

## *Research Article*

# **A New System of Generalized Nonlinear Mixed Variational Inclusions in Banach Spaces**

**Jian Wen Peng**

*College of Mathematics and Computer Science, Chongqing Normal University,  
Chongqing Sichuan 400047, China*

Correspondence should be addressed to Jian Wen Peng, [jwpeng6@yahoo.com.cn](mailto:jwpeng6@yahoo.com.cn)

Received 5 July 2009; Accepted 14 September 2009

Academic Editor: Mohamed A. Khamisi

Copyright © 2010 Jian Wen Peng. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

We introduce and study a new system of generalized nonlinear mixed variational inclusions in real  $q$ -uniformly smooth Banach spaces. We prove the existence and uniqueness of solution and the convergence of some new  $n$ -step iterative algorithms with or without mixed errors for this system of generalized nonlinear mixed variational inclusions. The results in this paper unify, extend, and improve some known results in literature.

## **1. Introduction**

Variational inclusion problems are among the most interesting and intensively studied classes of mathematical problems and have wide applications in the fields of optimization and control, economics and transportation equilibrium, as well as engineering science. For the past years, many existence results and iterative algorithms for various variational inequality and variational inclusion problems have been studied. For details, see [1–25] and the references therein.

Recently, some new and interesting problems, which are called to be system of variational inequality problems, were introduced and studied. Pang [1], Cohen and Chaplais [2], Bianchi [3], and Ansari and Yao [4] considered a system of scalar variational inequalities, and Pang showed that the traffic equilibrium problem, the spatial equilibrium problem, the Nash equilibrium, and the general equilibrium programming problem can be modeled as a system of variational inequalities. Ansari et al. [5] considered a system of vector variational inequalities and obtained its existence results. Allevi et al. [6] considered a system of generalized vector variational inequalities and established some existence results with relative pseudomonotonicity. Kassay and Kolumbán [7] introduced a system of variational inequalities and proved an existence theorem by the Ky Fan lemma. Kassay et al. [8] studied

Minty and Stampacchia variational inequality systems with the help of the Kakutani-Fan-Glicksberg fixed point theorem. Peng [9], Peng and Yang [10] introduced a system of quasivariational inequality problems and proved its existence theorem by maximal element theorems. Verma [11–15] introduced and studied some systems of variational inequalities and developed some iterative algorithms for approximating the solutions of system of variational inequalities in Hilbert spaces. J. K. Kim and D. S. Kim [16] introduced and studied a new system of generalized nonlinear quasivariational inequalities in Hilbert spaces. Cho et al. [17] introduced and studied a new system of nonlinear variational inequalities in Hilbert spaces. They proved some existence and uniqueness theorems of solutions for the system of nonlinear variational inequalities.

As generalizations of system of variational inequalities, Agarwal et al. [18] introduced a system of generalized nonlinear mixed quasivariational inclusions and investigated the sensitivity analysis of solutions for this system of generalized nonlinear mixed quasivariational inclusions in Hilbert spaces. Peng and Zhu [19] introduce a new system of generalized nonlinear mixed quasivariational inclusions in  $q$ -uniformly smooth Banach spaces and prove the existence and uniqueness of solutions and the convergence of several new two-step iterative algorithms with or without errors for this system of generalized nonlinear mixed quasivariational inclusions. Kazmi and Bhat [20] introduced a system of nonlinear variational-like inclusions and proved the existence of solutions and the convergence of a new iterative algorithm for this system of nonlinear variational-like inclusions. Fang and Huang [21], Verma [22], and Fang et al. [23] introduced and studied a new system of variational inclusions involving  $H$ -monotone operators,  $A$ -monotone operators and  $(H, \eta)$ -monotone operators, respectively. Yan et al. [24] introduced and studied a system of set-valued variational inclusions which is more general than the model in [21]. Peng and Zhu [25] introduced and studied a system of generalized mixed quasivariational inclusions involving  $(H, \eta)$ -monotone operators which contains those mathematical models in [11–16, 21–24] as special cases.

Inspired and motivated by the results in [1–25], the purpose of this paper is to introduce and study a new system of generalized nonlinear mixed quasivariational inclusions which contains some classes of system of variational inclusions and systems of variational inequalities in the literature as special cases. Using the resolvent technique for the  $m$ -accretive mappings, we prove the existence and uniqueness of solutions for this system of generalized nonlinear mixed quasivariational inclusions. We also prove the convergence of some new  $n$ -step iterative sequences with or without mixed errors to approximation the solution for this system of generalized nonlinear mixed quasivariational inclusions. The results in this paper unifies, extends, and improves some results in [11–16, 19] in several aspects.

## 2. Preliminaries

Throughout this paper we suppose that  $E$  is a real Banach space with dual space, norm and the generalized dual pair denoted by  $E^*$ ,  $\|\cdot\|$  and  $\langle \cdot, \cdot \rangle$ , respectively,  $2^E$  is the family of all the nonempty subsets of  $E$ ,  $\text{dom}(M)$  denotes the domain of the set-valued map  $M : E \rightarrow 2^E$ , and the generalized duality mapping  $J_q : E \rightarrow 2^{E^*}$  is defined by

$$J_q(x) = \left\{ f^* \in E^* : \langle x, f^* \rangle = \|f^*\| \cdot \|x\|, \|f^*\| = \|x\|^{q-1} \right\}, \quad \forall x \in E, \quad (2.1)$$

where  $q > 1$  is a constant. In particular,  $J_2$  is the usual normalized duality mapping. It is known that, in general,  $J_q(x) = \|x\|^{q-2}J_2(x)$ , for all  $x \neq 0$ , and  $J_q$  is single-valued if  $E^*$  is strictly convex.

The modulus of smoothness of  $E$  is the function  $\rho_E : [0, \infty) \rightarrow [0, \infty)$  defined by

$$\rho_E(t) = \sup \left\{ \frac{1}{2} (\|x + y\| + \|x - y\|) - 1 : \|x\| \leq 1, \|y\| \leq t \right\}. \quad (2.2)$$

A Banach space  $E$  is called uniformly smooth if

$$\lim_{t \rightarrow 0} \frac{\rho_E(t)}{t} = 0. \quad (2.3)$$

$E$  is called  $q$ -uniformly smooth if there exists a constant  $c > 0$ , such that

$$\rho_E(t) \leq ct^q, \quad q > 1. \quad (2.4)$$

Note that  $J_q$  is single-valued if  $E$  is uniformly smooth.

Xu [26] and Xu and Roach [27] proved the following result.

