Hindawi Publishing Corporation
Fixed Point Theory and Applications
Volume 2009, Article ID 402602, 16 pages
doi:10.1155/2009/402602

Research Article

Relaxed Composite Implicit Iteration Process for Common Fixed Points of a Finite Family of Strictly Pseudocontractive Mappings

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Received 26 November 2008; Accepted 28 May 2009

Recommended by Lai-Jiu Lin

We propose a relaxed composite implicit iteration process for finding approximate common fixed points of a finite family of strictly pseudocontractive mappings in Banach spaces. Several convergence results for this process are established.

1. Introduction and Preliminaries

Let E be a real Banach space, and let E^* be its dual space. Denote by J the normalized duality mapping from E into 2^{E^*} defined by

$$J(x) = \left\{\varphi \in E^* : \langle x, \varphi\rangle = \|x\|^2 = \|\varphi\|^2\right\}, \quad \forall x \in E, \tag{1.1}$$

where $\langle \cdot, \cdot\rangle$ is the generalized duality pairing between E and E^*. If E is smooth, then J is single valued and continuous from the norm topology of E to the weak* topology of E^*.

A mapping T with domain $D(T)$ and range $R(T)$ in E is called λ-strictly pseudocontractive in the terminology of Browder and Petryshyn [1], if there exists a constant $\lambda > 0$ such that

$$\langle Tx - Ty, j(x-y)\rangle \le \|x-y\|^2 - \lambda\|x - y - (Tx - Ty)\|^2 \tag{1.2}$$

for all $x, y \in D(T)$ and all $j(x-y) \in J(x-y)$. Without loss of generality, we may assume $\lambda \in (0,1)$. If I denotes the identity operator, then (1.2) can be written in the form

$$\langle (I-T)x-(I-T)y, j(x-y)\rangle \geq \lambda\|(I-T)x-(I-T)y\|^2 \tag{1.3}$$

for all $x, y \in D(T)$ and all $j(x-y) \in J(x-y)$. In (1.2) and (1.3), the positive number $\lambda > 0$ is said to be a strictly pseudocontractive constant.

The class of strictly pseudocontractive mappings has been studied by several authors (see, e.g., [1–10]). It is shown in [4] that a strictly pseudocontractive map is L-Lipschitzian (i.e., $\|Tx-Ty\| \leq L\|x-y\|$, $\forall x, y \in D(T)$ for some $L > 0$). Indeed, it follows immediately from (1.3) that

$$\|x-y\| \geq \lambda\|(I-T)x-(I-T)y\| \geq \lambda\big(\|Tx-Ty\|-\|x-y\|\big), \tag{1.4}$$

and hence $\|Tx-Ty\| \leq L\|x-y\|$, $\forall x, y \in D(T)$ where $L = 1+1/\lambda$. It is clear that in Hilbert spaces the important class of nonexpansive mappings (mappings T for which $\|Tx-Ty\| \leq \|x-y\|$, $\forall x, y \in D(T)$) is a subclass of the class of strictly pseudocontractive maps.

Let K be a nonempty convex subset of E, and let $\{T_i\}_{i=1}^N$ be a finite family of nonexpansive self-maps of K. In [11], Xu and Ori introduced the following implicit iteration process; for any initial $x_0 \in K$ and $\{\alpha_n\}_{n=1}^\infty \subset (0,1)$, the sequence $\{x_n\}_{n=1}^\infty$ is generated as follows:

$$\begin{aligned}
x_1 &= \alpha_1 x_0 + (1-\alpha_1)T_1x_1,\\
x_2 &= \alpha_2 x_1 + (1-\alpha_2)T_2x_2,\\
&\ \ \vdots\\
x_N &= \alpha_N x_{N-1} + (1-\alpha_N)T_Nx_N,\\
x_{N+1} &= \alpha_{N+1} x_N + (1-\alpha_{N+1})T_1x_{N+1},\\
&\ \ \vdots
\end{aligned} \tag{1.5}$$

The scheme is expressed in a compact form as

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n)T_nx_n, \quad n \geq 1, \tag{1.6}$$

where $T_n = T_{n \bmod N}$. Moreover, they proved the following convergence theorem in a Hilbert space.

Theorem 1.1 (see [11]). *Let H be a Hilbert space, and let K be a nonempty closed convex subset of H. Let $\{T_i\}_{i=1}^N$ be N nonexpansive self-maps of K such that $C = \bigcap_{i=1}^N F(T_i) \neq \varnothing$ where $F(T_i) = \{x \in K : T_ix = x\}$. Let $x_0 \in K$, and let $\{\alpha_n\}_{n=1}^\infty$ be a sequence in $(0,1)$, such that $\lim_{n\to\infty}\alpha_n = 0$. Then the sequence $\{x_n\}$ defined implicitly by (1.6) converges weakly to a common fixed point of the mappings $\{T_i\}_{i=1}^N$.*

Subsequently, Osilike [12] extended their results from nonexpansive mappings to strictly pseudocontractive mappings and derived the following convergence theorems in Hilbert and Banach spaces.

Theorem 1.2 (see [12]). *Let H be a real Hilbert space, and let K be a nonempty closed convex subset of H. Let $\{T_i\}_{i=1}^N$ be N strictly pseudocontractive self-maps of K such that $C = \bigcap_{i=1}^N F(T_i) \neq \emptyset$, where $F(T_i) = \{x \in K : T_i x = x\}$. Let $x_0 \in K$, and let $\{\alpha_n\}_{n=1}^\infty$ be a sequence in $(0,1)$ such that $\lim_{n\to\infty}\alpha_n = 0$. Then the sequence $\{x_n\}_{n=1}^\infty$ defined by* (1.6) *converges weakly to a common fixed point of the mappings $\{T_i\}_{i=1}^N$.*

Theorem 1.3 (see [12]). *Let E be a real Banach space, and let K be a nonempty closed convex subset of E. Let $\{T_i\}_{i=1}^N$ be N strictly pseudocontractive self-maps of K such that $C = \bigcap_{i=1}^N F(T_i) \neq \emptyset$, where $F(T_i) = \{x \in K : T_i x = x\}$, and let $\{\alpha_n\}_{n=1}^\infty$ be a real sequence satisfying the conditions:*

(i) $0 < \alpha_n < 1$;

(ii) $\sum_{n=1}^\infty (1-\alpha_n) = +\infty$;

(iii) $\sum_{n=1}^\infty (1-\alpha_n)^2 < +\infty$.

