6 of Choudhury cannot be applied.

Example 2.9. Let $X = \{4, 5, 6\}$ endowed with the usual metric $d(x, y) = |x - y|$ for all $x, y \in X$, and $\le := \{(4, 4), (5, 5), (6, 6), (6, 4)\}$. Clearly, $\le$ is a partial order on X.

Consider the mappings $T, S, R : X \to X$ defined by

$$T = S = \begin{pmatrix} 4 & 5 & 6 \\ 4 & 6 & 4 \end{pmatrix} \quad \text{and} \quad R = \begin{pmatrix} 4 & 5 & 6 \\ 4 & 5 & 6 \end{pmatrix}.$$

We will show that T and S are weakly increasing with respect to R. In the case under study, we have to check that $Tx \leq T(Tx)$ for all $x \in X$.

For $x = 4$, we have

$$T4 = 4 \preccurlyeq T(T4) = T4 = 4.$$

For $x = 5$, we have

$$T5 = 6 \preccurlyeq T(T5) = T6 = 4.$$

For $x = 6$, we have

$$T6 = 4 \preccurlyeq T(T6) = T4 = 4.$$

Thus, we have proved that T and S are weakly increasing with respect to R.

Now, we will show that $(X, \leq, d)$ is regular.

Let $\{z_n\}$ be a non-decreasing sequence in X with respect to $\leq$ such that $z_n \to z \in X$ as $n \to +\infty$. Then, we have $z_n \leq z_{n+1}$, for all $n \in \mathbb{N}$.

- If $z_0 = 4$, then $z_0 = 4 \leq z_1$. From the definition of $\leq$, we have $z_1 = 4$. By induction, we get $z_n = 4$ for all $n \in \mathbb{N}$ and $z = 4$. Then, $z_n \leq z$ for all $n \in \mathbb{N}$.
- If $z_0 = 5$, then $z_0 = 5 \leq z_1$. From the definition of $\leq$, we have $z_1 = 5$. By induction, we get $z_n = 5$ for all $n \in \mathbb{N}$ and $z = 5$. Then, $z_n \leq z$ for all $n \in \mathbb{N}$.
- If $z_0 = 6$, then $z_0 = 6 \leq z_1$. From the definition of $\leq$, we have $z_1 \in \{6, 4\}$. By induction, we get $z_n \in \{6, 4\}$ for all $n \in \mathbb{N}$. Suppose that there exists $p \geq 1$ such that $z_p = 4$. From the definition of $\leq$, we get $z_n = z_p = 4$ for all $n \geq p$. Thus, we have $z = 4$ and $z_n \leq z$ for all $n \in \mathbb{N}$. Now, suppose that $z_n = 6$ for all $n \in \mathbb{N}$. In this case, we get $z = 6$, and $z_n \leq z$ for all $n \in \mathbb{N}$. Thus, we proved that in all the cases considered, we have $z_n \leq z$ for all $n \in \mathbb{N}$. Then, $(X, \leq, d)$ is regular.

Now, define the functions $\psi_1, \psi_2 : [0, +\infty) \times [0, +\infty) \times [0, +\infty) \to [0, +\infty)$ by

$$\psi_1(t, u, v) = \frac{1}{4}(t + u + v), \quad \text{for all } t, u, v \geq 0$$

and

$$\psi_2(t, u, v) = \frac{1}{16}(t + u + v), \quad \text{for all } t, u, v \geq 0.$$

Clearly, ψ_1 and ψ_2 are the generalized altering distance functions, and for every $x, y \in X$ such that $Rx \leq Ry$, inequality (2.1) is satisfied.

Now, we can apply Theorem 2.8 to deduce that T, S, and R have a coincidence point $u = 4$. Note that u is also a fixed point of T since $S = T$, and R is the identity mapping.

On the other hand, taking $x = 4$ and $y = 5$, we get

$$\begin{aligned}
&\psi_1(d(4,5), d(4,S4), d(5,T5)) - \psi_2(d(4,5), d(4,S4), d(5,T5)) \\
&= \psi_1(1,0,1) - \psi_2(1,0,1) \\
&= \frac{1}{2} - \frac{1}{8} = \frac{3}{8} \\
&< \Phi_1(d(S4,T5)) = \Phi_1(2) = \frac{3}{2}.
\end{aligned}$$

Thus, Inequality (1.2) is not satisfied for $x = 4$ and $y = 5$. Then, Theorem 1.6 of Choudhury [5] cannot be applied in this case.

If $R : X \to X$ is the identity mapping, we can deduce easily the following common fixed point results.

The next result is an immediate consequence of Theorem 2.6.