**Lemma 2.1.** *Let  $E$  be a real uniformly smooth Banach space. Then,  $E$  is  $q$ -uniformly smooth if and only if there exists a constant  $c_q > 0$ , such that for all  $x, y \in E$ ,*

$$\|x + y\|^q \leq \|x\|^q + q \langle y, J_q(x) \rangle + c_q \|y\|^q. \quad (2.5)$$

*Definition 2.2* (see [28]). Let  $M : \text{dom}(M) \subseteq E \rightarrow 2^E$  be a multivalued mapping:

- (i)  $M$  is said to be accretive if, for any  $x, y \in \text{dom}(M)$ ,  $u \in M(x)$ ,  $v \in M(y)$ , there exists  $j_q(x - y) \in J_q(x - y)$  such that

$$\langle u - v, j_q(x - y) \rangle \geq 0; \quad (2.6)$$

- (ii)  $M$  is said to be  $m$ -accretive if  $M$  is accretive and  $(I + \rho M)(\text{dom}(M)) = E$  holds for every (equivalently, for some)  $\rho > 0$ , where  $I$  is the identity operator on  $E$ .

*Remark 2.3.* It is well known that, if  $E = \mathcal{H}$  is a Hilbert space, then  $M : \text{dom}(M) \subseteq E \rightarrow 2^E$  is  $m$ -accretive if and only if  $M$  is maximal monotone (see, e.g., [29]).

We recall some definitions needed later.

*Definition 2.4* (see [28]). Let the multivalued mapping  $M : \text{dom}(M) \subseteq E \rightarrow 2^E$  be  $m$ -accretive, for a constant  $\rho > 0$ , the mapping  $R_\rho^M : E \rightarrow \text{dom}(M)$  which is defined by

$$R_\rho^M(u) = (I + \rho M)^{-1}(u), \quad u \in E, \quad (2.7)$$

is called the resolvent operator associated with  $M$  and  $\rho$ .

*Remark 2.5.* It is well known that  $R_\rho^M$  is single-valued and nonexpansive mapping (see [28]).

*Definition 2.6.* Let  $E$  be a real uniformly smooth Banach space, and let  $T : E \rightarrow E$  be a single-valued operator. However,  $T$  is said to be

(i)  $r$ -strongly accretive if there exists a constant  $r > 0$  such that

$$\langle Tx - Ty, J_q(x - y) \rangle \geq r \|x - y\|^q, \quad \forall x, y \in E, \quad (2.8)$$

or equivalently,

$$\langle Tx - Ty, J_2(x - y) \rangle \geq r \|x - y\|^2, \quad \forall x, y \in E; \quad (2.9)$$

(ii)  $s$ -Lipschitz continuous if there exists a constant  $s > 0$  such that

$$\|T(x) - T(y)\| \leq s \|x - y\|, \quad \forall x, y \in E. \quad (2.10)$$

*Remark 2.7.* If  $T$  is  $r$ -strongly accretive, then  $T$  is  $r$ -expanding, that is,

$$\|T(x) - T(y)\| \geq r \|x - y\|, \quad \forall x, y \in E. \quad (2.11)$$

**Lemma 2.8** (see [30]). *Let  $\{a_n\}, \{b_n\}, \{c_n\}$  be three real sequences, satisfying*

$$a_{n+1} \leq (1 - t_n)a_n + b_n + c_n, \quad \forall n \geq 0, \quad (2.12)$$

where  $t_n \in (0, 1)$ ,  $\sum_{n=0}^{\infty} t_n = \infty$ , for all  $n \geq 0$ ,  $b_n = o(t_n)$ ,  $\sum_{n=0}^{\infty} c_n < \infty$ . Then  $a_n \rightarrow 0$ .

### 3. System of Generalized Nonlinear Mixed Variational Inequalities

In this section, we will introduce a new system of generalized nonlinear mixed variational inclusions which contains some classes of system of variational inclusions and systems of variational inequalities in literature as special cases.

In what follows, unless other specified, we always suppose that  $\theta$  is a zero element in  $E$ , and for each  $i = 1, 2, \dots, n$ ,  $T_i$  and  $S_i : E \rightarrow E$  are single-valued mappings,  $M_i : E \rightarrow 2^E$  is an  $m$ -accretive operator. We consider the following problem: find  $(x_1^*, x_2^*, \dots, x_n^*) \in E^n$  such that

$$\begin{aligned} \theta &\in \rho_1 T_1 x_2^* + \rho_1 S_1 x_2^* + x_1^* - x_2^* + \rho_1 M_1(x_1^*), \\ \theta &\in \rho_2 T_2 x_3^* + \rho_2 S_2 x_3^* + x_2^* - x_3^* + \rho_2 M_2(x_2^*), \\ &\vdots \\ \theta &\in \rho_{n-1} T_{n-1} x_n^* + \rho_{n-1} S_{n-1} x_n^* + x_{n-1}^* - x_n^* + \rho_{n-1} M_{n-1}(x_{n-1}^*), \\ \theta &\in \rho_n T_n x_1^* + \rho_n S_n x_1^* + x_n^* - x_1^* + \rho_n M_n(x_n^*), \end{aligned} \quad (3.1)$$

which is called the system of generalized nonlinear mixed variational inclusions, where  $\rho_i > 0$  ( $i = 1, 2, \dots, n$ ) are constants.

In what follows, there are some special cases of the problem (3.1).

(i) If  $n = 2$ , then problem (3.1) reduces to the system of nonlinear mixed quasivariational inclusions introduced and studied by Peng and Zhu [19].

If  $E = \mathcal{H}$  is a Hilbert space and  $n = 2$ , then problem (3.1) reduces to the system of nonlinear mixed quasivariational inclusions introduced and studied by Agarwal et al. [18].

(ii) If  $E = \mathcal{H}$  is a Hilbert space, and for each  $i = 1, 2, \dots, n$ ,  $M_i(x) = \partial\phi_i(x)$  for all  $x \in \mathcal{H}$ , where  $\phi_i : \mathcal{H} \rightarrow R \cup \{+\infty\}$  is a proper, convex, lower semicontinuous functional, and  $\partial\phi_i$  denotes the subdifferential operator of  $\phi_i$ , then problem (3.1) reduces to the following system of generalized nonlinear mixed variational inequalities, which is to find  $(x_1^*, x_2^*, \dots, x_n^*) \in \mathcal{H}^n$  such that

$$\begin{aligned} \langle \rho_1 T_1 x_2^* + \rho_1 S_1 x_2^* + x_1^* - x_2^*, x - x_1^* \rangle &\geq \rho_1 \phi_1(x_1^*) - \rho_1 \phi_1(x), \quad \forall x \in \mathcal{H}, \\ \langle \rho_2 T_2 x_3^* + \rho_2 S_2 x_3^* + x_2^* - x_3^*, x - x_2^* \rangle &\geq \rho_2 \phi_2(x_2^*) - \rho_2 \phi_2(x), \quad \forall x \in \mathcal{H}, \\ &\vdots \\ \langle \rho_{n-1} T_{n-1} x_n^* + \rho_{n-1} S_{n-1} x_n^* + x_{n-1}^* - x_n^*, x - x_{n-1}^* \rangle &\geq \rho_{n-1} \phi_{n-1}(x_{n-1}^*) - \rho_{n-1} \phi_{n-1}(x), \quad \forall x \in \mathcal{H}, \\ \langle \rho_n T_n x_1^* + \rho_n S_n x_1^* + x_n^* - x_1^*, x - x_n^* \rangle &\geq \rho_n \phi_n(x_n^*) - \rho_n \phi_n(x), \quad \forall x \in \mathcal{H}, \end{aligned} \quad (3.2)$$

where  $\rho_i > 0$  ( $i = 1, 2, \dots, n$ ) are constants.