Let $x_0 \in K$, and let $\{x_n\}_{n=1}^\infty$ be defined by (1.6). *Then*

(i) $\lim_{n\to\infty}\|x_n - p\|$ *exists for all* $p \in F$;

(ii) $\liminf_{n\to\infty}\|x_n - T_n x_n\| = 0$.

Let K be a nonempty closed convex subset of a real Banach space E. Very recently, Su and Li [13] introduced a new implicit iteration process for N strictly pseudocontractive self-maps $\{T_i\}_{i=1}^N$ of K:

$$\begin{gathered} x_n = \alpha_n x_{n-1} + (1-\alpha_n) T_n y_n, \\ y_n = \beta_n x_{n-1} + (1-\beta_n) T_n x_n, \quad n = 1,2,\ldots, \end{gathered} \tag{1.7}$$

that is,

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n) T_n \left(\beta_n x_{n-1} + (1-\beta_n) T_n x_n\right), \quad n \geq 1, \tag{1.8}$$

where $T_n = T_{n \bmod N}$ and $\{\alpha_n\}, \{\beta_n\} \subset [0,1]$. First, they established the following convergence theorem.

Theorem 1.4 ([13, Theorem 2.1]). *Let E be a real Banach space, and let K be a nonempty closed convex subset of E. Let $\{T_i\}_{i=1}^N$ be N strictly pseudocontractive self-maps of K such that $C = \bigcap_{i=1}^N F(T_i) \neq \emptyset$, where $F(T_i) = \{x \in K : T_i x = x\}$, and let $\{\alpha_n\}_{n=1}^\infty, \{\beta_n\}_{n=1}^\infty \subset [0,1]$ be two real sequences satisfying the conditions:*

(i) $\sum_{n=1}^\infty (1-\alpha_n) = +\infty$;

(ii) $\sum_{n=1}^\infty (1-\alpha_n)^2 < +\infty$;

(iii) $\sum_{n=1}^\infty (1-\beta_n) < +\infty$;

(iv) $(1-\alpha_n)(1-\beta_n)L^2 < 1$, $\forall n \geq 1$, *where $L \geq 1$ is common Lipschitz constant of $\{T_i\}_{i=1}^N$.*

For $x_0 \in K$, *let* $\{x_n\}_{n=1}^{\infty}$ *be defined by* (1.8). *Then*

(i) $\lim_{n\to\infty}\|x_n - p\|$ *exists for all* $p \in C$;

(ii) $\liminf_{n\to\infty}\|x_n - T_n x_n\| = 0$.

Second, they derived the following result by using Theorem 1.4.

Theorem 1.5 ([13, Theorem 2.2]). *Let* K *be a nonempty closed convex subset of a real Banach space* E, *let* T *be a semicompact strictly pseudocontractive self-map of* K *such that* $F(T) \neq \emptyset$, *where* $F(T) = \{x \in K : Tx = x\}$, *and let* $\{\alpha_n\} \subset [0,1]$ *be a real sequence satisfying the conditions:*

(i) $\sum_{n=1}^{\infty}(1-\alpha_n) = +\infty$;

(ii) $\sum_{n=1}^{\infty}(1-\alpha_n)^2 < +\infty$.

Then for $x_0 \in K$, *the sequence* $\{x_n\}$ *defined by Mann iterative process,*

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n) T x_{n-1}, \quad n \geq 1, \tag{1.9}$$

converges strongly to a fixed point of T.

On the other hand, Zeng and Yao [14] introduced a new implicit iteration scheme with perturbed mapping for approximation of common fixed points of a finite family of nonexpansive self-maps of a real Hilbert space H and established some convergence theorems for this implicit iteration scheme. To be more specific, let $\{T_i\}_{i=1}^{N}$ be a finite family of nonexpansive self-maps of H, and let $F : H \to H$ be a mapping such that for some constants $\kappa, \eta > 0$; F is a κ-Lipschitz and η-strongly monotone mapping. Let $\{\alpha_n\}_{n=1}^{\infty} \subset (0,1)$ and $\{\lambda_n\}_{n=1}^{\infty} \subset [0,1)$ and take a fixed number $\mu \in (0, 2\eta/\kappa^2)$. The authors proposed the following implicit iteration process with perturbed mapping F.

For an arbitrary initial point $x_0 \in H$, the sequence $\{x_n\}_{n=1}^{\infty}$ is generated as follows:

$$\begin{aligned}
x_1 &= \alpha_1 x_0 + (1-\alpha_1)\left[T_1 x_1 - \lambda_1 \mu F(T_1 x_1)\right],\\
x_2 &= \alpha_2 x_1 + (1-\alpha_2)\left[T_2 x_2 - \lambda_2 \mu F(T_2 x_2)\right],\\
&\ \vdots\\
x_N &= \alpha_N x_{N-1} + (1-\alpha_N)\left[T_N x_N - \lambda_N \mu F(T_N x_N)\right],\\
x_{N+1} &= \alpha_{N+1} x_N + (1-\alpha_{N+1})\left[T_1 x_{N+1} - \lambda_{N+1} \mu F(T_1 x_{N+1})\right],\\
&\ \vdots
\end{aligned} \tag{1.10}$$

The scheme is expressed in a compact form as

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n)\left[T_n x_n - \lambda_n \mu F(T_n x_n)\right], \quad n \geq 1. \tag{1.11}$$

It is clear that if $\lambda_n \equiv 0$, then the implicit iteration scheme (1.11) with perturbed mapping reduces to the implicit iteration process (1.6).

Theorem 1.6 ([14, Theorem 2.1]). *Let H be a real Hilbert space, and let $F : H \to H$ be a mapping such that for some constants $\kappa, \eta > 0$; F is κ-Lipschitz and η-strongly monotone. Let $\{T_i\}_{i=1}^N$ be N nonexpansive self-maps of H such that $C = \bigcap_{i=1}^N F(T_i) \neq \varnothing$. Let $\mu \in (0, 2\eta/\kappa^2)$, let $x_0 \in H$, $\{\lambda_n\}_{n=1}^\infty \subset [0,1)$, and let $\{\alpha_n\}_{n=1}^\infty \subset (0,1)$ satisfying the conditions: $\sum_{n=1}^\infty \lambda_n < \infty$ and $\alpha \leq \alpha_n \leq \beta$, $n \geq 1$, for some $\alpha, \beta \in (0,1)$. Then the sequence $\{x_n\}_{n=1}^\infty$ defined by (1.11) converges weakly to a common fixed point of the mappings $\{T_i\}_{i=1}^N$.*

The above Theorem 1.6 extends Theorem 1.1 from the implicit iteration process (1.6) to the implicit iteration scheme (1.11) with perturbed mapping.

