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Research Article

Iterative Algorithms with Variable Coefficients for Asymptotically Strict Pseudocontractions

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We introduce and study some new CQ-type iterative algorithms with variable coefficients for asymptotically strict pseudocontractions in real Hilbert spaces. General results for asymptotically strict pseudocontractions are established. The main result extends the previous results.

1. Introduction

Let H be a real Hilbert space, C a nonempty closed convex subset of H, $T : C \to C$ a self-mapping of C and $\mathrm{Fix}(T) := \{x \in C : Tx = x\}$.

Recall that a mapping $T : C \to C$ is called to be nonexpansive if

$$\|Tx - Ty\| \le \|x - y\|, \quad \forall x, y \in C. \tag{1.1}$$

T is called to be asymptotically nonexpansive [1] if there exists a sequence $\{k_n\}$ with $k_n \ge 1$ and $\lim_{n\to\infty} k_n = 1$ such that

$$\|T^n x - T^n y\| \le k_n \|x - y\|, \quad \forall x, y \in C, \text{ and all integers } n \ge 1. \tag{1.2}$$

T is called to be an asymptotically κ-strict pseudocontraction, if there exist $0 \le \kappa < 1$ and $0 \le \gamma_n \to 0$ $(n \to \infty)$ such that

$$\|T^n x - T^n y\|^2 \le (1 + \gamma_n)\|x - y\|^2 + \kappa\|(I - T^n)x - (I - T^n)y\|^2 \tag{1.3}$$

for all $x, y \in C$ and all integers $n \ge 1$.

As $\kappa = 0$, asymptotically κ-strict pseudocontraction T is asymptotically nonexpansive.

In [2], Nakajo and Takahashi studied the iterative approximation of fixed points of nonexpansive mappings and proved the following strong convergence theorem.

Theorem A. *Let C be a nonempty closed convex subset of a Hilbert space H and let T be a nonexpansive mapping of C into itself such that $\mathrm{Fix}(T) \neq \emptyset$. Suppose $\{x_n\}$ is given by*

$$\begin{aligned} &x_0 \in C \text{ chosen arbitrarily}, \\ &y_n = \alpha_n x_n + (1-\alpha_n) T x_n, \\ C_n &= \{ z \in C : \| y_n - z \| \leq \| x_n - z \| \}, \\ Q_n &= \{ z \in C : \langle x_n - z, x_0 - x_n \rangle \geq 0 \}, \\ &x_{n+1} = P_{C_n \cap Q_n} x_0, \quad n \in \mathbb{N}, \end{aligned} \tag{1.4}$$

where $P_{C_n \cap Q_n}$ is the metric projection from C onto $C_n \cap Q_n$ and α_n is chosen so that $0 \leq \alpha_n \leq a < 1$. Then, $\{x_n\}$ converges strongly to $P_{\mathrm{Fix}(T)} x_0$, where $P_{\mathrm{Fix}(T)}$ is the metric projection from C onto $\mathrm{Fix}(T)$.

Such algorithm in (1.4) is referred to be the (CQ) algorithm in [3], due to the fact that each iterate x_{n+1} is obtained by projecting x_0 onto the intersection of the suitably constructed closed convex sets C_n and Q_n. It is known that the (CQ) algorithm in (1.4) is of independent interest, and the (CQ) algorithm has been extended to various mappings by many authors (cf., e.g., [3–11]).

Very recently, by extending the (CQ) algorithm, Takahashi et al. [9] studied a family of nonexpansive mappings and gave some good strong convergence theorems. Kim and Xu [5] extended the (CQ) algorithm to study asymptotically κ-strict pseudocontractions and established the following interesting result with the help of some boundedness conditions.

Theorem B. *Let C be a closed convex subset of a Hilbert space H and let $T : C \longrightarrow C$ be an asymptotically κ-strict pseudocontractions for some $0 \leq \kappa < 1$. Assume that the fixed point set $\mathrm{Fix}(T)$ of T is nonempty and bounded. Let $\{x_n\}_{n=0}^{\infty}$ be the sequence generated by the following (CQ) algorithm:*

$$\begin{aligned} &x_0 \in C, \text{ chosen arbitrarily}, \\ &y_n = \alpha_n x_n + (1-\alpha_n) T^n x_n, \\ C_n &= \left\{ z \in C : \| y_n - z \|^2 \leq \| x_n - z \|^2 + [\kappa - \alpha_n (1-\alpha_n)] \| x_n - T x_n \|^2 + \theta_n \right\}, \\ Q_n &= \{ z \in C : \langle x_n - z, x_0 - x_n \rangle \geq 0 \}, \\ &x_{n+1} = P_{C_n \cap Q_n} x_0, \end{aligned} \tag{1.5}$$

where

$$\theta_n = \Delta_n^2 (1-\alpha_n) \gamma_n \longrightarrow 0 \quad (n \longrightarrow \infty), \qquad \Delta_n = \sup\{ \| x_n - z \| : z \in \mathrm{Fix}(T) \} < \infty. \tag{1.6}$$

