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Research Article

An Iteration Method for Nonexpansive Mappings in Hilbert Spaces

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In real Hilbert space H, from an arbitrary initial point $x_0 \in H$, an explicit iteration scheme is defined as follows: $x_{n+1} = \alpha_n x_n + (1-\alpha_n)T^{\lambda_{n+1}}x_n, n \geq 0$, where $T^{\lambda_{n+1}}x_n = Tx_n - \lambda_{n+1}\mu F(Tx_n)$, $T : H \to H$ is a nonexpansive mapping such that $F(T) = \{x \in K : Tx = x\}$ is nonempty, $F : H \to H$ is a η-strongly monotone and k-Lipschitzian mapping, $\{\alpha_n\} \subset (0,1)$, and $\{\lambda_n\} \subset [0,1)$. Under some suitable conditions, the sequence $\{x_n\}$ is shown to converge strongly to a fixed point of T and the necessary and sufficient conditions that $\{x_n\}$ converges strongly to a fixed point of T are obtained.

1. Introduction

Let H be a Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$. A mapping $T : H \to H$ is said to be nonexpansive if $\|Tx - Ty\| \leq \|x - y\|$ for any $x, y \in H$. A mapping $F : H \to H$ is said to be η-strongly monotone if there exists constant $\eta > 0$ such that $\langle Fx - Fy, x - y\rangle \geq \eta\|x - y\|^2$ for any $x, y \in H$. $F : H \to H$ is said to be k-Lipschitzian if there exists constant $k > 0$ such that $\|Fx - Fy\| \leq k\|x - y\|$ for any $x, y \in H$.

The interest and importance of construction of fixed points of nonexpansive mappings stem mainly from the fact that it may be applied in many areas, such as imagine recovery and signal processing (see, e.g., [1–3]). Iterative techniques for approximating fixed points of nonexpansive mappings have been studied by various authors (see, e.g., [1, 4–10], etc.), using famous Mann iteration method, Ishikawa iteration method, and many other iteration methods such as, viscosity approximation method [6] and CQ method [7].

Let $F : H \to H$ be a nonlinear mapping and K nonempty closed convex subset of H. The variational inequality problem is formulated as finding a point $u^* \in K$ such that

$$(\mathrm{VI}(F,K))\,\langle F(u^*),v-u^*\rangle \geq 0, \quad \forall v \in K. \tag{1.1}$$

The variational inequalities were initially studied by Kinderlehrer and Stampacchia [11], and ever since have been widely studied. It is well known that the VI(F,K) is equivalent to the fixed point equation

$$u^* = P_K(u^* - \mu F(u^*)), \tag{1.2}$$

where P_K is the projection from H onto K and μ is an arbitrarily fixed constant. In fact, when F is an η-strongly monotone and Lipschitzian mapping on K and $\mu > 0$ small enough, then the mapping defined by the right-hand side of (1.2) is a contraction.

For reducing the complexity of computation caused by the projection P_K, Yamada [12] proposed an iteration method to solve the variational inequalities VI(F,K). For arbitrary $u_0 \in H$,

$$u_{n+1} = Tu_n - \lambda_{n+1}\mu F(T(u_n)), \quad n \geq 0, \tag{1.3}$$

where T is a nonexpansive mapping from H into itself, K is the fixed point set of T, F is an η-strongly monotone and k-Lipschitzian mapping on K, $\{\lambda_n\}$ is a real sequence in $[0,1)$, and $0 < \mu < 2\eta/k^2$. Then Yamada [12] proved that $\{u_n\}$ converges strongly to the unique solution of the VI(F,K) as $\{\lambda_n\}$ satisfies the following conditions:

(1) $\lim_{n\to\infty}\lambda_n = 0$,

(2) $\sum_{n=0}^{\infty}\lambda_n = \infty$,

(3) $\lim_{n\to\infty}(\lambda_n - \lambda_{n+1})/\lambda_{n+1}^2 = 0$.

Motivated by the above work, we propose a new explicit iteration scheme with mapping F to approximate the fixed point of nonexpansive mapping T in Hilbert space. The strong and weak convergence theorems to a fixed point of T are obtained. The necessary and sufficient conditions for strong convergence of this iteration scheme are obtained, too.