(iii) If  $n = 2$ , then (3.2) reduces to the problem of finding  $(x_1^*, x_2^*) \in \mathcal{H} \times \mathcal{H}$  such that

$$\begin{aligned} \langle \rho_1 T_1 x_2^* + \rho_1 S_1 x_2^* + x_1^* - x_2^*, x - x_1^* \rangle &\geq \rho_1 \phi_1(x_1^*) - \rho_1 \phi_1(x), \quad \forall x \in \mathcal{H}, \\ \langle \rho_2 T_2 x_1^* + \rho_2 S_2 x_1^* + x_2^* - x_1^*, x - x_2^* \rangle &\geq \rho_2 \phi_2(x_2^*) - \rho_2 \phi_2(x), \quad \forall x \in \mathcal{H}. \end{aligned} \quad (3.3)$$

Moreover, if  $\phi_1 = \phi_2 = \varphi$ , then problem (3.3) becomes the system of generalized nonlinear mixed variational inequalities introduced and studied by J. K. Kim and D. S. Kim in [16].

(iv) For  $i = 1, 2, \dots, n$ , if  $\phi_i = \delta_{K_i}$  (the indicator function of a nonempty closed convex subset  $K_i \subset \mathcal{H}$ ) and  $T_i = 0$ , then (3.2) reduces to the problem of finding  $(x_1^*, x_2^*, \dots, x_n^*) \in \prod_{i=1}^n K_i$ , such that

$$\begin{aligned} \langle \rho_1 S_1 x_2^* + x_1^* - x_2^*, x - x_1^* \rangle &\geq 0, \quad \forall x \in K_1, \\ \langle \rho_2 S_2 x_3^* + x_2^* - x_3^*, x - x_2^* \rangle &\geq 0, \quad \forall x \in K_1, \\ &\vdots \\ \langle \rho_{n-1} S_{n-1} x_n^* + x_{n-1}^* - x_n^*, x - x_{n-1}^* \rangle &\geq 0, \quad \forall x \in K_{n-1}, \\ \langle \rho_n S_n x_1^* + x_n^* - x_1^*, x - x_n^* \rangle &\geq 0, \quad \forall x \in K_n. \end{aligned} \quad (3.4)$$

Problem (3.4) is called the system of nonlinear variational inequalities. Moreover, if  $n = 2$ , then problem (3.4) reduces to the following system of nonlinear variational inequalities, which is to find  $(x_1^*, x_2^*) \in K_1 \times K_2$  such that

$$\begin{aligned} \langle \rho_1 S_1 x_2^* + x_1^* - x_2^*, x - x_1^* \rangle &\geq 0, \quad \forall x \in K_1, \\ \langle \rho_2 S_2 x_1^* + x_2^* - x_1^*, x - x_2^* \rangle &\geq 0, \quad \forall x \in K_2. \end{aligned} \quad (3.5)$$

If  $S_1 = S_2$  and  $K_1 = K_2 = K$ , then (3.5) reduces to the problem introduced and researched by Verma [11–13].

**Lemma 3.1.** *For any given  $x_i^* \in E (i = 1, 2, \dots, n)$ ,  $(x_1^*, x_2^*, \dots, x_n^*)$  is a solution of the problem (3.1) if and only if*

$$\begin{aligned} x_1^* &= R_{\rho_1}^{M_1} [x_2^* - \rho_1 (T_1 x_2^* + S_1 x_2^*)], \\ x_2^* &= R_{\rho_2}^{M_2} [x_3^* - \rho_2 (T_2 x_3^* + S_2 x_3^*)], \\ &\vdots \\ x_{n-1}^* &= R_{\rho_{n-1}}^{M_{n-1}} [x_n^* - \rho_{n-1} (T_{n-1} x_n^* + S_{n-1} x_n^*)], \\ x_n^* &= R_{\rho_n}^{M_n} [x_1^* - \rho_n (T_n x_1^* + S_n x_1^*)], \end{aligned} \quad (3.6)$$

where  $R_{\rho_i}^{M_i} = (I + \rho_i M_i)^{-1}$  is the resolvent operators of  $M_i$  for  $i = 1, 2, \dots, n$ .

*Proof.* It is easy to know that Lemma 3.1 follows from Definition 2.4 and so the proof is omitted.  $\square$

#### 4. Existence and Uniqueness

In this section, we will show the existence and uniqueness of solution for problems (3.1).

**Theorem 4.1.** *Let  $E$  be a real  $q$ -uniformly smooth Banach spaces. For  $i = 1, 2, \dots, n$ , let  $S_i : E \rightarrow E$  be strongly accretive and Lipschitz continuous with constants  $k_i$  and  $\mu_i$ , respectively, let  $T_i : E \rightarrow E$  be Lipschitz continuous with constant  $\nu_i$ , and let  $M_i : E \rightarrow E$  be an  $m$ -accretive mapping. If for each  $i = 1, 2, \dots, n$ ,*

$$0 < \sqrt[q]{1 - q\rho_i k_i + c_q \rho_i^q \mu_i^q} + \rho_i \nu_i < 1, \quad (4.1)$$

then (3.1) has a unique solution  $(x_1^*, x_2^*, \dots, x_n^*) \in E^n$ .