Assume that control sequence $\{\alpha_n\}_{n=0}^{\infty}$ is chosen so that $\limsup_{n \to \infty} \alpha_n < 1 - \kappa$. Then $\{x_n\}$ converges strongly to $P_{\mathrm{Fix}(T)} x_0$.

It is our purpose in this paper to try to obtain some new fixed point theorems for asymptotically strict pseudocontractions without the boundedness conditions as in Theorem B. Motivated by Nakajo and Takahashi [2], Takahashi et al. [9], and Kim and Xu [5], we introduce and study certain new CQ-type iterative algorithms with variable coefficients for asymptotically strict pseudocontractions in real Hilbert spaces. Our results improve essentially the corresponding results of [5].

2. Results and Proofs

Throughout this paper,

(i) $x_n \rightharpoonup x$ means that $\{x_n\}$ converges weakly to x.

(ii) $x_n \to x$ means that $\{x_n\}$ converges strongly to x.

(iii) $\omega_w(x_n) := \{x : \exists\, x_{n_j} \rightharpoonup x\}$, that is, the weak ω-limit set of $\{x_n\}$.

(iv) $B_r(x_0) := \{x \in H : \|x - x_0\| \le r\}$.

(v) $\mathbb{N}$ is the set of nonnegative integers.

The following lemmas are basic (cf., e.g., [6] for Lemma 2.1, and [5] for Lemmas 2.2-2.3).

Lemma 2.1. *Let K be a closed convex subset of a real Hilbert space H. Given $x \in H, z \in K$. Then $z = P_K x$ if and only if*

$$\langle x - z, y - z\rangle \le 0, \quad \forall y \in K, \tag{2.1}$$

where $P_K x$ is the unique point in K with the property

$$\|x - P_K x\| \le \|x - y\|, \quad \forall y \in K. \tag{2.2}$$

Lemma 2.2. *Let K be a closed convex subset of a real Hilbert space H, $\{x_n\} \subset H, u \in H$, and $q = P_K u$. Suppose that $\{x_n\}$ satisfies*

$$\|x_n - u\| \le \|u - q\|, \quad \forall n \in \mathbb{N}, \tag{2.3}$$

and $\omega_w(x_n) \subset K$. Then $x_n \to q$.

Lemma 2.3. *Let C be a closed convex subset of a Hilbert space H and $T : C \to C$ an asymptotically κ-strict pseudocontraction. Then*

(I) *for each $n \ge 1$, T^n satisfies the Lipschitz condition:*

$$\|T^n x - T^n y\| \le L_n \|x - y\|, \quad \forall x, y \in C, \tag{2.4}$$

where

$$L_n = \frac{\kappa + \sqrt{1 + \gamma_n(1-\kappa)}}{1-\kappa}, \quad \{\gamma_n\} \text{ is as in (1.3)}; \tag{2.5}$$

(II) *if* $\{x_n\}$ *is a sequence in* C *such that* $x_n \rightharpoonup \tilde{x}$ *and*

$$\limsup_{m\to\infty}\limsup_{n\to\infty}\|x_n - T^m x_n\| = 0, \tag{2.6}$$

then

$$(I-T)x_n \longrightarrow 0 \Longrightarrow (I-T)\tilde{x} = 0. \tag{2.7}$$

In particular,

$$x_n \rightharpoonup \tilde{x}, \qquad (I-T)x_n \longrightarrow 0 \Longrightarrow (I-T)\tilde{x} = 0. \tag{2.8}$$

(III) $\mathrm{Fix}(T)$ *is closed and convex so that the projection* $P_{\mathrm{Fix}(T)}$ *is well defined.*