Let E be a real Banach space, and let K be a nonempty convex subset of E. Recall that a mapping $F : K \to K$ is said to be δ-strongly accretive if there exists a constant $\delta \in (0,1)$ such that

$$\langle Fx - Fy, j(x-y) \rangle \geq \delta \|x-y\|^2 \tag{1.12}$$

for all $x, y \in K$ and all $j(x-y) \in J(x-y)$.

Proposition 1.7. *Let X be a real Banach space, and let $F : K \to K$ be a mapping:*

(i) *if F is λ-strictly pseudocontractive, then F is a Lipschitz mapping with constant $L = 1 + 1/\lambda$.*

(ii) *if F is both λ-strictly pseudocontractive and δ-strongly accretive with $\lambda + \delta \geq 1$, then $I - F$ is nonexpansive.*

Proof. It is easy to see that statement (i) immediately follows from the definition of strict pseudocontraction. Now utilizing the definitions of strict pseudocontraction and strong accretivity, we obtain

$$\lambda \|(I-F)x - (I-F)y\|^2 \leq \|x-y\|^2 - \langle Fx - Fy, j(x-y) \rangle \leq (1-\delta)\|x-y\|^2. \tag{1.13}$$

Since $\lambda + \delta \geq 1$,

$$\|(I-F)x - (I-F)y\| \leq \sqrt{\frac{1-\delta}{\lambda}} \|x-y\| \leq \|x-y\|, \tag{1.14}$$

and hence $I - F$ is nonexpansive. □

Let E be a real Banach space, and let K be a nonempty convex subset of E such that $K - K \subset K$. Let $\{T_i\}_{i=1}^N$ be N strictly pseudocontractive self-maps of K, and let $F : K \to K$ be a perturbed mapping which is both δ-strongly accretive and λ-strictly pseudocontractive with $\delta + \lambda \geq 1$. In this paper we introduce a general implicit iteration process as follows:

$$\begin{aligned} x_n &= \alpha_n x_{n-1} + (1-\alpha_n)\left[T_n y_n - \lambda_n F(T_n y_n)\right], \\ y_n &= \beta_n x_{n-1} + (1-\beta_n) T_n x_n, \quad n = 1, 2, \ldots, \end{aligned} \tag{1.15}$$

where $T_n = T_{n \bmod N}$, and $\{\alpha_n\}, \{\beta_n\}, \{\lambda_n\} \subset [0,1]$. In particular, whenever $\lambda_n \equiv 0$, it is easy to see that (1.15) reduces to (1.8).

Let $L \geq 1$ denote common Lipschitz constant of N strictly pseudocontractive self-maps $\{T_i\}_{i=1}^{N}$ of K. Since K is a nonempty convex subset of E such that $K - K \subset K$, for each $n \geq 1$, the operator

$$\begin{aligned} S_n x &= \alpha_n x_{n-1} + (1-\alpha_n)\{T_n[\beta_n x_{n-1} + (1-\beta_n)T_n x] - \lambda_n F T_n[\beta_n x_{n-1} + (1-\beta_n)T_n x]\} \\ &= \alpha_n x_{n-1} + (1-\alpha_n)(I - \lambda_n F)T_n[\beta_n x_{n-1} + (1-\beta_n)T_n x] \\ &= \alpha_n x_{n-1} + (1-\alpha_n)[(1-\lambda_n)I + \lambda_n(I-F)]T_n[\beta_n x_{n-1} + (1-\beta_n)T_n x] \end{aligned} \tag{1.16}$$

maps K into itself.

Utilizing Proposition 1.7, we have

$$\begin{aligned} &\langle S_n x - S_n y, j(x-y)\rangle \\ &\quad = (1-\alpha_n)\langle[(1-\lambda_n)I + \lambda_n(I-F)]T_n[\beta_n x_{n-1} + (1-\beta_n)T_n x] \\ &\qquad\qquad - [(1-\lambda_n)I + \lambda_n(I-F)]T_n[\beta_n x_{n-1} + (1-\beta_n)T_n y], j(x-y)\rangle \\ &\quad \leq (1-\alpha_n)\|[(1-\lambda_n)I + \lambda_n(I-F)]T_n[\beta_n x_{n-1} + (1-\beta_n)T_n x] \\ &\qquad\qquad - [(1-\lambda_n)I + \lambda_n(I-F)]T_n[\beta_n x_{n-1} + (1-\beta_n)T_n y]\|\|x-y\| \\ &\quad \leq (1-\alpha_n)\{(1-\lambda_n)\|T_n[\beta_n x_{n-1} + (1-\beta_n)T_n x] - T_n[\beta_n x_{n-1} + (1-\beta_n)T_n y]\| \\ &\qquad\qquad + \lambda_n\|(I-F)T_n[\beta_n x_{n-1} + (1-\beta_n)T_n x] - (I-F)T_n[\beta_n x_{n-1} + (1-\beta_n)T_n y]\|\} \\ &\qquad \times \|x-y\| \\ &\quad \leq (1-\alpha_n)\|T_n[\beta_n x_{n-1} + (1-\beta_n)T_n x] - T_n[\beta_n x_{n-1} + (1-\beta_n)T_n y]\|\|x-y\| \\ &\quad \leq (1-\alpha_n)L\|\beta_n x_{n-1} + (1-\beta_n)T_n x - [\beta_n x_{n-1} + (1-\beta_n)T_n y]\|\|x-y\| \\ &\quad = (1-\alpha_n)(1-\beta_n)L\|T_n x - T_n y\|\|x-y\| \\ &\quad \leq (1-\alpha_n)(1-\beta_n)L^2\|x-y\|^2 \end{aligned} \tag{1.17}$$

for all $x, y \in K$. Thus, S_n is strongly pseudocontractive, if $(1-\alpha_n)(1-\beta_n)L^2 < 1$ for each $n \geq 1$. Since S_n is also Lipschitz mapping, it follows from [12, 15, 16] that S_n has a unique fixed point $x_n \in K$, that is, for each $n \geq 1$

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n)[(1-\lambda_n)I + \lambda_n(I-F)]T_n[\beta_n x_{n-1} + (1-\beta_n)T_n x_n]. \tag{1.18}$$

Therefore, if $(1-\alpha_n)(1-\beta_n)L^2 < 1$, $\forall n \geq 1$, then the composite implicit iteration process (1.15) with perturbed mapping can be employed for the approximation of common fixed points of N strictly pseudocontractive self-maps of K.