*Proof.* First, we prove the existence of the solution. Define a mapping  $F : E \rightarrow E$  as follows:

$$\begin{aligned}
F(x) &= R_{\rho_1}^{M_1} [x_2 - \rho_1(T_1x_2 + S_1x_2)], \\
x_2 &= R_{\rho_2}^{M_2} [x_3 - \rho_2(T_2x_3 + S_2x_3)], \\
&\vdots \\
x_{n-1} &= R_{\rho_{n-1}}^{M_{n-1}} [x_n - \rho_{n-1}(T_{n-1}x_n + S_{n-1}x_n)], \\
x_n &= R_{\rho_n}^{M_n} [x - \rho_n(T_nx + S_nx)].
\end{aligned} \tag{4.2}$$

For  $i = 1, 2, \dots, n$ , since  $R_{\rho_i}^{M_i}$  is a nonexpansive mapping,  $S_i$  is strongly accretive and Lipschitz continuous with constants  $k_i$  and  $\mu_i$ , respectively, and  $T_i$  is Lipschitz continuous with constant  $\nu_i$ , for any  $x, y \in E$ , we have

$$\begin{aligned}
&\|F(x) - F(y)\| \\
&= \left\| R_{\rho_1}^{M_1} [x_2 - \rho_1(T_1x_2 + S_1x_2)] - R_{\rho_1}^{M_1} [y_2 - \rho_1(T_1y_2 + S_1y_2)] \right\| \\
&\leq \|(x_2 - y_2) - \rho_1((T_1x_2 + S_1x_2) - (T_1y_2 + S_1y_2))\| \\
&\leq \|(x_2 - y_2) - \rho_1(S_1x_2 - S_1y_2)\| + \rho_1\|T_1x_2 - T_1y_2\| \\
&\leq \sqrt[q]{\|x_2 - y_2\|^q - q\rho_1\langle S_1x_2 - S_1y_2, J_q(x_2 - y_2) \rangle + c_q\rho_1^q\|S_1x_2 - S_1y_2\|^q} + \rho_1\nu_1\|x_2 - y_2\| \\
&= \left( \sqrt[q]{1 - q\rho_1k_1 + c_q\rho_1^q\mu_1^q} + \rho_1\nu_1 \right) \|x_2 - y_2\| \\
&= \left( \sqrt[q]{1 - q\rho_1k_1 + c_q\rho_1^q\mu_1^q} + \rho_1\nu_1 \right) \left\| R_{\rho_2}^{M_2} [x_3 - \rho_2(T_2x_3 + S_2x_3)] - R_{\rho_2}^{M_2} [y_3 - \rho_2(T_2y_3 + S_2y_3)] \right\| \\
&\leq \left( \sqrt[q]{1 - q\rho_1k_1 + c_q\rho_1^q\mu_1^q} + \rho_1\nu_1 \right) \|x_3 - y_3 - \rho_2[(S_2x_3 - S_2y_3) + (T_2x_3 - T_2y_3)]\| \\
&\leq \left( \sqrt[q]{1 - q\rho_1k_1 + c_q\rho_1^q\mu_1^q} + \rho_1\nu_1 \right) \|x_3 - y_3 - \rho_2(S_2x_3 - S_2y_3)\| + \rho_2\|T_2x_3 - T_2y_3\| \\
&\leq \left( \sqrt[q]{1 - q\rho_1k_1 + c_q\rho_1^q\mu_1^q} + \rho_1\nu_1 \right) \\
&\quad \times \left[ \sqrt[q]{\|x_3 - y_3\|^q - q\rho_2\langle S_2x_3 - S_2y_3, J_q(x_3 - y_3) \rangle + c_q\rho_2^q\|S_2x_3 - S_2y_3\|^q} + \rho_2\nu_2\|x_3 - y_3\| \right] \\
&\leq \left( \sqrt[q]{1 - q\rho_1k_1 + c_q\rho_1^q\mu_1^q} + \rho_1\nu_1 \right) \left( \sqrt[q]{1 - q\rho_2k_2 + c_q\rho_2^q\mu_2^q} + \rho_2\nu_2 \right) \|x_3 - y_3\| \\
&\leq \dots \leq \prod_{i=1}^{n-1} \left( \sqrt[q]{1 - q\rho_ik_i + c_q\rho_i^q\mu_i^q} + \rho_i\nu_i \right) \|x_n - y_n\|
\end{aligned}$$

$$\begin{aligned}
&= \prod_{i=1}^{n-1} \left\| R_{\rho_n}^{M_n} [x - \rho_n(T_n x + S_n x)] - R_{\rho_n}^{M_n} [y - \rho_n(T_n y + S_n y)] \right\| \\
&\leq \prod_{i=1}^{n-1} \left( \sqrt[q]{1 - q\rho_i k_i + c_q \rho_i^q \mu_i^q} + \rho_i \nu_i \right) \left[ \|x - y - \rho_n(S_n x) - S_n y\| + \rho_n \|T_n x - T_n y\| \right] \\
&\leq \prod_{i=1}^{n-1} \left( \sqrt[q]{1 - q\rho_i k_i + c_q \rho_i^q \mu_i^q} + \rho_i \nu_i \right) \\
&\quad \times \left[ \sqrt[q]{\|x - y\|^q - q\rho_n \langle S_n x - S_n y, J_q(x - y) \rangle + c_q \rho_n^q \|S_n x - S_n y\|^q} + \rho_n \nu_n \|x - y\| \right] \\
&\leq \prod_{i=1}^n \left( \sqrt[q]{1 - q\rho_i k_i + c_q \rho_i^q \mu_i^q} + \rho_i \nu_i \right) \|x - y\|.
\end{aligned} \tag{4.3}$$

It follows from (4.1) that

$$0 < \prod_{i=1}^n \left( \sqrt[q]{1 - q\rho_i k_i + c_q \rho_i^q \mu_i^q} + \rho_i \nu_i \right) < 1. \tag{4.4}$$

Thus, (4.3) implies that  $F$  is a contractive mapping and so there exists a point  $x_1^* \in E$  such that

$$x_1^* = F(x_1^*). \tag{4.5}$$

Let

$$\begin{aligned}
x_i^* &= R_{\rho_i}^{M_i} [x_{i+1}^* - \rho_i(T_i(x_{i+1}^*) + S_i(x_{i+1}^*))], \quad i = 1, 2, \dots, n-1 \\
x_n^* &= R_{\rho_n}^{M_n} [x_1^* - \rho_n(T_n(x_1^*) + S_n(x_1^*))]
\end{aligned} \tag{4.6}$$

then by the definition of  $F$ , we have

$$\begin{aligned}
x_1^* &= R_{\rho_1}^{M_1} [x_2^* - \rho_1(T_1 x_2^* + S_1 x_2^*)], \\
x_2^* &= R_{\rho_2}^{M_2} [x_3^* - \rho_2(T_2 x_3^* + S_2 x_3^*)], \\
&\vdots \\
x_{n-1}^* &= R_{\rho_{n-1}}^{M_{n-1}} [x_n^* - \rho_{n-1}(T_{n-1} x_n^* + S_{n-1} x_n^*)], \\
x_n^* &= R_{\rho_n}^{M_n} [x_1^* - \rho_n(T_n x_1^* + S_n x_1^*)],
\end{aligned} \tag{4.7}$$

that is,  $(x_1^*, x_2^*, \dots, x_n^*)$  is a solution of problem (3.1).