Theorem 2.4. *Let* C *be a closed convex subset of a Hilbert space* H, $T : C \to C$ *an asymptotically* κ*-strict pseudocontraction for some* $0 \le \kappa < 1$, *and* $\mathrm{Fix}(T) \neq \emptyset$. *Let* $\{x_n\}$ *be the sequence generated by the following CQ-type algorithm with variable coefficients:*

$$\begin{aligned}
&x_0 \in C \text{ chosen arbitrarily},\\
&y_n = \left(1-\hat{\beta}_n\right)x_n + \hat{\beta}_n T^n x_n,\\
&C_n = \left\{z \in C : \left\|y_n - z\right\|^2 \le \|x_n - z\|^2 + \hat{\beta}_n\left(\kappa + \hat{\beta}_n - 1\right)\|x_n - T^n x_n\|^2 + \theta_n\right\},\\
&Q_n = \{z \in C : \langle x_n - z, x_0 - x_n\rangle \ge 0\},\\
&x_{n+1} = P_{C_n\cap Q_n}x_0, \quad n \in \mathbb{N},
\end{aligned} \tag{2.9}$$

where

$$\hat{\beta}_n = \frac{\beta_n}{1+\|x_n - x_0\|^2}, \qquad \beta_n \in \left[\frac{1}{2}, 1\right], \qquad \theta_n = 2\left(1+r_0^2\right)\beta_n\gamma_n, \tag{2.10}$$

the sequence $\{\beta_n\}$ *is chosen so that* $\beta_n \to 1$ $(n \to \infty)$, *the positive real number* r_0 *is chosen so that* $B_{r_0}(x_0) \cap \mathrm{Fix}(T) \neq \emptyset$, *and* $\{\gamma_n\}$ *is as in* (1.3). *Then* $\{x_n\}$ *converges strongly to* $P_{\mathrm{Fix}(T)}x_0$.

Proof. We divide the proof into five steps.

Step 1. We prove that $C_n \cap Q_n$ is nonempty, convex and closed.

Clearly, both Q_n and C_n are convex and closed, so is $C_n \cap Q_n$. Since $T : C \to C$ is an asymptotically κ-strict pseudocontraction, we have by (1.3),

$$\begin{aligned}
\left\|T^n x - p\right\|^2 &\le (1+\gamma_n)\left\|x-p\right\|^2 + \kappa\left\|(I-T^n)x - (I-T^n)p\right\|^2\\
&\le (1+\gamma_n)\left\|x-p\right\|^2 + \kappa\|x - T^n x\|^2,
\end{aligned} \tag{2.11}$$

for all $x \in C$, $p \in \mathrm{Fix}(T)$, and all integers $n \ge 1$.

By (2.9) and (2.11), we deduce that for each $p \in B_{r_0}(x_0) \cap \mathrm{Fix}(T), n \in \mathbb{N}$,

$$\begin{aligned}\|y_n - p\|^2 &= \left\|\left(1-\widehat{\beta}_n\right)(x_n - p) + \widehat{\beta}_n(T^n x_n - p)\right\|^2 \\ &= \left(1-\widehat{\beta}_n\right)\|x_n - p\|^2 + \widehat{\beta}_n\|T^n x_n - p\|^2 - \widehat{\beta}_n\left(1-\widehat{\beta}_n\right)\|x_n - T^n x_n\|^2 \\ &= \left(1-\widehat{\beta}_n\right)\|x_n - p\|^2 + \widehat{\beta}_n\left[(1+\gamma_n)\|x_n - p\|^2 + \kappa\|x_n - T^n x_n\|^2\right] \\ &\quad - \widehat{\beta}_n\left(1-\widehat{\beta}_n\right)\|x_n - T^n x_n\|^2 \\ &\le \|x_n - p\|^2 + \widehat{\beta}_n\left(\kappa + \widehat{\beta}_n - 1\right)\|x_n - T^n x_n\|^2 + \beta_n\gamma_n \frac{2\left(\|x_n - x_0\|^2 + \|x_0 - p\|^2\right)}{1 + \|x_n - x_0\|^2} \\ &\le \|x_n - p\|^2 + \widehat{\beta}_n\left(\kappa + \widehat{\beta}_n - 1\right)\|x_n - T^n x_n\|^2 + 2\left(1 + r_0^2\right)\beta_n\gamma_n \\ &= \|x_n - p\|^2 + \widehat{\beta}_n\left(\kappa + \widehat{\beta}_n - 1\right)\|x_n - T^n x_n\|^2 + \theta_n.\end{aligned} \tag{2.12}$$

Therefore,

$$B_r(x_0) \cap \mathrm{Fix}(T) \subset C_n, \quad \forall n \in \mathbb{N}. \tag{2.13}$$

Next, we prove by induction that

$$B_{r_0}(x_0) \cap \mathrm{Fix}(T) \subset Q_n, \quad \forall n \in \mathbb{N}. \tag{2.14}$$

Obviously, $B_{r_0}(x_0) \cap \mathrm{Fix}(T) \subset C = Q_0$, that is, (2.14) holds for $n = 0$. Assume that $B_{r_0}(x_0) \cap \mathrm{Fix}(T) \subset Q_n$ for some $n \in \mathbb{N}$. Then, (2.13) implies that $B_{r_0}(x_0) \cap \mathrm{Fix}(T) \subset C_n \cap Q_n \neq \emptyset$ and $x_{n+1} = P_{C_n \cap Q_n} x_0$ is well defined.