The purpose of this paper is to investigate the problem of approximating common fixed points of strictly pseudocontractive mappings of Browder-Petryshyn in an arbitrary real Banach space by this general implicit iteration process (1.15). To this end, we need the following lemma and definition.

Lemma 1.8 (see [8]). *Let $\{a_n\}_{n=1}^{\infty}$, $\{b_n\}_{n=1}^{\infty}$, and $\{\epsilon_n\}_{n=1}^{\infty}$ be sequences of nonnegative real numbers satisfying the inequality*

$$a_{n+1} \leq (1+\epsilon_n)a_n + b_n, \quad n \geq 1. \tag{1.19}$$

If

$$\sum_{n=1}^{\infty} \epsilon_n < +\infty, \qquad \sum_{n=1}^{\infty} b_n < +\infty, \tag{1.20}$$

then $\lim_{n\to\infty} a_n$ *exists.*

The following definition can be found, for example, in [13].

Definition 1.9. Let D be a closed subset of a real Banach space E, and let $T : D \to D$ be a mapping. T is said to be semicompact if, for any bounded sequence $\{x_n\}$ in D such that $\|x_n - Tx_n\| \to 0\,(n \to \infty)$, there must exist a subsequence $\{x_{n_i}\} \subset \{x_n\}$ such that $x_{n_i} \to x^* \in D$.

2. Main Results

We are now in a position to prove our main results in this paper.

Theorem 2.1. *Let E be a real Banach space, and let K be a nonempty closed convex subset of E such that $K - K \subset K$. Let $F : K \to K$ be a perturbed mapping which is both δ-strongly accretive and λ-strictly pseudocontractive with $\delta + \lambda \geq 1$. Let $\{T_i\}_{i=1}^{N}$ be N strictly pseudocontractive self-maps of K such that $C = \bigcap_{i=1}^{N} F(T_i) \neq \emptyset$, where $F(T_i) = \{x \in K : T_i x = x\}$, and let $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$, and $\{\lambda_n\}_{n=1}^{\infty}$ be three real sequences in $[0,1]$ satisfying the conditions:*

(i) $\sum_{n=1}^{\infty}(1-\alpha_n) = +\infty$;

(ii) $\sum_{n=1}^{\infty}(1-\alpha_n)^2 < +\infty$;

(iii) $\sum_{n=1}^{\infty}(1-\beta_n) < +\infty$;

(iv) $\sum_{n=1}^{\infty}\lambda_n(1-\alpha_n) < +\infty$;

(v) $(1-\alpha_n)(1-\beta_n)L^2 < 1$, $\forall n \geq 1$, *where $L \geq 1$ is the common Lipschitz constant of $\{T_i\}_{i=1}^{N}$.*

For $x_0 \in K$, let $\{x_n\}_{n=1}^{\infty}$ be defined by

$$\begin{aligned} x_n &= \alpha_n x_{n-1} + (1-\alpha_n)\left[T_n y_n - \lambda_n F(T_n y_n)\right], \\ y_n &= \beta_n x_{n-1} + (1-\beta_n)T_n x_n, \quad n = 1,2,\ldots, \end{aligned} \tag{2.1}$$

where $T_n = T_{n \bmod N}$, then

(i) $\lim_{n\to\infty}\|x_n - p\|$ *exists for all $p \in C$;*

(ii) $\liminf_{n\to\infty}\|x_n - T_n x_n\| = 0$.

Proof. First, since each strictly pseudocontractive mapping is a Lipschitz mapping, there exists a constant $L \geq 1$ such that

$$\|T_i x - T_i y\| \leq L\|x - y\|, \quad \forall x, y \in K, \forall i = 1,2,\ldots,N. \tag{2.2}$$

It is now well known (see, e.g., [15]) that

$$\|x+y\|^2 \leq \|x\|^2 + 2\langle y, j(x+y)\rangle \tag{2.3}$$

for all $x, y \in E$ and all $j(x+y) \in J(x+y)$. Take $p \in C$ arbitrarily. Then it follows from (2.1) that

$$\begin{aligned}
x_n - p &= \alpha_n x_{n-1} + (1-\alpha_n)\left[T_n y_n - \lambda_n F(T_n y_n)\right] - p \\
&= \alpha_n(x_{n-1}-p) + (1-\alpha_n)\left\{\left[T_n y_n - \lambda_n F(T_n y_n)\right] - p\right\} \\
&= \alpha_n(x_{n-1}-p) + (1-\alpha_n)\left\{\left[(1-\lambda_n)I + \lambda_n(I-F)\right]T_n y_n - p\right\} \\
&= \alpha_n(x_{n-1}-p) + (1-\alpha_n)\left\{(1-\lambda_n)(T_n y_n - p) + \lambda_n\left[(I-F)T_n y_n - p\right]\right\} \\
&= \alpha_n(x_{n-1}-p) + (1-\alpha_n)\left\{(1-\lambda_n)(T_n y_n - T_n p) + \lambda_n\left[(I-F)T_n y_n - (I-F)T_n p\right]\right. \\
&\qquad\qquad \left. + \lambda_n\left[(I-F)T_n p - p\right]\right\} \\
&= \alpha_n(x_{n-1}-p) + (1-\alpha_n)\left\{(1-\lambda_n)(T_n y_n - T_n p) + \lambda_n\left[(I-F)T_n y_n - (I-F)T_n p\right]\right\} \\
&\quad - (1-\alpha_n)\lambda_n F(p).
\end{aligned} \tag{2.4}$$