Then, we show the uniqueness of the solution. Let  $(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n)$  be another solution of problem (3.1). It follows from Lemma 3.1 that

$$\begin{aligned}\bar{x}_1 &= R_{\rho_1}^{M_1} [\bar{x}_2 - \rho_1(T_1\bar{x}_2 + S_1\bar{x}_2)], \\ \bar{x}_2 &= R_{\rho_2}^{M_2} [\bar{x}_3 - \rho_2(T_2\bar{x}_3 + S_2\bar{x}_3)], \\ &\vdots \\ \bar{x}_{n-1} &= R_{\rho_{n-1}}^{M_{n-1}} [\bar{x}_n - \rho_{n-1}(T_{n-1}\bar{x}_n + S_{n-1}\bar{x}_n)], \\ \bar{x}_n &= R_{\rho_n}^{M_n} [\bar{x}_1 - \rho_n(T_n\bar{x}_1 + S_n\bar{x}_1)].\end{aligned}\tag{4.8}$$

As the proof of (4.3), we have

$$\|x_1^* - \bar{x}_1\| \leq \prod_{i=1}^n \left( \sqrt[q]{1 - q\rho_i k_i + c_q \rho_i^q \mu_i^q} + \rho_i v_i \right) \|x_1^* - \bar{x}_1\|.\tag{4.9}$$

It follows from (4.1) that

$$0 < \prod_{i=1}^n \left( \sqrt[q]{1 - q\rho_i k_i + c_q \rho_i^q \mu_i^q} + \rho_i v_i \right) < 1.\tag{4.10}$$

Hence,

$$x_1^* = \bar{x}_1,\tag{4.11}$$

and so for  $i = 2, 3, \dots, n$ , we have

$$x_i^* = \bar{x}_i.\tag{4.12}$$

This completes the proof.  $\square$

*Remark 4.2.* (i) If  $E$  is a 2-uniformly smooth space, and there exist  $\rho_i > 0$  ( $i = 1, 2, \dots, n$ ) such that

$$0 < \rho_i < \min \left\{ \frac{2(k_i - v_i)}{c_2 \mu_i^2 - v_i^2}, \frac{1}{v_i} \right\}, \quad v_i < c_2 \mu_i.\tag{4.13}$$

Then (4.1) holds. We note that the Hilbert spaces and  $L_p$  (or  $l_q$ ) spaces ( $2 \leq q < \infty$ ) are 2-uniformly smooth.

(ii) Let  $n = 2$ , by Theorem 4.1, we recover [19, Theorem 3.1]. So Theorem 4.1 unifies, extends, and improves [19, Theorem 3.1, Corollaries 3.2 and 3.3], [16, Theorems 2.1–2.4] and the main results in [13].

## 5. Algorithms and Convergence

This section deals with an introduction of some  $n$ -step iterative sequences with or without mixed errors for problem (3.1) that can be applied to the convergence analysis of the iterative sequences generated by the algorithms.

*Algorithm 5.1.* For any given point  $x_0 \in E$ , define the generalized  $N$ -step iterative sequences  $\{x_{1,k}\}, \{x_{2,k}\}, \dots, \{x_{n,k}\}$  as follows:

$$\begin{aligned}
 x_{1,k+1} &= (1 - \alpha_k)x_{1,k} + \alpha_k R_{\rho_1}^{M_1} [x_{2,k} - \rho_1(T_1x_{2,k} + S_1x_{2,k})] + \alpha_k u_{1,k} + w_k, \\
 x_{2,k} &= R_{\rho_2}^{M_2} [x_{3,k} - \rho_2(T_2x_{3,k} + S_2x_{3,k})] + u_{2,k}, \\
 &\vdots \\
 x_{n-1,k} &= R_{\rho_{n-1}}^{M_{n-1}} [x_{n,k} - \rho_{n-1}(T_{n-1}x_{n,k} + S_{n-1}x_{n,k})] + u_{n-1,k}, \\
 x_{n,k} &= R_{\rho_n}^{M_n} [x_{1,k} - \rho_n(T_nx_{1,k} + S_nx_{1,k})] + u_{n,k},
 \end{aligned} \tag{5.1}$$

where  $x_{1,1} = x_0$ ,  $\{\alpha_k\}$  is a sequence in  $[0, 1]$ , and  $\{u_{i,k}\} \subset E$  ( $i = 1, 2, \dots, n$ ),  $\{w_k\} \subset E$  are the sequences satisfying the following conditions:

$$\sum_{k=1}^{\infty} \alpha_k = +\infty; \quad \sum_{k=1}^{\infty} \|w_k\| < +\infty; \quad \lim_{k \rightarrow \infty} \|u_{i,k}\| = 0, \quad i = 1, 2, \dots, n. \tag{5.2}$$

**Theorem 5.2.** Let  $T_i$ ,  $S_i$ , and  $M_i$  be the same as in Theorem 4.1, and suppose that the sequences  $\{x_{1,k}\}, \{x_{2,k}\}, \dots, \{x_{n,k}\}$  are generated by Algorithm 5.1. If the condition (4.1) holds, then  $(x_{1,k}, x_{2,k}, \dots, x_{n,k})$  converges strongly to the unique solution  $(x_1^*, x_2^*, \dots, x_n^*)$  of the problem (3.1).

*Proof.* By the Theorem 4.1, we know that problem (3.1) has a unique solution  $(x_1^*, x_2^*, \dots, x_n^*)$ , it follows from Lemma 3.1 that

$$\begin{aligned}
 x_1^* &= R_{\rho_1}^{M_1} [x_2^* - \rho_1(T_1x_2^* + S_1x_2^*)], \\
 x_2^* &= R_{\rho_2}^{M_2} [x_3^* - \rho_2(T_2x_3^* + S_2x_3^*)], \\
 &\vdots \\
 x_{n-1}^* &= R_{\rho_{n-1}}^{M_{n-1}} [x_n^* - \rho_{n-1}(T_{n-1}x_n^* + S_{n-1}x_n^*)], \\
 x_n^* &= R_{\rho_n}^{M_n} [x_1^* - \rho_n(T_nx_1^* + S_nx_1^*)].
 \end{aligned} \tag{5.3}$$

By (5.1) and (5.3), we have

$$\begin{aligned}
& \|x_{1,k+1} - x_1^*\| \\
&= \left\| (1 - \alpha_k)x_{1,k} + \alpha_k R_{\rho_1}^{M_1} [x_{2,k} - \rho_1(T_1x_{2,k} + S_1x_{2,k})] + \alpha_k u_{1,k} + w_k - x_1^* \right\| \\
&\leq (1 - \alpha_k) \|x_{1,k} - x_1^*\| + \alpha_k \left\| R_{\rho_1}^{M_1} [x_{2,k} - \rho_1(T_1x_{2,k} + S_1x_{2,k})] - R_{\rho_1}^{M_1} [x_2^* - \rho_1(T_1x_2^* + S_1x_2^*)] \right\| \\
&\quad + \alpha_k \|u_{1,k}\| + \|w_k\| \\
&\leq (1 - \alpha_k) \|x_{1,k} - x_1^*\| + \alpha_k \left\| (x_{2,k} - x_2^*) - \rho_1[(T_1x_{2,k} + S_1x_{2,k}) - (T_1x_2^* + S_1x_2^*)] \right\| \\
&\quad + \alpha_k \|u_{1,k}\| + \|w_k\|.
\end{aligned} \tag{5.4}$$