2. Preliminaries

Let T be a nonexpansive mapping from H into itself, $F: H \to H$ an η-strongly monotone and k-Lipschitzian mapping, $\{\lambda_n\} \subset (0,1)$, $\{\lambda_n\} \subset [0,1)$, and μ a fixed constant in $(0,2\eta/k^2)$. Starting with an initial point $x_0 \in H$, the explicit iteration scheme with mapping F is defined as follows:

$$x_{n+1} = \alpha_n x_n + (1-\alpha_n)(Tx_n - \lambda_{n+1}\mu F(Tx_n)), \quad n \geq 0. \tag{2.1}$$

For simplicity, we define a mapping $T^{\lambda}: H \to H$ by

$$T^{\lambda}x = Tx - \lambda\mu F(Tx), \quad \forall x \in H. \tag{2.2}$$

Then (2.1) may be written as follows:

$$x_{n+1} = \alpha_n x_n + (1-\alpha_n)T^{\lambda_{n+1}}x_n, \quad n \geq 0. \tag{2.3}$$

In fact, as $\lambda_n = 0$, $n \geq 1$, then the iteration scheme (2.3) reduces to the famous Mann iteration scheme.

A Banach space E is said to satisfy Opial's condition if for any sequence $\{x_n\}$ in E, $x_n \rightharpoonup x$ implies that $\limsup_{n\to\infty} \|x_n - x\| < \limsup_{n\to\infty} \|x_n - y\|$ for all $y \in E$ with $y \neq x$, where $x_n \rightharpoonup x$ denotes that $\{x_n\}$ converges weakly to x. It is well known that every Hilbert space satisfies Opial's condition.

A mapping $T : K \to E$ is said to be semicompact if, for any sequence $\{x_n\}$ in K such that $\|x_n - Tx_n\| \to 0$ $(n \to \infty)$, there exists subsequence $\{x_{n_j}\}$ of $\{x_n\}$ such that $\{x_{n_j}\}$ converges strongly to $x^* \in K$.

A mapping T with domain $D(T)$ and range $R(T)$ in E is said to be demiclosed at p; if whenever $\{x_n\}$ is a sequence in $D(T)$ such that $\{x_n\}$ converges weakly to $x^* \in D(T)$ and $\{Tx_n\}$ converges strongly to p, then $Tx^* = p$.

Lemma 2.1 [13]. *Let $\{\alpha_n\}$ and $\{t_n\}$ be two nonnegative sequences satisfying*

$$\alpha_{n+1} \leq (1 + a_n)\alpha_n + b_n, \quad \forall n \geq 1. \tag{2.4}$$

If $\sum_{n=1}^{\infty} a_n < \infty$ and $\sum_{n=1}^{\infty} b_n < \infty$, then $\lim_{n\to\infty} \alpha_n$ exists.

Lemma 2.2 [12]. *Let $T^\lambda x = Tx - \lambda\mu F(Tx)$, where $T : H \to H$ is a nonexpansive mapping from H into itself and F is an η-strongly monotone and k-Lipschitzian mapping from H into itself. If $0 \leq \lambda < 1$ and $0 < \mu < 2\eta/k^2$, then T^λ is a contraction and satisfies*

$$||T^\lambda x - T^\lambda y|| \leq (1 - \lambda\tau)\|x - y\|, \quad \forall x, y \in H, \tag{2.5}$$

where $\tau = 1 - \sqrt{1 - \mu(2\eta - \mu k^2)}$.

Lemma 2.3 [14]. *Let K be a nonempty closed convex subset of a real Hilbert space H and T a nonexpansive mapping from K into itself. If T has a fixed point, then $I - T$ is demiclosed at zero, where I is the identity mapping of H, that is, whenever $\{x_n\}$ is a sequence in K weakly converging to some $x \in K$ and the sequence $\{(I - T)x_n\}$ strongly converges to some y, it follows that $(I - T)x = y$.*

3. Main results

Lemma 3.1. *Let H be a Hilbert space, $T : H \to H$ a nonexpansive mapping with $F(T) \neq \phi$, and $F : H \to H$ an η-strongly monotone and k-Lipschitzian mapping. For any given $x_0 \in H$, $\{x_n\}$ is defined by*

$$x_{n+1} = \alpha_n x_n + (1 - \alpha_n) T^{\lambda_{n+1}} x_n, \quad n \geq 0, \tag{3.1}$$

where $\{\alpha_n\}$ and $\{\lambda_n\} \subset [0,1)$ satisfy the following conditions:

(1) $\alpha \leq \alpha_n \leq \beta$ *for some* $\alpha, \beta \in (0,1)$;

(2) $\sum_{n=1}^{\infty} \lambda_n < \infty$;

(3) $0 < \mu < 2\eta/k^2$.