Utilizing (2.3), we obtain

$$\begin{aligned}
\|x_n - p\|^2 &\leq \alpha_n^2\|x_{n-1}-p\|^2 + 2(1-\alpha_n)(1-\lambda_n)\langle T_n y_n - T_n p, j(x_n - p)\rangle \\
&\quad + 2(1-\alpha_n)\lambda_n\langle (I-F)T_n y_n - (I-F)T_n p, j(x_n-p)\rangle \\
&\quad - 2(1-\alpha_n)\lambda_n\langle F(p), j(x_n-p)\rangle \\
&\leq \alpha_n^2\|x_{n-1}-p\|^2 + 2(1-\alpha_n)(1-\lambda_n) \\
&\quad \times \left[\langle T_n y_n - T_n x_n, j(x_n-p)\rangle + \langle T_n x_n - T_n p, j(x_n-p)\rangle\right] \\
&\quad + 2(1-\alpha_n)\lambda_n\left[\langle (I-F)T_n y_n - (I-F)T_n x_n, j(x_n-p)\rangle\right. \\
&\qquad\qquad \left. + \langle (I-F)T_n x_n - (I-F)T_n p, j(x_n-p)\rangle\right] \\
&\quad - 2(1-\alpha_n)\lambda_n\langle F(p), j(x_n-p)\rangle.
\end{aligned} \tag{2.5}$$

Since each T_i, $i = 1, 2, \ldots, N$, is strictly pseudocontractive, there exists $\lambda \in (0,1)$ such that

$$\langle T_i x - T_i y, j(x-y)\rangle \leq \|x-y\|^2 - \lambda\|x - T_i x - (y - T_i y)\|^2, \quad \forall x, y \in K. \tag{2.6}$$

Thus, utilizing Proposition 1.7(ii) we know from (2.5) that

$$
\begin{aligned}
\|x_n - p\|^2 \le{}& \alpha_n^2 \|x_{n-1} - p\|^2 + 2(1-\alpha_n)(1-\lambda_n) \\
& \times \left[L\|y_n - x_n\|\|x_n - p\| + \|x_n - p\|^2 - \lambda\|x_n - T_n x_n\|^2\right] \\
& + 2(1-\alpha_n)\lambda_n\left[L\|y_n - x_n\|\|x_n - p\| + L\|x_n - p\|^2\right] \\
& - 2(1-\alpha_n)\lambda_n\langle F(p), j(x_n - p)\rangle \\
\le{}& \alpha_n^2 \|x_{n-1} - p\|^2 + 2(1-\alpha_n) \\
& \times \left[L\|y_n - x_n\|\|x_n - p\| + \|x_n - p\|^2 - \lambda\|x_n - T_n x_n\|^2\right] \\
& + 2(1-\alpha_n)\lambda_n L\|x_n - p\|^2 + 2(1-\alpha_n)\lambda_n\|F(p)\|\|x_n - p\| \\
\le{}& \alpha_n^2 \|x_{n-1} - p\|^2 + 2(1-\alpha_n) \\
& \times \left[L\|y_n - x_n\|\|x_n - p\| + \|x_n - p\|^2 - \lambda\|x_n - T_n x_n\|^2\right] \\
& + 2(1-\alpha_n)\lambda_n L\|x_n - p\|^2 + (1-\alpha_n)\lambda_n\left(\|F(p)\|^2 + \|x_n - p\|^2\right) \\
\le{}& \alpha_n^2 \|x_{n-1} - p\|^2 + 2(1-\alpha_n) \\
& \times \left[L\|y_n - x_n\|\|x_n - p\| + \|x_n - p\|^2 - \lambda\|x_n - T_n x_n\|^2\right] \\
& + 3L(1-\alpha_n)\lambda_n\|x_n - p\|^2 + (1-\alpha_n)\lambda_n\|F(p)\|^2.
\end{aligned}
\tag{2.7}
$$

From (2.1), we also have that

$$
\begin{aligned}
&\|y_n - x_n\| \\
&\quad \le \beta_n\|x_n - x_{n-1}\| + (1-\beta_n)\|x_n - T_n x_n\| \\
&\quad \le \beta_n(1-\alpha_n)\|T_n y_n - \lambda_n F(T_n y_n) - x_{n-1}\| + (1-\beta_n)\|x_n - T_n x_n\| \\
&\|T_n y_n - \lambda_n F(T_n y_n) - x_{n-1}\| \\
&\quad \le \|T_n y_n - \lambda_n F(T_n y_n) - p\| + \|x_{n-1} - p\| \\
&\quad = \|(1-\lambda_n)(T_n y_n - p) + \lambda_n((I-F)T_n y_n - p)\| + \|x_{n-1} - p\| \\
&\quad = \|(1-\lambda_n)(T_n y_n - p) + \lambda_n((I-F)T_n y_n - (I-F)p) - \lambda_n F(p)\| + \|x_{n-1} - p\| \\
&\quad \le (1-\lambda_n)\|T_n y_n - p\| + \lambda_n\|T_n y_n - p\| + \lambda_n\|F(p)\| + \|x_{n-1} - p\| \\
&\quad = \|T_n y_n - p\| + \lambda_n\|F(p)\| + \|x_{n-1} - p\| \\
&\quad \le L\|y_n - p\| + \lambda_n\|F(p)\| + \|x_{n-1} - p\| \\
&\quad \le (L\beta_n + 1)\|x_{n-1} - p\| + L^2(1-\beta_n)\|x_n - p\| + \lambda_n\|F(p)\|.
\end{aligned}
\tag{2.8}
$$

Since T_i is a Lipschitz mapping with constant L, we have

$$\|x_n - T_n x_n\| \le \|x_n - p\| + \|T_n x_n - p\| \le (L+1)\|x_n - p\|. \tag{2.9}$$