For  $i = 1, 2, \dots, n$ , since  $S_i$  is strongly monotone and Lipschitz continuous with constants  $k_i$  and  $\mu_i$ , respectively, and  $T_i$  is Lipschitz continuous with constant  $\nu_i$ , we get for  $i = 1, 2, \dots, n - 1$ ,

$$\begin{aligned}
& \left\| (x_{i+1,k} - x_{i+1}^*) - \rho_i[(T_i x_{i+1,k} + S_i x_{i+1,k}) - (T_i x_{i+1}^* + S_i x_{i+1}^*)] \right\| \\
&\leq \left\| (x_{i+1,k} - x_{i+1}^*) - \rho_i(S_i x_{i+1,k} - S_i x_{i+1}^*) \right\| + \rho_i \|T_i x_{i+1,k} - T_i x_{i+1}^*\| \\
&\leq \sqrt[q]{\|x_{i+1,k} - x_{i+1}^*\|^q - q\rho_i \langle S_i x_{i+1,k} - S_i x_{i+1}^*, J_q(x_{i+1,k} - x_{i+1}^*) \rangle} + c_q \rho_i^q \|S_i x_{i+1,k} - S_i x_{i+1}^*\|^q \\
&\quad + \rho_i \nu_i \|x_{i+1,k} - x_{i+1}^*\| \\
&\leq \xi_i \|x_{i+1,k} - x_{i+1}^*\|,
\end{aligned} \tag{5.5}$$

where  $\xi_i = \sqrt[q]{1 - q\rho_i k_i + c_q \rho_i^q \mu_i^q} + \rho_i \nu_i$ .

It follows from (5.4) and (5.5) that

$$\|x_{1,k+1} - x_1^*\| \leq (1 - \alpha_k) \|x_{1,k} - x_1^*\| + \alpha_k \xi_1 \|x_{2,k} - x_2^*\| + \alpha_k \|u_{1,k}\| + \|w_k\|. \tag{5.6}$$

By (5.1), (5.3), and (5.5), we have

$$\begin{aligned}
& \|x_{2,k} - x_2^*\| \\
&= \left\| R_{\rho_2}^{M_2} [x_{3,k} - \rho_2(T_2x_{3,k} + S_2x_{3,k})] - R_{\rho_2}^{M_2} [x_3^* - \rho_2(T_2x_3^* + S_2x_3^*)] + u_{2,k} \right\| \\
&\leq \left\| (x_{3,k} - x_3^*) - \rho_2[(T_2x_{3,k} + S_2x_{3,k}) - (T_2x_3^* + S_2x_3^*)] \right\| + \|u_{2,k}\| \\
&\leq \xi_2 \|x_{3,k} - x_3^*\| + \|u_{2,k}\|,
\end{aligned}$$

$$\begin{aligned}
& \|x_{3,k} - x_3^*\| \\
&= \left\| R_{\rho_3}^{M_3} [x_{4,k} - \rho_3(T_3x_{4,k} + S_3x_{4,k})] - R_{\rho_3}^{M_3} [x_4^* - \rho_3(T_3x_4^* + S_3x_4^*)] + u_{3,k} \right\| \\
&\leq \left\| (x_{4,k} - x_4^*) - \rho_3[(T_3x_{4,k} + S_3x_{4,k}) - (T_3x_4^* + S_3x_4^*)] \right\| + \|u_{3,k}\| \\
&\leq \xi_3 \|x_{4,k} - x_4^*\| + \|u_{3,k}\|, \dots, \\
& \|x_{n-1,k} - x_{n-1}^*\| \\
&= \left\| R_{\rho_{n-1}}^{M_{n-1}} [x_{n,k} - \rho_{n-1}(T_{n-1}x_{n,k} + S_{n-1}x_{n,k})] - R_{\rho_{n-1}}^{M_{n-1}} [x_n^* - \rho_{n-1}(T_{n-1}x_n^* + S_{n-1}x_n^*)] + u_{n-1,k} \right\| \\
&\leq \left\| (x_{n,k} - x_n^*) - \rho_{n-1}[(T_{n-1}x_{n,k} + S_{n-1}x_{n,k}) - (T_{n-1}x_n^* + S_{n-1}x_n^*)] \right\| + \|u_{n-1,k}\| \\
&\leq \xi_{n-1} \|x_{n,k} - x_n^*\| + \|u_{n-1,k}\|,
\end{aligned} \tag{5.7}$$

since  $S_n$  is strongly accretive and Lipschitz continuous with constants  $k_n$  and  $\mu_n$ , respectively, and  $T_n$  is Lipschitz continuous with constant  $\nu_n$ , we get

$$\begin{aligned}
& \|x_{n,k} - x_n^*\| \\
&= \left\| R_{\rho_n}^{M_n} [x_{1,k} - \rho_n(T_nx_{1,k} + S_nx_{1,k})] - R_{\rho_n}^{M_n} [x_1^* - \rho_n(T_nx_1^* + S_nx_1^*)] + u_{n,k} \right\| \\
&\leq \left\| (x_{1,k} - x_1^*) - \rho_n[(T_nx_{1,k} + S_nx_{1,k}) - (T_nx_1^* + S_nx_1^*)] \right\| + \|u_{n,k}\| \\
&\leq \sqrt[q]{\|x_{1,k} - x_1^*\|^q - q\rho_n \langle S_nx_{1,k} - S_nx_1^*, J_q(x_{1,k} - x_1^*) \rangle + c_q\rho_n^q \|S_nx_{1,k} - S_nx_1^*\|^q} \\
&\quad + \rho_n \|T_nx_{1,k} - T_nx_1^*\| \\
&\leq \xi_n \|x_{1,k} - x_1^*\| + \|u_{n,k}\|,
\end{aligned} \tag{5.8}$$

where  $\xi_n = \sqrt[q]{1 - q\rho_n k_n + c_q\rho_n^q \mu_n^q + \rho_n \nu_n}$ .