Then,

(1) $\lim_{n\to\infty} \|x_n - q\|$ *exists for each* $q \in F(T)$;

(2) $\lim_{n\to\infty} \|x_n - Tx_n\| = 0$.

Proof. (1) For any $q \in F(T)$, we have

$$\begin{aligned}||x_{n+1}-q||^2 &= ||\alpha_n(x_n-q)+(1-\alpha_n)(T^{\lambda_{n+1}}x_n-q)||^2\\ &= \alpha_n||x_n-q||^2+(1-\alpha_n)||T^{\lambda_{n+1}}x_n-q||^2-\alpha_n(1-\alpha_n)||x_n-T^{\lambda_{n+1}}x_n||^2,\end{aligned} \tag{3.2}$$

where (by Lemma 2.2)

$$\begin{aligned}||T^{\lambda_{n+1}}x_n-q|| &= ||T^{\lambda_{n+1}}x_n-T^{\lambda_{n+1}}q+T^{\lambda_{n+1}}q-q||\\ &\le ||T^{\lambda_{n+1}}x_n-T^{\lambda_{n+1}}q||+||T^{\lambda_{n+1}}q-q||\\ &\le (1-\lambda_{n+1}\tau)||x_n-q||+\lambda_{n+1}\mu||F(q)||.\end{aligned} \tag{3.3}$$

Furthermore,

$$||T^{\lambda_{n+1}}x_n-q||^2 \le (1-\lambda_{n+1}\tau)||x_n-q||^2+\frac{\lambda_{n+1}\mu^2}{\tau}||F(q)||^2. \tag{3.4}$$

Thus,

$$\begin{aligned}||x_{n+1}-q||^2 &\le \alpha_n||x_n-q||^2+(1-\alpha_n)(1-\lambda_{n+1}\tau)||x_n-q||^2\\ &\quad +(1-\alpha_n)\frac{\lambda_{n+1}\mu^2}{\tau}||F(q)||^2-\alpha_n(1-\alpha_n)||x_n-T^{\lambda_{n+1}}x_n||^2\\ &\le \alpha_n||x_n-q||^2+(1-\alpha_n)(1-\lambda_{n+1}\tau)||x_n-q||^2\\ &\quad +(1-\alpha_n)\frac{\lambda_{n+1}\mu^2}{\tau}||F(q)||^2-\alpha_n||x_{n+1}-x_n||^2\\ &\le ||x_n-q||^2+\frac{\lambda_{n+1}\mu^2}{\tau}||F(q)||^2-\alpha_n||x_{n+1}-x_n||^2.\end{aligned} \tag{3.5}$$

Since $\sum_{n=1}^{\infty}\lambda_n<\infty$, it follows from Lemma 2.1 that $\lim_{n\to\infty}\|x_n-q\|$ exists for each $q\in F(T)$. It also implies that $\{x_n\}$ is bounded.

(2) From (3.5), we have

$$\alpha||x_{n+1}-x_n||^2 \le \alpha_n||x_{n+1}-x_n||^2 \le ||x_n-q||^2-||x_{n+1}-q||^2+\frac{\lambda_{n+1}\mu^2}{\tau}||F(q)||^2. \tag{3.6}$$

Therefore, $\lim_{n\to\infty}\|x_{n+1}-x_n\|=0$. In addition,

$$(1-\beta)||x_n-T^{\lambda_{n+1}}x_n|| \le (1-\alpha_n)||x_n-T^{\lambda_{n+1}}x_n|| = ||x_{n+1}-x_n||. \tag{3.7}$$

