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Letter to the Editor

Comment on "A Strong Convergence of a Generalized Iterative Method for Semigroups of Nonexpansive Mappings in Hilbert Spaces"

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Piri and Vaezi (2010) introduced an iterative scheme for finding a common fixed point of a semigroup of nonexpansive mappings in a Hilbert space. Here, we present that their conclusions are not original and most parts of their paper are picked up from Saeidi and Naseri (2010), though it has not been cited.

Let S be a semigroup and $B(S)$ the Banach space of all bounded real-valued functions on S with supremum norm. For each $s \in S$, the left translation operator $l(s)$ on $B(S)$ is defined by $(l(s)f)(t) = f(st)$ for each $t \in S$ and $f \in B(S)$. Let X be a subspace of $B(S)$ containing 1 and let X^* be its topological dual. An element μ of X^* is said to be a mean on X if $\|\mu\| = \mu(1) = 1$. Let X be l_s-invariant, that is, $l_s(X) \subset X$ for each $s \in S$. A mean μ on X is said to be left invariant if $\mu(l_s f) = \mu(f)$ for each $s \in S$ and $f \in X$. A net $\{\mu_\alpha\}$ of means on X is said to be asymptotically left invariant if $\lim_\alpha(\mu_\alpha(l_s f) - \mu_\alpha(f)) = 0$ for each $f \in X$ and $s \in S$, and it is said to be strongly left regular if $\lim_\alpha \|l_s^* \mu_\alpha - \mu_\alpha\| = 0$ for each $s \in S$, where l_s^* is the adjoint operator of l_s. Let C be a nonempty closed and convex subset of E. A mapping $T : C \to C$ is said to be nonexpansive if $\|Tx - Ty\| \le \|x - y\|$, for all $x, y \in C$. Then $\varphi = \{T(t) : t \in S\}$ is called a representation of S as nonexpansive mappings on C if $T(s)$ is nonexpansive for each $s \in S$ and $T(st) = T(s)T(t)$ for each $s, t \in S$. The set of common fixed points of φ is denoted by $\mathrm{Fix}(\varphi)$.

If, for each $x^* \in E^*$, the function $t \mapsto \langle T(t)x, x^* \rangle$ is contained in X and C is weakly compact, then, there exists a unique point x_0 of E such that $\mu_t \langle T(t)x, x^* \rangle = \langle x_0, x^* \rangle$ for each $x^* \in E^*$. Such a point x_0 is denoted by $T(\mu)x$. Note that $T(\mu)$ is a nonexpansive mapping of C into itself and $T(\mu)z = z$, for each $z \in \mathrm{Fix}(\varphi)$.

Recall that a mapping F with domain $D(F)$ and range $R(F)$ in a normed space E is called δ-strongly accretive if for each $x, y \in D(F)$, there exists $j(x-y) \in J(x-y)$ such that

$$\langle Fx - Fy, j(x-y) \rangle \geq \delta \|x-y\|^2 \quad \text{for some } \delta \in (0,1). \tag{1}$$

F is called λ-strictly pseudocontractive if for each $x, y \in D(F)$, there exists $j(x-y) \in J(x-y)$ such that

$$\langle Fx - Fy, j(x-y) \rangle \leq \|x-y\|^2 - \lambda \|x - y - (Fx - Fy)\|^2, \tag{2}$$

for some $\lambda \in (0,1)$.

In [1], Saeidi and Naseri established a strong convergence theorem for a semigroup of nonexpansive mappings, as follows.

Theorem 1 (Saeidi and Naseri [1]). *Let $= \{T(t) : t \in S\}$ be a nonexpansive semigroup on H such that $F(\varphi) \neq \varnothing$. Let X be a left invariant subspace of $B(S)$ such that $1 \in X$, and the function $t \mapsto \langle T(t)x, y \rangle$ is an element of X for each $x, y \in H$. Let $\{\mu_n\}$ be a left regular sequence of means on X and let $\{\alpha_n\}$ be a sequence in $(0,1)$ such that $\alpha_n \to 0$ and $\sum_{n=0}^{\infty} \alpha_n = \infty$. Let $x_0 \in H$, $0 < \gamma < \overline{\gamma}/\alpha$ and let $\{x_n\}$ be generated by the iterative algorithm*