It follows from (5.6)–(5.8) that

$$\begin{aligned}
& \|x_{1,k+1} - x_1^*\| \\
&\leq (1 - \alpha_k) \|x_{1,k} - x_1^*\| + \alpha_k \xi_1 \|x_{2,k} - x_2^*\| + \alpha_k \|u_{1,k}\| + \|w_k\| \\
&\leq (1 - \alpha_k) \|x_{1,k} - x_1^*\| + \alpha_k \xi_1 [\xi_2 \|x_{3,k} - x_3^*\| + \|u_{2,k}\|] + \alpha_k \|u_{1,k}\| + \|w_k\| \\
&\leq (1 - \alpha_k) \|x_{1,k} - x_1^*\| + \alpha_k \xi_1 \xi_2 \|x_{3,k} - x_3^*\| + \alpha_k \xi_1 \|u_{2,k}\| + \alpha_k \|u_{1,k}\| + \|w_k\| \\
&\leq (1 - \alpha_k) \|x_{1,k} - x_1^*\| + \alpha_k \xi_1 \xi_2 \xi_3 \|x_{4,k} - x_4^*\| + \alpha_k \xi_1 \xi_2 \|u_{3,k}\| + \alpha_k \xi_1 \|u_{2,k}\| + \alpha_k \|u_{1,k}\| + \|w_k\| \\
&\leq \dots \leq (1 - \alpha_k) \|x_{1,k} - x_1^*\| + \alpha_k \xi_1 \xi_2 \dots \xi_{n-1} \|x_{n,k} - x_n^*\| + \alpha_k \xi_1 \xi_2 \dots \xi_{n-2} \|u_{n-1,k}\| \\
&\quad + \dots + \alpha_k \xi_1 \|u_{2,k}\| + \alpha_k \|u_{1,k}\| + \|w_k\|
\end{aligned}$$

$$\begin{aligned}
&\leq (1 - \alpha_k) \|x_{1,k} - x_1^*\| + \alpha_k \xi_1 \xi_2 \cdots \xi_n \|x_{1,k} - x_1^*\| + \alpha_k \xi_1 \xi_2 \cdots \xi_{n-1} \|u_{n,k}\| + \alpha_k \xi_1 \xi_2 \cdots \xi_{n-2} \|u_{n-1,k}\| \\
&\quad + \cdots + \alpha_k \xi_1 \|u_{2,k}\| + \alpha_k \|u_{1,k}\| + \|w_k\| \\
&= [1 - \alpha_k (1 - \xi_1 \xi_2 \cdots \xi_n)] \|x_{1,k} - x_1^*\| \\
&\quad + \alpha_k (1 - \xi_1 \xi_2 \cdots \xi_n) \frac{1}{1 - \xi_1 \xi_2 \cdots \xi_n} (\|u_{1,k}\| + \xi_1 \|u_{2,k}\| + \cdots + \xi_1 \xi_2 \cdots \xi_{n-1} \|u_{n,k}\|) + \|w_k\|.
\end{aligned} \tag{5.9}$$

Let

$$\begin{aligned}
a_k &= \|x_{1,k} - x_1^*\|, \quad t_k = \alpha_k (1 - \xi_1 \xi_2 \cdots \xi_n), \quad c_k = \|w_k\|, \\
b_k &= \frac{1}{1 - \xi_1 \xi_2 \cdots \xi_n} (\|u_{1,k}\| + \xi_1 \|u_{2,k}\| + \cdots + \xi_1 \xi_2 \cdots \xi_{n-1} \|u_{n,k}\|).
\end{aligned} \tag{5.10}$$

Then (5.9) can be written as follows:

$$a_{k+1} \leq (1 - t_k) a_k + b_k t_k + c_k. \tag{5.11}$$

From the assumption (5.2), we know that  $\{a_k\}, \{b_k\}, \{t_k\}, \{c_k\}$  satisfy the conditions of Lemma 2.8.

Thus  $a_k \rightarrow 0$  ( $k \rightarrow \infty$ ), that is,  $\|x_{1,k} - x_1^*\| \rightarrow 0$  ( $k \rightarrow \infty$ ). It follows from (5.6)–(5.8) that  $\|x_{n,k} - x_n^*\| \rightarrow 0$  ( $k \rightarrow \infty$ ),  $\|x_{n-1,k} - x_{n-1}^*\| \rightarrow 0$  ( $k \rightarrow \infty$ ),  $\dots$ ,  $\|x_{2,k} - x_2^*\| \rightarrow 0$  ( $k \rightarrow \infty$ ). So  $x_{i,k} \rightarrow x_i^*$  ( $k \rightarrow \infty$ ) for  $i = 1, 2, \dots, n$ . That is,  $(x_{1,k}, x_{2,k}, \dots, x_{n,k})$  converges strongly to the unique solution  $(x_1^*, x_2^*, \dots, x_n^*)$  of (3.1).

For  $i = 1, 2, \dots, n$ , let  $u_{i,k} = 0$  and  $w_k = 0$ , by Algorithm 5.1 and Theorem 5.2, it is easy to obtain the following Algorithm 5.3 and Theorem 5.4.  $\square$

*Algorithm 5.3.* For any given point  $x_0 \in E$ , define the generalized  $N$ -step iterative sequences  $\{x_{1,k}\}, \{x_{2,k}\}, \dots, \{x_{n,k}\}$  as follows:

$$\begin{aligned}
x_{1,k+1} &= (1 - \alpha_k) x_{1,k} + \alpha_k R_{\rho_1}^{M_1} [x_{2,k} - \rho_1 (T_1 x_{2,k} + S_1 x_{2,k})], \\
x_{2,k} &= R_{\rho_2}^{M_2} [x_{3,k} - \rho_2 (T_2 x_{3,k} + S_2 x_{3,k})], \\
&\quad \vdots \\
x_{n-1,k} &= R_{\rho_{n-1}}^{M_{n-1}} [x_{n,k} - \rho_{n-1} (T_{n-1} x_{n,k} + S_{n-1} x_{n,k})], \\
x_{n,k} &= R_{\rho_n}^{M_n} [x_{1,k} - \rho_n (T_n x_{1,k} + S_n x_{1,k})],
\end{aligned} \tag{5.12}$$

where  $x_{1,1} = x_0$ ,  $\{\alpha_k\}$  is a sequence in  $[0, 1]$ , satisfying  $\sum_{k=1}^{\infty} \alpha_k = +\infty$ .

**Theorem 5.4.** Let  $T_i, S_i$ , and  $M_i$  be the same as in Theorem 4.1, and suppose that the sequences  $\{x_{1,k}\}, \{x_{2,k}\}, \dots, \{x_{n,k}\}$  are generated by Algorithm 5.3. If (4.1) holds, then  $(x_{1,k}, x_{2,k}, \dots, x_{n,k})$  converges strongly to the unique solution  $(x_1^*, x_2^*, \dots, x_n^*)$  of (3.1).

*Remark 5.5.* Theorem 5.4 unifies and generalizes [19, Theorems 4.3 and 4.4] and the main results in [11, 12]. So Theorem 5.2 unifies, extends, and improves the corresponding results in [11–14, 16, 19].