Hence, $\lim_{n\to\infty}\|x_n-T^{\lambda_{n+1}}\|=0$. Thus,

$$\begin{aligned}||x_n-Tx_n|| &= ||x_n-T^{\lambda_{n+1}}x_n+T^{\lambda_{n+1}}x_n-Tx_n||\\ &\le ||x_n-T^{\lambda_{n+1}}x_n||+\lambda_{n+1}\mu||F(Tx_n)||.\end{aligned} \tag{3.8}$$

Since $\{x_n\}$ is bounded, then $\{Tx_n\}$ and $\{F(Tx_n)\}$ are bounded, as well. Therefore, $\lim_{n\to\infty}\|x_n-Tx_n\|=0$. The proof is completed. □

Theorem 3.2. *Let H be a Hilbert space, $T : H \to H$ a nonexpansive mapping with $F(T) \neq \phi$, and $F : H \to H$ an η-strongly monotone and k-Lipschitzian mapping. For any given $x_0 \in H$, $\{x_n\}$ is defined by*

$$x_{n+1} = \alpha_n x_n + (1-\alpha_n) T^{\lambda_{n+1}} x_n, \quad n \geq 0, \tag{3.9}$$

where $\{\alpha_n\}$ and $\{\lambda_n\} \subset [0,1)$ satisfy the following conditions:

(1) $\alpha \leq \alpha_n \leq \beta$ *for some* $\alpha, \beta \in (0,1)$;

(2) $\sum_{n=1}^{\infty} \lambda_n < \infty$;

(3) $0 < \mu < 2\eta/k^2$.

Then,

(1) $\{x_n\}$ *converges weakly to a fixed point of* T;

(2) $\{x_n\}$ *converges strongly to a fixed point of T if and only if* $\liminf_{n\to\infty} d(x_n, F(T)) = 0$.

Proof. (1) It follows from Lemma 3.1 that $\{x_n\}$ is bounded. Thus, let q_1 and q_2 be weak limits of subsequences $\{x_{n_k}\}$ and $\{x_{n_j}\}$ of $\{x_n\}$, respectively. It follows from Lemmas 2.3 and 3.1 that $q_1, q_2 \in F(T)$. Assume $q_1 \neq q_2$, then by Opial's condition, we obtain

$$\begin{aligned} \lim_{n\to\infty} ||x_n - q_1|| &= \lim_{k\to\infty} ||x_{n_k} - q_1|| < \lim_{k\to\infty} ||x_{n_k} - q_2|| \\ &= \lim_{j\to\infty} ||x_{n_j} - q_2|| < \lim_{k\to\infty} ||x_{n_k} - q_1|| = \lim_{n\to\infty} ||x_n - q_1||, \end{aligned} \tag{3.10}$$

which is a contradiction; hence, $q_1 = q_2$. Then, $\{x_n\}$ converges weakly to a common fixed point of T.

(2) Suppose that $\{x_n\}$ converges strongly to a fixed point q of T, then $\lim_{n\to\infty} \|x_n - q\| = 0$. Since $0 \leq d(x_n, F(T)) \leq \|x_n - q\|$, we have $\liminf_{n\to\infty} d(x_n, F(T)) = 0$.

Conversely, suppose that $\liminf_{n\to\infty} d(x_n, F(T)) = 0$. For any $p \in F(T)$, $\|F(p)\| \leq \|F(p) - F(x_n)\| + \|F(x_n)\| \leq k\|x_n - p\| + \|F(x_n)\|$. Since $\{x_n\}$ and $\{F(x_n)\}$ are bounded, $\|F(p)\|$ is bounded for any $p \in F(T)$, that is, there exists constant $M > 0$ such that $\|F(p)\| \leq M$ for all $p \in F(T)$. In addition, it follows from (3.5) that

$$||x_{n+1} - p||^2 \leq ||x_n - p||^2 + \frac{\lambda_{n+1}\mu^2}{\tau} ||F(p)||^2. \tag{3.11}$$

So,

$$\begin{aligned} ||x_{n+1} - p||^2 &\leq ||x_n - p||^2 + \frac{\lambda_{n+1}\mu^2}{\tau} \left(2k^2 ||x_n - p||^2 + 2||F(x_n)||^2\right) \\ &= \left(1 + 2k^2 \frac{\lambda_{n+1}\mu^2}{\tau}\right) ||x_n - p||^2 + 2\frac{\lambda_{n+1}\mu^2}{\tau} ||F(x_n)||^2. \end{aligned} \tag{3.12}$$