Substituting (2.8) and (2.9) into (2.7), we deduce that

$$\begin{aligned}
\|x_n - p\|^2 \le{}& \alpha_n^2\|x_{n-1} - p\|^2 + 2(1-\alpha_n)^2 L\beta_n(L\beta_n + 1)\|x_{n-1} - p\|\|x_n - p\| \\
&+ 2(1-\alpha_n)^2 L^3\beta_n(1-\beta_n)\|x_n - p\|^2 \\
&+ 2(1-\alpha_n)(1-\beta_n)L(L+1)\|x_n - p\|^2 \\
&+ 2(1-\alpha_n)\|x_n - p\|^2 - 2(1-\alpha_n)\lambda\|x_n - T_n x_n\|^2 \\
&+ 2L(1-\alpha_n)^2\lambda_n\beta_n\|F(p)\|\|x_n - p\| \\
&+ 3L(1-\alpha_n)\lambda_n\|x_n - p\|^2 + (1-\alpha_n)\lambda_n\|F(p)\|^2 \\
\le{}& \alpha_n^2\|x_{n-1} - p\|^2 + 2(1-\alpha_n)^2 L\beta_n(L\beta_n + 1)\|x_{n-1} - p\|\|x_n - p\| \\
&+ 2(1-\alpha_n)^2 L^3\beta_n(1-\beta_n)\|x_n - p\|^2 \\
&+ 2(1-\alpha_n)(1-\beta_n)L(L+1)\|x_n - p\|^2 \\
&+ 2(1-\alpha_n)\|x_n - p\|^2 - 2(1-\alpha_n)\lambda\|x_n - T_n x_n\|^2 \\
&+ 4L(1-\alpha_n)\lambda_n\|x_n - p\|^2 + 2L(1-\alpha_n)\lambda_n\|F(p)\|^2,
\end{aligned} \tag{2.10}$$

and hence

$$\begin{aligned}
&\left[1 - 2(1-\alpha_n)^2 L^3\beta_n(1-\beta_n) - 2(1-\alpha_n)(1-\beta_n)L(L+1) - 4L(1-\alpha_n)\lambda_n - 2(1-\alpha_n)\right]\|x_n - p\|^2 \\
&\quad\le \alpha_n^2\|x_{n-1} - p\|^2 + 2(1-\alpha_n)^2 L\beta_n(L\beta_n + 1)\|x_{n-1} - p\|\|x_n - p\| \\
&\qquad - 2(1-\alpha_n)\lambda\|x_n - T_n x_n\|^2 + 2L(1-\alpha_n)\lambda_n\|F(p)\|^2.
\end{aligned} \tag{2.11}$$

Setting

$$b_n = 2(1-\alpha_n)^2 L^3\beta_n(1-\beta_n) + 2(1-\alpha_n)(1-\beta_n)L(L+1) + 4L(1-\alpha_n)\lambda_n, \tag{2.12}$$

we conclude from (2.11) that

$$\|x_n - p\|^2 \le \frac{\alpha_n^2}{1-2(1-\alpha_n)-b_n}\|x_{n-1}-p\|^2 + \frac{2(1-\alpha_n)^2 L\beta_n(L\beta_n+1)}{1-2(1-\alpha_n)-b_n}\|x_{n-1}-p\|\|x_n-p\| - \frac{2(1-\alpha_n)\lambda}{1-2(1-\alpha_n)-b_n}\|x_n - T_n x_n\|^2 + \frac{(1-\alpha_n)\lambda_n}{1-2(1-\alpha_n)-b_n}2L\|F(p)\|^2. \tag{2.13}$$

Thus

$$\|x_n - p\|^2 \le \left[1 + \frac{(1-\alpha_n)^2 + b_n}{1-2(1-\alpha_n)-b_n}\right]\|x_{n-1}-p\|^2 + \frac{2(1-\alpha_n)^2 L\beta_n(L\beta_n+1)}{1-2(1-\alpha_n)-b_n}\|x_{n-1}-p\|\|x_n-p\| - 2(1-\alpha_n)\lambda\|x_n - T_n x_n\|^2 + \frac{(1-\alpha_n)\lambda_n}{1-2(1-\alpha_n)-b_n}2L\|F(p)\|^2. \tag{2.14}$$

Since

$$1-2(1-\alpha_n)-b_n = 1-(1-\alpha_n)\left[2 + 2(1-\alpha_n)L^3\beta_n(1-\beta_n) + 2(1-\beta_n)L(L+1) + 4L\lambda_n\right] \tag{2.15}$$

and $\{\alpha_n\}_{n=1}^{\infty}, \{\beta_n\}_{n=1}^{\infty}, \{\lambda_n\}_{n=1}^{\infty} \subset [0,1]$, we get

$$\left[2 + 2(1-\alpha_n)L^3\beta_n(1-\beta_n) + 2(1-\beta_n)L(L+1) + 4L\lambda_n\right] \le 6L + 2L^3 + 2L(L+1). \tag{2.16}$$

Setting $M_1 = 6L + 2L^3 + 2L(L+1)$, it follows from condition (ii) that $\lim_{n\to\infty}(1-\alpha_n) = 0$ and so there must exist a natural number N_1 such that for all $n \ge N_1$,

$$\frac{1}{1-2(1-\alpha_n)-b_n} < 2. \tag{2.17}$$

Therefore, it follows from (2.14) that

$$\|x_n - p\|^2 \le \left[1 + 2\left((1-\alpha_n)^2 + b_n\right)\right]\|x_{n-1}-p\|^2 + 2\left[2(1-\alpha_n)^2 L\beta_n(L\beta_n+1)\right]\|x_{n-1}-p\|\|x_n-p\| - 2(1-\alpha_n)\lambda\|x_n - T_n x_n\|^2 + 4L\|F(p)\|^2(1-\alpha_n)\lambda_n. \tag{2.18}$$

In order to consider the second term on the right-hand side of (2.18), we will prove that $\{x_n\}$ is bounded. Indeed, utilizing (2.8) and (2.9) and simplifying these inequalities, we have

