

Erratum

Erratum to “Some Fixed Point Theorems of Integral Type Contraction in Cone Metric Space”

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We regret making following mistake in the above-mentioned paper [1]. We would like to correct it and explain some notations.

(1) In [1] we introduced a new concept of integral type contraction in cone metric spaces and generalized Branciari and Meir-Keeler theorems in such spaces. [1, Theorem 2.9] is an extension of Branciari’s theorem, and [1, Theorem 3.2] is an extension of Branciari and Meir-Keeler’s results. We asserted the following in [1, Theorem 2.9].

- (i) “Let (X, d) be a complete cone metric space and P be a normal cone. Suppose $\phi : P \rightarrow P$ is a non-vanishing map and a sub-additive cone integrable on each $[a, b] \subset P$ such that for each $\epsilon \gg 0$, $\int_0^\epsilon \phi d_p \gg 0$. If $f : X \rightarrow X$ is a map such that for all $x, y \in X$

$$\int_0^{d(f(x), f(y))} \phi d_p \leq \alpha \int_0^{d(x, y)} \phi d_p \quad (1)$$

for some $\alpha \in (0, 1)$, then f has a unique fixed point in X .”

Also, we asserted in [1, Theorem 3.2] the following.

- (ii) “Let (X, d) be a complete regular cone metric space and f be a mapping on X . Assume that there exists a function θ from P into itself satisfying the following:

(B1) $\theta(0) = 0$ and $\theta(t) \gg 0$ for all $t \gg 0$.

(B2) θ is nondecreasing and continuous function. Moreover, its inverse is continuous.

(B3) For all $0 \neq \varepsilon \in P$, there exists $\delta \gg 0$ such that for all $x, y \in X$

$$\theta(d(x, y)) < \varepsilon + \delta \quad \text{implies} \quad \theta(d(fx, fy)) < \varepsilon. \quad (2)$$

(B4) For all $x, y \in X$

$$\theta(x + y) \leq \theta(x) + \theta(y). \quad (3)$$

Then f has a unique fixed point."

After this theorem, we asserted the following in [1, Remark 3.3] that:

- (iii) "If $\phi : P \rightarrow P$ is a non-vanishing map and a sub-additive cone integrable on each $[a, b] \subset P$ such that for each $\varepsilon \gg 0$, $\int_0^\varepsilon \phi d_p \gg 0$ and $\theta(x) = \int_0^x \phi d_p$, then θ satisfies in all conditions of [1, Theorem 3.2]. Equivalently [1, Theorem 2.9] is concluded from [1, Theorem 3.2]."

Note that, in (B2) of [1, Theorem 3.2] and [1, Remark 3.3], we have emphasized that the map $\theta(x) = \int_0^x \phi d_p$ must have the continuous inverse, but unfortunately this assumption has been forgotten mistakenly in [1, Theorem 2.9]. Note that this assumption is a necessary condition to prove [1, Theorem 2.9].

(2) To prove [1, Theorem 3.2] and [1, Theorem 2.9], it is sufficient that $\theta(x) = \int_0^x \phi d_p$ satisfy the following: for each sequence $\{x_n\} \subset P$

$$\theta(x_n) \rightarrow 0 \quad \text{implies} \quad x_n \rightarrow 0. \quad (4)$$

On the other hand, (4) is equivalent to continuity of θ^{-1} at zero.

(3) In [2] the authors gave a counterexample on [1, Theorem 2.9] only for our misprint that we have asserted it in the above as you have seen. They also gave a comment for us at the end of their paper to correct such misprint and emphasized that θ must have the continuous inverse. As you have seen, we have asserted and emphasized such note in (B2) of [1, Theorem 3.2] and [1, Remark 3.3] before the authors in [2] mentioned it.

Nevertheless, we do apologize to the readers for this mistake.

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References

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