Thus,

$$[d(x_{n+1}, F(T))]^2 \leq \left(1 + 2k^2 \frac{\lambda_{n+1}\mu^2}{\tau}\right) [d(x_n, F(T))]^2 + 2\frac{\lambda_{n+1}\mu^2}{\tau} ||F(x_n)||^2. \tag{3.13}$$

In addition, we obtain that $\sum_{n=1}^{\infty} 2k^2(\lambda_{n+1}\mu^2/\tau) < \infty$ and $\sum_{n=1}^{\infty} 2(\lambda_{n+1}\mu^2/\tau)\|F(x_n)\|^2 < \infty$ since $\sum_{n=1}^{\infty} \lambda_n < \infty$ and $\{F(x_n)\}$ is bounded. It follows from Lemma 2.1 that

$\lim_{n\to\infty} d(x_n, F(T))$ exists. Furthermore, since $\liminf_{n\to\infty} d(x_n, F(T)) = 0$, we have $\lim_{n\to\infty} d(x_n, F(T)) = 0$. We now prove that $\{x_n\}$ is a Cauchy sequence.

Taking $M_1 = \max\{2e^{(2\mu^2k^2/\tau)\sum_{i=1}^{\infty}\lambda_i}, 4(\mu^2M^2/\tau)e^{(2\mu^2k^2/\tau)\sum_{i=1}^{\infty}\lambda_i}\}$, for any $\epsilon > 0$, there exists positive integer N such that $d(x_n, F(T)) < \sqrt{\epsilon/4M_1}$ and $\sum_{i=n}^{\infty}\lambda_i < \epsilon/4M_1$ as $n \geq N$. Taking $q \in F(T)$, for any $n, m \geq N$, it follows from (3.12) that

$$\begin{aligned}
\frac{||x_n - x_m||^2}{2} &\leq ||x_n - q||^2 + ||x_m - q||^2 \\
&\leq \left(1 + 2k^2\frac{\lambda_n\mu^2}{\tau}\right)||x_{n-1} - q||^2 + 2\frac{\lambda_n\mu^2}{\tau}||F(x_{n-1})||^2 \\
&\quad + \left(1 + 2k^2\frac{\lambda_m\mu^2}{\tau}\right)||x_{m-1} - q||^2 + 2\frac{\lambda_m\mu^2}{\tau}||F(x_{m-1})||^2 \\
&\leq \left(1 + 2k^2\frac{\lambda_n\mu^2}{\tau}\right)||x_{n-1} - q||^2 + 2\frac{\lambda_n\mu^2}{\tau}M^2 \\
&\quad + \left(1 + 2k^2\frac{\lambda_m\mu^2}{\tau}\right)||x_{m-1} - q||^2 + 2\frac{\lambda_m\mu^2}{\tau}M^2 \\
&\leq \prod_{i=N+1}^{n}\left(1 + 2k^2\frac{\lambda_i\mu^2}{\tau}\right)||x_N - q||^2 + \sum_{i=N+1}^{n-1} 2\frac{\lambda_i\mu^2}{\tau}M^2\prod_{j=i+1}^{n}\left(1 + 2k^2\frac{\lambda_j\mu^2}{\tau}\right) \\
&\quad + 2\frac{\lambda_n\mu^2}{\tau}M^2 + \prod_{i=N+1}^{m}\left(1 + 2k^2\frac{\lambda_i\mu^2}{\tau}\right)||x_N - q||^2 \\
&\quad + \sum_{i=N+1}^{m-1} 2\frac{\lambda_i\mu^2}{\tau}M^2\prod_{j=i+1}^{m}\left(1 + 2k^2\frac{\lambda_j\mu^2}{\tau}\right) + 2\frac{\lambda_m\mu^2}{\tau}M^2 \\
&\leq 2e^{(2\mu^2k^2/\tau)\sum_{i=N+1}^{\infty}\lambda_i}||x_N - q||^2 + 4\frac{\mu^2M^2}{\tau}e^{(2\mu^2k^2/\tau)\sum_{i=N+1}^{\infty}\lambda_i}\sum_{i=N+1}^{\infty}\lambda_i.
\end{aligned} \tag{3.14}$$