$$
\begin{aligned}
&\|x_n - p\|^2 \\
&\quad = \langle x_n - p, j(x_n - p)\rangle \\
&\quad = \alpha_n \langle x_{n-1} - p, j(x_n - p)\rangle + (1-\alpha_n)\langle T_n y_n - \lambda_n F(T_n y_n) - p, j(x_n - p)\rangle \\
&\quad = \alpha_n \langle x_{n-1} - p, j(x_n - p)\rangle + (1-\alpha_n)(1-\lambda_n) \\
&\qquad \times \left[\langle T_n y_n - T_n x_n, j(x_n - p)\rangle + \langle T_n x_n - T_n p, j(x_n - p)\rangle\right] \\
&\qquad + (1-\alpha_n)\lambda_n \langle (I-F)T_n y_n - (I-F)T_n p, j(x_n - p)\rangle - (1-\alpha_n)\lambda_n \langle F(p), j(x_n - p)\rangle \\
&\quad \leq \alpha_n \|x_{n-1} - p\| \|x_n - p\| + (1-\alpha_n)(1-\lambda_n)\left[L\|y_n - x_n\|\|x_n - p\| + L\|x_n - p\|^2\right] \\
&\qquad + (1-\alpha_n)\lambda_n \left[L\|y_n - x_n\|\|x_n - p\| + L\|x_n - p\|^2\right] + (1-\alpha_n)\lambda_n \|F(p)\|\|x_n - p\| \\
&\quad \leq \alpha_n \|x_{n-1} - p\| \|x_n - p\| + (1-\alpha_n)\left[L\|y_n - x_n\|\|x_n - p\| + L\|x_n - p\|^2\right] \\
&\qquad + (1-\alpha_n)\lambda_n \|F(p)\|\|x_n - p\| \\
&\quad \leq \left[\alpha_n + L(1-\alpha_n)^2\beta_n(L\beta_n + 1)\right]\|x_{n-1} - p\|\|x_n - p\| \\
&\qquad + \left[(1-\alpha_n)L + L^3(1-\alpha_n)^2\beta_n(1-\beta_n) + L(1-\alpha_n)(1-\beta_n)(L+1)\right]\|x_n - p\|^2 \\
&\qquad + L\beta_n(1-\alpha_n)^2\lambda_n\|F(p)\|\|x_n - p\| + (1-\alpha_n)\lambda_n\|F(p)\|\|x_n - p\| \\
&\quad \leq \left[\alpha_n + L(1-\alpha_n)^2\beta_n(L\beta_n + 1)\right]\|x_{n-1} - p\|\|x_n - p\| \\
&\qquad + \left[(1-\alpha_n)L + L^3(1-\alpha_n)^2\beta_n(1-\beta_n) + L(1-\alpha_n)(1-\beta_n)(L+1)\right]\|x_n - p\|^2 \\
&\qquad + (L+1)(1-\alpha_n)\lambda_n\|F(p)\|\|x_n - p\|,
\end{aligned}
\tag{2.19}
$$

and hence

$$
\begin{aligned}
&\left[1 - (1-\alpha_n)L - L^3(1-\alpha_n)^2\beta_n(1-\beta_n) - L(1-\alpha_n)(1-\beta_n)(L+1)\right]\|x_n - p\|^2 \\
&\quad \leq \left[\alpha_n + L(1-\alpha_n)^2\beta_n(L\beta_n + 1)\right]\|x_{n-1} - p\|\|x_n - p\| \\
&\qquad + (L+1)(1-\alpha_n)\lambda_n\|F(p)\|\|x_n - p\|.
\end{aligned}
\tag{2.20}
$$

This implies that

$$
\begin{aligned}
&\|x_n - p\| \\
&\quad\le \frac{\alpha_n + L(1-\alpha_n)^2\beta_n(L\beta_n+1)}{1-(1-\alpha_n)L - L^3(1-\alpha_n)^2\beta_n(1-\beta_n) - L(1-\alpha_n)(1-\beta_n)(L+1)}\|x_{n-1}-p\| \\
&\qquad + \frac{(1-\alpha_n)\lambda_n}{1-(1-\alpha_n)L - L^3(1-\alpha_n)^2\beta_n(1-\beta_n) - L(1-\alpha_n)(1-\beta_n)(L+1)}(L+1)\|F(p)\| \\
&\quad\le \left[1+\frac{L^3(1-\alpha_n)^2\beta_n(1-\beta_n) + L(1-\alpha_n)(1-\beta_n)(L+1) + L(1-\alpha_n)^2\beta_n(L\beta_n+1)}{1-(1-\alpha_n)L - L^3(1-\alpha_n)^2\beta_n(1-\beta_n) - L(1-\alpha_n)(1-\beta_n)(L+1)}\right]\|x_{n-1}-p\| \\
&\qquad + \frac{(1-\alpha_n)\lambda_n}{1-(1-\alpha_n)L - L^3(1-\alpha_n)^2\beta_n(1-\beta_n) - L(1-\alpha_n)(1-\beta_n)(L+1)}(L+1)\|F(p)\|.
\end{aligned}
\tag{2.21}
$$

Now, we consider the second term on the right-hand side of (2.21). Since $\{\alpha_n\}, \{\beta_n\} \subset [0,1]$, we have

$$
(1-\alpha_n)\left[L + L^3(1-\alpha_n)\beta_n(1-\beta_n) + L(1-\beta_n)(L+1)\right] \le (1-\alpha_n)\left[L + L^3 + L(L+1)\right]. \tag{2.22}
$$

Since $\lim_{n\to\infty}(1-\alpha_n) = 0$, there exists a natural number $N_2(\ge N_1)$ such that for all $n \ge N_2$,

$$
1-(1-\alpha_n)L - L^3(1-\alpha_n)^2\beta_n(1-\beta_n) - L(1-\alpha_n)(1-\beta_n)(L+1) \ge \frac{1}{2}. \tag{2.23}
$$

Again, it follows from condition $\{\alpha_n\}, \{\beta_n\} \subset [0,1]$ that

$$
\begin{aligned}
&L^3(1-\alpha_n)^2\beta_n(1-\beta_n) + L(1-\alpha_n)(1-\beta_n)(L+1) + L(1-\alpha_n)^2\beta_n(L\beta_n+1) \\
&\quad\le L^3(1-\alpha_n)^2 + L(1-\beta_n)(L+1) + L(1-\alpha_n)^2(L+1).
\end{aligned}
\tag{2.24}
$$

Therefore, it follows from (2.21) that

$$
\begin{aligned}
\|x_n - p\| \le{}& \left\{1 + 2\left[L^3(1-\alpha_n)^2 + L(1-\beta_n)(L+1) + L(1-\alpha_n)^2(L+1)\right]\right\}\|x_{n-1}-p\| \\
&+ 2(1-\alpha_n)\lambda_n(L+1)\|F(p)\|.
\end{aligned}
\tag{2.25}
$$

According to conditions (ii)–(iv), we can readily see that

$$
\begin{gathered}
\sum_{n=1}^{\infty}\left\{2\left[L^3(1-\alpha_n)^2 + L(1-\beta_n)(L+1) + L(1-\alpha_n)^2(L+1)\right]\right\} < +\infty, \\
\sum_{n=1}^{\infty}\left\{2(1-\alpha_n)\lambda_n(L+1)\|F(p)\|\right\} < +\infty.
\end{gathered}
\tag{2.26}
$$

Thus, in terms of Lemma 1.8 we deduce that $\lim_{n\to\infty}\|x_n - p\|$ exists, and hence $\{x_n\}$ is bounded.