By Lemma 2.1, we get $\langle x_{n+1} - z, x_0 - x_{n+1}\rangle \ge 0, \forall z \in C_n \cap Q_n$. In particular, for each $z \in B_{r_0}(x_0) \cap \mathrm{Fix}(T)$, we have $\langle x_{n+1} - z, x_0 - x_{n+1}\rangle \ge 0$.This together with the definition of Q_{n+1}, the inequality (2.14) holds for $n+1$. So (2.14) is true.

Step 2. We prove that $\lim_{n\to\infty}\|x_{n+1} - x_n\| = 0$.

By the definition of Q_n and Lemma 2.1, we get $x_n = P_{Q_n} x_0$. Hence,

$$\|x_n - x_0\| \le \|p - x_0\|, \quad \forall p \in B_{r_0}(x_0) \cap \mathrm{Fix}(T). \tag{2.15}$$

Denoting $M := \|x_0\| + \|p - x_0\|$, we have $\|x_n\| \le M$, for all $n \in \mathbb{N}$, and

$$\|x_n - x_0\| \le \|q - x_0\|, \quad \forall n \in \mathbb{N}, \tag{2.16}$$

where $q = P_{\mathrm{Fix}(T)}x_0 \subset B_{r_0}(x_0) \cap \mathrm{Fix}(T)$. The definition of x_{n+1} shows that $x_{n+1} \in Q_n$, that is, $\langle x_{n+1} - x_n, x_n - x_0 \rangle \geq 0$. This implies that

$$\begin{aligned} \|x_{n+1} - x_n\|^2 &= \|x_{n+1} - x_0\|^2 - \|x_n - x_0\|^2 - 2\langle x_{n+1} - x_n, x_n - x_0 \rangle \\ &\leq \|x_{n+1} - x_0\|^2 - \|x_n - x_0\|^2. \end{aligned} \tag{2.17}$$

Thus $\{\|x_n - x_0\|\}$ is increasing. Since $\{x_n\}$ is bounded, $\lim_{n\to\infty}\|x_n - x_0\|$ exists and

$$\lim_{n\to\infty} \|x_{n+1} - x_n\| = 0. \tag{2.18}$$

Step 3. We prove that $\lim_{n\to\infty}\|x_n - T^n x_n\| = 0$.

The definition of x_{n+1} shows that $x_{n+1} \in C_n$, that is,

$$\left\|y_n - x_{n+1}\right\|^2 \leq \|x_n - x_{n+1}\|^2 + \widehat{\beta}_n\left(\kappa + \widehat{\beta}_n - 1\right)\|x_n - T^n x_n\|^2 + \theta_n. \tag{2.19}$$

By (2.19) and the definition of y_n in (2.9), we deduce that

$$\begin{aligned} \widehat{\beta}_n^2\|x_n - T^n x_n\|^2 &= \left\|y_n - x_n\right\|^2 \\ &\leq \left\|y_n - x_{n+1}\right\|^2 + \|x_{n+1} - x_n\|^2 + 2\left\|y_n - x_{n+1}\right\| \cdot \|x_{n+1} - x_n\| \\ &\leq \widehat{\beta}_n\left(\kappa + \widehat{\beta}_n - 1\right)\|x_n - T^n x_n\|^2 + \theta_n + 2\|x_{n+1} - x_n\|^2 + 2\left\|y_n - x_{n+1}\right\| \cdot \|x_{n+1} - x_n\|. \end{aligned} \tag{2.20}$$

Further, we have

$$(1-\kappa)\widehat{\beta}_n\|x_n - T^n x_n\|^2 \leq 2\|x_{n+1} - x_n\|^2 + 2\|x_{n+1} - x_n\| \cdot \left\|y_n - x_{n+1}\right\| + \theta_n. \tag{2.21}$$