Thus,

$$||x_n - x_m||^2 \leq 2M_1||x_N - q||^2 + 2M_1\sum_{i=N+1}^{\infty}\lambda_i. \tag{3.15}$$

Taking the infimum for all $q \in F(T)$, we have

$$||x_n - x_m||^2 \leq 2M_1[d(x_N, F(T))]^2 + 2M_1\sum_{i=N+1}^{\infty}\lambda_i < \epsilon. \tag{3.16}$$

This implies that $\{x_n\}$ is a Cauchy sequence. Therefore, there exists $p \in H$ such that $\{x_n\}$ converges strongly to p. It follows from Lemma 3.1 that

$$\|p - Tp\| \leq ||p - x_n|| + ||x_n - Tx_n|| \longrightarrow 0, \quad \text{as } n \longrightarrow \infty. \tag{3.17}$$

Hence, $p \in F(T)$. The proof is completed. □

Corollary 3.3. *Under the conditions of Lemma 3.1, if T is completely continuous, then $\{x_n\}$ converges strongly to a fixed point of T.*

Proof. By Lemma 3.1, $\{x_n\}$ is bounded and $\lim_{n\to\infty}\|x_n - Tx_n\| = 0$, then $\{Tx_n\}$ is also bounded. Since T is completely continuous, there exists subsequence $\{Tx_{n_j}\}$ of $\{Tx_n\}$ such that $Tx_{n_j} \to p$ as $j \to \infty$. It follows from Lemma 3.1 that $\lim_{j\to\infty}\|x_{n_j} - Tx_{n_j}\| = 0$. So by the continuity of T and Lemma 2.3, we have $\lim_{j\to\infty}\|x_{n_j} - p\| = 0$ and $p \in F(T)$. Furthermore, by Lemma 3.1, we get that $\lim_{n\to\infty}\|x_n - p\|$ exists. Thus, $\lim_{n\to\infty}\|x_n - p\| = 0$. The proof is completed. □

Corollary 3.4. *Under the conditions of Lemma 3.1, if T is demicompact, then $\{x_n\}$ converges strongly to a fixed point of T.*

Proof. Since T is demicompact, $\{x_n\}$ is bounded and $\lim_{n\to\infty}\|x_n - Tx_n\| = 0$, then there exists subsequence $\{x_{n_j}\}$ of $\{x_n\}$ such that $\{x_{n_j}\}$ converges strongly to $q \in H$. It follows from Lemma 2.3 that $q \in F(T)$. Thus, $\lim_{n\to\infty}\|x_n - q\|$ exists by Lemma 3.1. Since the subsequence $\{x_{n_j}\}$ of $\{x_n\}$ such that $\{x_{n_j}\}$ converges strongly to q, then $\{x_n\}$ converges strongly to the common fixed point $q \in F(T)$. The proof is completed. □

For studying the strong convergence of fixed points of a nonexpansive mapping, Senter and Dotson [9] introduced Condition (A). Later on, Maiti and Ghosh [5] well as Tan and Xu [10] studied Condition (A) and pointed out that Condition (A) is weaker than the requirement of demicompactness for nonexpansive mappings. A mapping $T : K \to K$ with $F(T) = \{x \in K : Tx = x\} \neq \phi$ is said to satisfy condition (A) if there exists a nondecreasing function $f : [0,\infty) \to [0,\infty)$ with $f(0) = 0$ and $f(t) > 0$ for all $t \in (0,\infty)$ such that $\|x - Tx\| \geq f(d(x,F(T)))$ for all $x \in K$, where $d(x,F(T)) = \inf\{\|x - q\| : q \in F(T)\}$.

Theorem 3.5. *Under the conditions of Lemma 3.1, if T satisfies condition (A), then $\{x_n\}$ converges strongly to a fixed point of T.*

Proof. Since T satisfies condition (A), then $f(d(x_n,F(T))) \leq \|x_n - Tx_n\|$. It follows from Lemma 3.1 that $\liminf_{n\to\infty} d(x_n,F(T)) = 0$. Thus, it follows from Theorem 3.2 that $\{x_n\}$ converges strongly to a fixed point of T. The proof is completed. □

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