## Acknowledgments

This research was supported by the National Natural Science Foundation of China (Grants 10771228 and 10831009) and the Research Project of Chongqing Normal University (Grant 08XLZ05).

## References

- [1] J.-S. Pang, "Asymmetric variational inequality problems over product sets: applications and iterative methods," *Mathematical Programming*, vol. 31, no. 2, pp. 206–219, 1985.
- [2] G. Cohen and F. Chaplais, "Nested monotony for variational inequalities over product of spaces and convergence of iterative algorithms," *Journal of Optimization Theory and Applications*, vol. 59, no. 3, pp. 369–390, 1988.
- [3] M. Bianchi, "Pseudo P-monotone operators and variational inequalities," Tech. Rep. 6, Istituto di econometria e Matematica per le Decisioni Economiche, Universita Cattolica del Sacro Cuore, Milan, Italy, 1993.
- [4] Q. H. Ansari and J.-C. Yao, "A fixed point theorem and its applications to a system of variational inequalities," *Bulletin of the Australian Mathematical Society*, vol. 59, no. 3, pp. 433–442, 1999.
- [5] Q. H. Ansari, S. Schaible, and J. C. Yao, "System of vector equilibrium problems and its applications," *Journal of Optimization Theory and Applications*, vol. 107, no. 3, pp. 547–557, 2000.
- [6] E. Allevi, A. Gnudi, and I. V. Konnov, "Generalized vector variational inequalities over product sets," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 47, no. 1, pp. 573–582, 2001.
- [7] G. Kassay and J. Kolumbán, "System of multi-valued variational variational inequalities," *Publicaciones Mathematicae Debrecen*, vol. 54, pp. 267–279, 1999.
- [8] G. Kassay, J. Kolumbán, and Z. Páles, "Factorization of Minty and Stampacchia variational inequality systems," *European Journal of Operational Research*, vol. 143, no. 2, pp. 377–389, 2002.
- [9] J. W. Peng, "System of generalised set-valued quasi-variational-like inequalities," *Bulletin of the Australian Mathematical Society*, vol. 68, no. 3, pp. 501–515, 2003.
- [10] J. W. Peng and X. M. Yang, "On existence of a solution for the system of generalized vector quasi-equilibrium problems with upper semicontinuous set-valued maps," *International Journal of Mathematics and Mathematical Sciences*, no. 15, pp. 2409–2420, 2005.
- [11] R. U. Verma, "Projection methods, algorithms, and a new system of nonlinear variational inequalities," *Computers & Mathematics with Applications*, vol. 41, no. 7-8, pp. 1025–1031, 2001.
- [12] R. U. Verma, "Iterative algorithms and a new system of nonlinear quasivariational inequalities," *Advances in Nonlinear Variational Inequalities*, vol. 4, no. 1, pp. 117–124, 2001.
- [13] R. U. Verma, "General convergence analysis for two-step projection methods and applications to variational problems," *Applied Mathematics Letters*, vol. 18, no. 11, pp. 1286–1292, 2005.
- [14] R. U. Verma, "On a new system of nonlinear variational inequalities and associated iterative algorithms," *Mathematical Sciences Research Hot-Line*, vol. 3, no. 8, pp. 65–68, 1999.
- [15] R. U. Verma, "Generalized system for relaxed cocoercive variational inequalities and projection methods," *Journal of Optimization Theory and Applications*, vol. 121, no. 1, pp. 203–210, 2004.
- [16] J. K. Kim and D. S. Kim, "A new system of generalized nonlinear mixed variational inequalities in Hilbert spaces," *Journal of Convex Analysis*, vol. 11, no. 1, pp. 235–243, 2004.
- [17] Y. J. Cho, Y. P. Fang, N. J. Huang, and H. J. Hwang, "Algorithms for systems of nonlinear variational inequalities," *Journal of the Korean Mathematical Society*, vol. 41, no. 3, pp. 489–499, 2004.
- [18] R. P. Agarwal, N.-J. Huang, and M.-Y. Tan, "Sensitivity analysis for a new system of generalized nonlinear mixed quasi-variational inclusions," *Applied Mathematics Letters*, vol. 17, no. 3, pp. 345–352, 2004.

- [19] J. W. Peng and D. L. Zhu, "Existence of solutions and convergence of iterative algorithms for a system of generalized nonlinear mixed quasi-variational inclusions," *Computers & Mathematics with Applications*, vol. 53, no. 5, pp. 693–705, 2007.
- [20] K. R. Kazmi and M. I. Bhat, "Iterative algorithm for a system of nonlinear variational-like inclusions," *Computers & Mathematics with Applications*, vol. 48, no. 12, pp. 1929–1935, 2004.
- [21] Y. P. Fang and N. J. Huang, " $H$ -monotone operators and system of variational inclusions," *Communications on Applied Nonlinear Analysis*, vol. 11, no. 1, pp. 93–101, 2004.
- [22] R. U. Verma, "A-monotonicity and applications to nonlinear variational inclusion problems," *Journal of Applied Mathematics and Stochastic Analysis*, no. 2, pp. 193–195, 2004.
- [23] Y.-P. Fang, N.-J. Huang, and H. B. Thompson, "A new system of variational inclusions with  $(H, \eta)$ -monotone operators in Hilbert spaces," *Computers & Mathematics with Applications*, vol. 49, no. 2-3, pp. 365–374, 2005.
- [24] W.-Y. Yan, Y.-P. Fang, and N.-J. Huang, "A new system of set-valued variational inclusions with  $H$ -monotone operators," *Mathematical Inequalities & Applications*, vol. 8, no. 3, pp. 537–546, 2005.
- [25] J. W. Peng and D. L. Zhu, "A new system of generalized mixed quasi-variational inclusions with  $(H, \eta)$ -monotone operators," *Journal of Mathematical Analysis and Applications*, vol. 327, no. 1, pp. 175–187, 2007.
- [26] H. K. Xu, "Inequalities in Banach spaces with applications," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 16, no. 12, pp. 1127–1138, 1991.
- [27] Z. B. Xu and G. F. Roach, "Characteristic inequalities of uniformly convex and uniformly smooth Banach spaces," *Journal of Mathematical Analysis and Applications*, vol. 157, no. 1, pp. 189–210, 1991.
- [28] K. Deimling, *Nonlinear Functional Analysis*, Springer, Berlin, Germany, 1985.
- [29] V. Barbu, *Nonlinear Semigroups and Differential Equations in Banach Spaces*, Noordhoff, Leyden, The Netherlands, 1979.
- [30] L. S. Liu, "Ishikawa and Mann iterative process with errors for nonlinear strongly accretive mappings in Banach spaces," *Journal of Mathematical Analysis and Applications*, vol. 194, no. 1, pp. 114–125, 1995.