Corollary 2.10. *Let $(X, \leq)$ be a partially ordered set, and suppose that there exists a metric d on X such that (X, d) is a complete metric space. Let T, $S : X \to X$ be given mappings satisfying for every pair $(x, y) \in X \times X$ such that x and y are comparable. Then,*

$$\Phi_1(d(Sx,Ty)) \leq \psi_1(d(x,y), d(x,Sx), d(y,Ty)) - \psi_2(d(x,y), d(x,Sx), d(y,Ty)),$$

where ψ_1 and ψ_2 are generalized altering distance functions and $\Phi_1(x) = \psi_1(x, x, x)$. We assume the following hypotheses:

(i) *T and S are continuous.*
(ii) *T and S are weakly increasing.*

Then, T and S have a common fixed point, that is, there exists $u \in X$ such that $u = Tu = Su$.

The following result is an immediate consequence of Theorem 2.8.

Corollary 2.11. *Let $(X, \leq)$ be a partially ordered set and suppose that there exists a metric d on X such that (X, d) is a complete metric space. Let T, $S : X \to X$ be given mappings satisfying for every pair $(x, y) \in X \times X$ such that x and y are comparable. Then,*

$$\Phi_1(d(Sx,Ty)) \leq \psi_1(d(x,y), d(x,Sx), d(y,Ty)) - \psi_2(d(x,y), d(x,Sx), d(y,Ty)),$$

where ψ_1 and ψ_2 are generalised altering distance functions and $\Phi_1(x) = \psi_1(x, x, x)$. We assume the following hypotheses:

(i) *X is regular.*
(ii) *T and S are weakly increasing.*

Then, T and S have a common fixed point.

A number of fixed point results may be obtained by assuming different forms for the functions ψ_1 and ψ_2. In particular, fixed point results under various contractive conditions may be derived from the above theorems. For example, if we consider

$$\begin{aligned}
&\psi_1(x,y,z) = k_1x^s + k_2y^s + k_3z^s, \\
&\psi_2(x,y,z) = (1-k)[k_1x^s + k_2y^s + k_3z^s],
\end{aligned}$$

where $s > 0$ and $0 < k = k_1 + k_2 + k_3 < 1$, then we obtain the following results.

The next result is an immediate consequence of Corollary 2.10.

Corollary 2.12. *Let $(X, \leq)$ be a partially ordered set, and suppose that there exists a metric d on X such that (X, d) is a complete metric space. Let $T, S : X \to X$ be given mappings satisfying for every pair $(x, y) \in X \times X$ such that x and y are comparable. Then,*

$$[d(Sx, Ty)]^s \leq k_1 [d(x, y)]^s + k_2 [d(x, Sx)]^s + k_3 [d(y, Ty)]^s,$$

where $s > 0$ and $0 < k = k_1 + k_2 + k_3 < 1$. We assume the following hypotheses:

(i) *T and S are continuous.*
(ii) *T and S are weakly increasing.*

Then, T and S have a common fixed point, that is, there exists $u \in X$ such that $u = Tu = Su$.

The next result is an immediate consequence of Corollary 2.11.

Corollary 2.13. *Let $(X, \leq)$ be a partially ordered set, and suppose that there exists a metric d on X such that (X, d) is a complete metric space. Let $T, S : X \to X$ be given mappings satisfying for every pair $(x, y) \in X \times X$ such that x and y are comparable. Then,*

$$[d(Sx, Ty)]^s \leq k_1 [d(x, y)]^s + k_2 [d(x, Sx)]^s + k_3 [d(y, Ty)]^s, \tag{2.34}$$

where $s > 0$ and $0 < k = k_1 + k_2 + k_3 < 1$. We assume the following hypotheses:

(i) *X is regular.*
(ii) *T and S are weakly increasing.*

Then, T and S have a common fixed point.

Remark 2.14. Other fixed point results may also be obtained under specific choices of ψ_1 and ψ_2.

Application

Consider the integral equations:

$$\begin{aligned} u(t) &= \int_0^T K_1(t, s, u(s))\, ds + g(t),\ t \in [0, T], \\ u(t) &= \int_0^T K_2(t, s, u(s))\, ds + g(t),\ t \in [0, T], \end{aligned} \tag{3.1}$$

where $T > 0$.

The purpose of this section is to give an existence theorem for common solution of (3.1) using Corollary 2.13. This application is inspired in [9].

Previously, we have considered the space $C(I)(I = [0, T])$ of continuous functions defined on I. Obviously, this space with the metric given by

$$d(x, y) = \sup_{t \in I} |x(t) - y(t)|, \quad \forall x, y \in C(I),$$

is a complete metric space. $C(I)$ can also be equipped with the partial order ≤ given by

$$x, y \in C(I), \qquad x \preccurlyeq y \Leftrightarrow x(t) \le y(t), \quad \forall\, t \in I.$$

Moreover, in [20], it is proved that ($C(I)$, ≤) is regular.