Now, we consider the second term on the right-hand side of (2.18). Since $\{x_n\}$ is bounded, and $\{\beta_n\}_{n=1}^{\infty} \subset [0,1]$, there exists a constant $M_2 > 0$ and a natural number $N_3(\geq N_2)$ such that for all $n \geq N_3$,

$$2\left[2(1-\alpha_n)^2 L\beta_n(L\beta_n+1)\right]\|x_{n-1}-p\|\|x_n-p\| \leq 2(1-\alpha_n)^2 M_2. \tag{2.27}$$

Thus, it follows from (2.18) that

$$\begin{aligned}\|x_n-p\|^2 \leq{}& \left[1+2\left((1-\alpha_n)^2+b_n\right)\right]\|x_{n-1}-p\|^2+2(1-\alpha_n)^2M_2\\&-2(1-\alpha_n)\lambda\|x_n-T_nx_n\|^2+4L\|F(p)\|^2(1-\alpha_n)\lambda_n.\end{aligned} \tag{2.28}$$

Since $\{x_n\}$ is bounded, there exists a constant $M_3 > 0$ such that $\|x_n - p\|^2 \leq M_3$. It follows from (2.28) that

$$\begin{aligned}2\lambda\sum_{j=N+1}^{n}(1-\alpha_j)\|x_j-T_jx_j\|^2 \leq{}& \|x_N-p\|^2+M_3\sum_{j=N+1}^{n}2\left[(1-\alpha_j)^2+b_j\right]\\&+2M_2\sum_{j=N+1}^{n}(1-\alpha_j)^2+4L\|F(p)\|^2\sum_{j=N+1}^{n}(1-\alpha_j)\lambda_j,\end{aligned} \tag{2.29}$$

and hence

$$\begin{aligned}2\lambda\sum_{n=N+1}^{\infty}(1-\alpha_n)\|x_n-T_nx_n\|^2 \leq{}& \|x_N-p\|^2+M_3\sum_{n=N+1}^{\infty}2\left[(1-\alpha_n)^2+b_n\right]\\&+2M_2\sum_{n=N+1}^{\infty}(1-\alpha_n)^2+4L\|F(p)\|^2\sum_{n=N+1}^{\infty}(1-\alpha_n)\lambda_n.\end{aligned} \tag{2.30}$$

Utilizing conditions (ii)–(iv), we know from (2.30) that

$$2\lambda\sum_{n=1}^{\infty}(1-\alpha_n)\|x_n-T_nx_n\|^2 < +\infty. \tag{2.31}$$

Since $\sum_{n=1}^{\infty}(1-\alpha_n) = +\infty$, we have

$$\liminf_{n\to\infty}\|x_n-T_nx_n\| = 0. \tag{2.32}$$

This completes the proof of Theorem 2.1. □

The iterative scheme (1.15) becomes the explicit version as follows, whenever $\beta_n \equiv 1$:

$$x_n = \alpha_n x_{n-1} + (1 - \alpha_n)[T_n x_{n-1} - \lambda_n F(T_n x_{n-1})], \quad n \geq 1. \tag{2.33}$$

In the case when $N = 1$, (2.33) is the Mann iteration process as follows:

$$x_n = \alpha_n x_{n-1} + (1 - \alpha_n)[T x_{n-1} - \lambda_n F(T x_{n-1})], \quad n \geq 1. \tag{2.34}$$

The conclusion of Theorem 2.1 remains valid for the iteration processes (2.33) and (2.34). Furthermore, we have the following result.

Theorem 2.2. *Let E be a real Banach space, and let K be a nonempty closed convex subset of E such that $K - K \subset K$. Let $F : K \longrightarrow K$ be a perturbed mapping which is both δ-strongly accretive and λ-strictly pseudocontractive with $\delta + \lambda \geq 1$. Let T be a semicompact strictly pseudocontractive self-map of K such that $F(T) \neq \emptyset$, where $F(T) = \{x \in K : Tx = x\}$, and let $\{\alpha_n\}_{n=1}^{\infty}$ and $\{\lambda_n\}_{n=1}^{\infty}$ be two real sequences in $[0, 1]$ satisfying the conditions:*

(i) $\sum_{n=1}^{\infty}(1 - \alpha_n) = +\infty$;

(ii) $\sum_{n=1}^{\infty}(1 - \alpha_n)^2 < +\infty$;

(iii) $\sum_{n=1}^{\infty}\lambda_n(1 - \alpha_n) < +\infty$.

Then Mann iteration process (2.34) *converges strongly to a fixed point of T.*

Proof. Since

$$\liminf_{n \to \infty} \|x_n - T x_n\| = 0, \tag{2.35}$$

there exists a subsequence $\{n_k\}$ of $\{n\}$ such that

$$\lim_{k \to \infty} \|x_{n_k} - T x_{n_k}\| = 0. \tag{2.36}$$

By the semicompactness of T, there must exist a subsequence $\{x_{n_{k_i}}\}$ of $\{x_{n_k}\}$ such that

$$\lim_{i \to \infty} x_{n_{k_i}} = p_0. \tag{2.37}$$

It follows from (2.36) that $p_0 = T p_0$, and hence $p_0 \in F(T)$. Since $\lim_{n \to \infty}\|x_n - p_0\|$ exists, we have

$$\lim_{n \to \infty} \|x_n - p_0\| = \lim_{i \to \infty} \|x_{n_{k_i}} - p_0\| = 0. \tag{2.38}$$

This completes the proof of Theorem 2.2. □

Acknowledgments

The first author was partially supported by the National Science Foundation of China (10771141), Ph.D. Program Foundation of Ministry of Education of China (20070270004), and Science and Technology Commission of Shanghai Municipality grant (075105118), Leading Academic Discipline Project of Shanghai Normal University (DZL707), Shanghai Leading Academic Discipline Project (S30405) and Innovation Program of Shanghai Municipal Education Commission (09ZZ133). The third author was partially supported by the Grant NSF 97-2115-M-110-001

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