Thus, (2.19) and (2.21) imply that

$$\begin{aligned} (1-\kappa)\widehat{\beta}_n\|x_n - T^n x_n\|^2 \leq{}& 4\|x_{n+1} - x_n\|^2 + 2\|x_{n+1} - x_n\| \cdot \|x_n - T^n x_n\|\sqrt{\widehat{\beta}_n\left|\kappa + \widehat{\beta}_n - 1\right|} \\ &+ 2\|x_{n+1} - x_n\|\sqrt{\theta_n} + \theta_n. \end{aligned} \tag{2.22}$$

Noticing $\|x_n\| \leq M, \beta_n \in [1/2, 1]$, we get

$$\widehat{\beta}_n = \frac{\beta_n}{1 + \|x_n\|^2} \geq \frac{1}{2(1+M^2)} > 0. \tag{2.23}$$

From $\lim_{n\to\infty}\|x_{n+1} - x_n\| = 0$, $\lim_{n\to\infty}\theta_n = 0$, and (2.22), it follows that

$$\lim_{n\to\infty} \|x_n - T^n x_n\| = 0. \tag{2.24}$$

Step 4. We prove that

$$\lim_{n\to\infty}\|x_n - Tx_n\| = 0. \tag{2.25}$$

By Lemma 2.3 and the definition of T, we obtain

$$\begin{aligned}\|x_n - Tx_n\| &\le \|x_n - x_{n+1}\| + \left\|x_{n+1} - T^{n+1}x_{n+1}\right\| + \left\|T^{n+1}x_{n+1} - T^{n+1}x_n\right\| + \left\|T^{n+1}x_n - Tx_n\right\| \\ &\le (1 + L_{n+1})\|x_{n+1} - x_n\| + \left\|x_{n+1} - T^{n+1}x_{n+1}\right\| + L_1\|x_n - T^n x_n\|,\end{aligned} \tag{2.26}$$

where

$$L_n = \frac{\kappa + \sqrt{1 + \gamma_n(1-\kappa)}}{1-\kappa}, \quad \{\gamma_n\} \text{ is as in (1.3).} \tag{2.27}$$

By (2.18), (2.24), and (2.26), we know that (2.25) holds.

Step 5. Finally, by Lemma 2.3 and (2.25), we have $\omega_w(x_n) \subset \mathrm{Fix}(T)$. Furthermore, it follows from (2.16) and Lemma 2.2 that the sequence $\{x_n\}$ converges strongly to $q = P_{\mathrm{Fix}(T)}x_0$. □

Remark 2.5. Theorem 2.4 improves [5, Theorem 4.1] since the condition that $\theta_n \to 0$ is satisfied and the boundedness of $\mathrm{Fix}(T)$ is dropped off.

Theorem 2.6. *Let C be a closed convex subset of a Hilbert space H, $T : C \to C$ an asymptotically κ-strict pseudocontraction for some $0 \le \kappa < 1$, and $\mathrm{Fix}(T)$ be nonempty and bounded. Let $\{x_n\}$ the sequence generated by the following CQ-type algorithm with variable coefficients:*

$$\begin{aligned} &x_0 \in C \text{ chosen arbitrarily}, \\ &y_n = \left(1 - \widehat{\beta}_n\right)x_n + \widehat{\beta}_n T^n x_n, \\ &C_n = \left\{z \in C : \|y_n - z\|^2 \le \|x_n - z\|^2 + \widehat{\beta}_n\left(\kappa + \widehat{\beta}_n - 1\right)\|x_n - T^n x_n\|^2 + \theta_n\right\}, \\ &Q_n = \{z \in C : \langle x_n - z, x_0 - x_n\rangle \ge 0\}, \\ &x_{n+1} = P_{C_n\cap Q_n}x_0, \quad n \in \mathbb{N}, \end{aligned} \tag{2.28}$$

where

$$\widehat{\beta}_n = \frac{\beta_n}{1 + \|x_n - x_0\|^2}, \qquad \beta_n \in \left[\frac{1}{2}, 1\right], \qquad \theta_n = \left\{\sup_{z\in\mathrm{Fix}(T)}\|x_n - z\|\right\}^2 \widehat{\beta}_n\gamma_n, \tag{2.29}$$

the sequence $\{\beta_n\}$ is chosen so that $\beta_n \to 1$ $(n \to \infty)$, and $\{\gamma_n\}$ is as in (1.3). Then $\{x_n\}$ converges strongly to $P_{\mathrm{Fix}(T)}x_0$.

Proof. It is easy to see that $\theta_n \to 0$ in Theorem 2.6. Following the reasoning in the proof of Theorem 2.4 and using $\mathrm{Fix}(T)$ instead of $B_{r_0}(x_0) \cap \mathrm{Fix}(T)$, we deduce the conclusion of Theorem 2.6. □

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