Now, we will prove the following result.

Theorem 3.1. *Suppose that the following hypotheses hold:*

(i) $K_1, K_2 : I \times I \times \mathbb{R} \to \mathbb{R}$, *and* $g : \mathbb{R} \to \mathbb{R}$ *are continuous;*

(ii) *for all* $t, s \in I$, *we have*

$$K_1(t, s, u(t)) \le K_2\left(t, s, \int_0^T K_1(s, \tau, u(\tau))\, d\tau + g(s)\right),$$
$$K_2(t, s, u(t)) \le K_1\left(t, s, \int_0^T K_2(s, \tau, u(\tau))\, d\tau + g(s)\right);$$

(iii) *there exists a continuous function* $p : I \times I \to \mathbb{R}_+$ *such that*

$$|K_1(t, s, x) - K_2(t, s, y)| \le p(t, s)(x - y)$$

for all $t, s \in I$ *and* $x, y \in \mathbb{R}$ *such that* $x \ge y$;

(iv) $\sup_{t \in I} \int_0^T p(t, s)\, ds = \alpha < 1$.

Then, the integral equations (3.1) have a solution $u^* \in C(I)$.

Proof. Define $T, S : C(I) \to C(I)$ by

$$Tx(t) = \int_0^T K_1(t, s, x(s))\, ds + g(t), \quad t \in I,$$

and

$$Sx(t) = \int_0^T K_2(t, s, x(s))\, ds + g(t), \quad t \in I.$$

Now, we will prove that T and S are weakly increasing. From (ii), for all $t \in I$, we have

$$\begin{aligned} Tx(t) &= \int_0^T K_1(t, s, x(s))\, ds + g(t) \\ &\le \int_0^T K_2\left(t, s, \int_0^T K_1(s, \tau, x(\tau))\, d\tau + g(s)\right) ds + g(t) \\ &= \int_0^T K_2(t, s, Tx(s))\, ds + g(t) \\ &= STx(t) \end{aligned}$$

Similarly,

$$\begin{aligned} Sx(t) &= \int_0^T K_2(t,s,x(s))\,ds + g(t) \\ &\leq \int_0^T K_1\left(t,s,\int_0^T K_2(s,\tau,x(\tau))\,d\tau + g(s)\right)\,ds + g(t) \\ &= \int_0^T K_1(t,s,Sx(s))\,ds + g(t) \\ &= TSx(t). \end{aligned}$$

Then, we have $Tx \leq STx$ and $Sx \leq TSx$ for all $x \in C(I)$. This implies that T and S are weakly increasing.

Now, for all $x, y \in C(I)$ such that $x \leq y$, by (iii) and (iv), we have

$$\begin{aligned} |Sx(t) - Ty(t)| &\leq \int_0^T |K_2(t,s,x(s)) - K_1(t,s,y(s))|\,ds \\ &\leq \int_0^T p(t,s)(y(s) - x(s))\,ds \\ &\leq d(x,y)\int_0^T p(t,s)\,ds \\ &\leq \alpha d(x,y). \end{aligned}$$

This implies that for all $x, y \in C(I)$ such that $x \leq y$,

$$d(Sx, Ty) \leq \alpha d(x,y).$$

Hence, the contractive condition required by Corollary 2.13 is satisfied with $s = 1$, $k_1 = \alpha$, and $k_2 = k_3 = 0$.

Now, all the required hypotheses of Corollary 2.13 are satisfied. Then, there exists u^* $\in C(I)$, a common fixed point of T and S, that is, u^* is a solution to (3.1).

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Author details
[1]Department of Mathematics, Disha Institute of Management and Technology, Satya Vihar, Vidhansabha-Chandrakhuri Marg, Naradha, Mandir Hasaud, Raipur-492101, Chhattisgarh, India [2]Département de Mathématiques, Université de Tunis, Ecole Supérieure des Sciences et Techniques de Tunis, 5, avenue Taha Hussein-Tunis, B.P. 56, Bab Menara-1008, Tunisie [3]Department of Mathematics, Kyungnam University, Masan, Kyungnam 631-701, Korea

Authors' contributions
JKK and HKN conceived the study and participated in its design and coordination. JKK suggested many good ideas that are useful for achievement this paper and made the revision. HKN and BS prepared the manuscript initially and performed all the steps of proof in this research. All authors read and approved the final manuscript.

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The authors declare that we have no competing interests.

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