$$x_{n+1} = \alpha_n \gamma f(x_n) + (I - \alpha_n A) T(\mu_n) x_n, \quad n \geq 0, \tag{3}$$

where: $H \to H$ is a contraction with constant $0 \leq \alpha < 1$ and $A : H \to H$ is strongly positive with constant $\overline{\gamma} > 0$ (i.e., $\langle Ax, x \rangle \geq \overline{\gamma}\|x\|^2$, for all $x \in H$). Then, $\{x_n\}$ converges in norm to $x^* \in \mathrm{Fix}(\varphi)$ which is a unique solution of the variational inequality $\langle (A - \gamma f)x^*, x - x^* \rangle \geq 0$, $x \in \mathrm{Fix}(\varphi)$. Equivalently, one has $P_{\mathrm{Fix}(\varphi)}(I - A + \gamma f)x^* = x^*$.

Afterward, Piri and Vaezi [2] gave the following theorem, which is a minor variation of that given originally in [1], though they are not cited [1] in their paper.

Theorem 2 (Piri and Vaezi [2]). *Let $= \{T(t) : t \in S\}$ be a nonexpansive semigroup on H such that $F(\varphi) \neq \varnothing$. Let X be a left invariant subspace of $B(S)$ such that $1 \in X$, and the function $t \mapsto \langle T(t)x, y \rangle$ is an element of X for each $x, y \in H$. Let $\{\mu_n\}$ be a left regular sequence of means on X and let $\{\alpha_n\}$ be a sequence in $(0,1)$ such that $\alpha_n \to 0$ and $\sum_{n=0}^{\infty} \alpha_n = \infty$. Let $x_0 \in H$ and $\{x_n\}$ be generated by the iteration algorithm*

$$x_{n+1} = \alpha_n \gamma f(x_n) + (I - \alpha_n F) T(\mu_n) x_n, \quad n \geq 0, \tag{4}$$

where: $H \to H$ is a contraction with constant $0 \leq \alpha < 1$ and $F : H \to H$ is δ-strongly accretive and λ-strictly pseudocontractive with $0 \leq \delta, \lambda < 1$, $\delta + \lambda > 1$ and $\gamma \in (0, 1 - \sqrt{(1-\delta)/\lambda}/\alpha)$. Then, $\{x_n\}$ converges in norm to $x^* \in \mathrm{Fix}(\varphi)$ which is a unique solution of the variational inequality $\langle (F - \gamma f)x^*, x - x^* \rangle \geq 0$, $x \in \mathrm{Fix}(\varphi)$. Equivalently, one has $P_{\mathrm{Fix}(\varphi)}(I - F + \gamma f)x^* = x^*$.

The following are some comments on Piri and Vaezi's paper.

(i) It is well known that for small enough α_n's, both of the mappings $I - \alpha_n A$ and $I - \alpha_n F$ in Theorems 1 and 2 are contractive with constants $1 - \alpha_n \overline{\gamma}$ and $1 - \alpha_n(1 -$

$\sqrt{(1-\delta)/\lambda})$, respectively. In fact what differentiates the proofs of these theorems is their use of different constants, and the whole proof of Theorem 1 has been repeated for Theorem 2.

(ii) In Hilbert spaces, accretive operators are called monotone, though, it has not been considered, in Piri and Vaezi's paper.

(iii) Repeating the proof of Theorem 1, one may see that the same result holds for a strongly monotone and Lipschitzian mapping. A λ-strict pseudocontractive mapping is Lipschitzian with constant $(1+1/\lambda)$.

(iv) The proof of Corollary 3.2 of Piri and Vaezi's paper is false. To correct, one may impose the condition $\|A\| \leq 1$.

(v) The constant γ, used in Theorem 2, should be chosen in $(0, (1-\sqrt{(1-\delta)/\lambda})/\alpha)$.

References

[1] S. Saeidi and S. Naseri, "Iterative methods for semigroups of nonexpansive mappings and variational inequalities," *Mathematical Reports*, vol. 12(62), no. 1, pp. 59–70, 2010.

[2] H. Piri and H. Vaezi, "A strong convergence of a generalized iterative method for semigroups of nonexpansive mappings in Hilbert spaces," *Fixed Point Theory and Applications*, vol. 2010, Article ID 907275, 16 pages, 2010.