Fixed Point Theory and Algorithms for Sciences and Engineering

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Existence of common fixed point in Kannan F-contractive mappings in quasi-partial b-metric space with an application

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Abstract

The purpose of this study is to demonstrate results on fixed point theory in quasi-partial b-metric space recognizing a new type of mapping, which is a blend of F-contraction and Kannan contraction, and to establish the fixed point results in F-expanding type mappings. Additionally, the obtained results are the application of the contractive mappings to functional equations. Furthermore, Mathematica software is used to demonstrate the 3D shapes of the examples discussed here.

MSC: 47H09; 47H10

Keywords: F-contraction; Kannan contraction; F-expanding type mapping; Fixed point; Quasi-partial b-metric space

1 Introduction

In the early years of the twentieth century, Banach [1] commenced the concept of fixed point theorem for metric space acknowledged as the Banach contraction principle. After this classical approach, generalizations of the contraction principle came into existence with several fixed point results and their applications in different spaces as a remarkable contribution by several authors [2, 3]. It is well known that every contraction is continuous. In 1968, Kannan [4] established a new generalization in which he proved that there exists a contraction mapping with a unique fixed point which is not necessarily continuous. In 1989, Bhaktin [5] introduced the notion of b-metric space, which was followed up by Czerwik [6] in 1993. In 1994, Matthews [7] established a new space called partial metric space and proved the fixed point theorem. In 2012, Wardowski [8] demonstrated fixed point results for F-contraction mapping in a complete metric space. This approach was extended by several authors [9–12]. In the year 2013, Karapinar et al. [13] initiated the concept of quasi-partial metric space. Later, Gupta and Gautam [14, 15] introduced quasi-partial b-metric space and proved some fixed point results for such spaces. Due to its significance, many researchers [16–27] have broadened its scope by obtaining various extensions of fixed point theory. In a similar manner, Górnicki [28] demonstrated F-expanding type mappings followed up by Goswami et al. [29] to establish a new type of

F-expanding type mapping. Very recently, Lucas et al. [30] defined the Kannan mapping in a metric space and proved the fixed point results.

In this paper, we have settled the fixed point results in a quasi-partial b-metric space by adopting a new type of mapping into consideration, which is a combination of F and Kannan contractive type mapping. The existence of a fixed point in F-expanding type mapping is also proved. Applications of the results are obtained to find the solution of a functional equation.

2 Preliminaries

First, let us recall some basic definitions concerning our main results. Throughout this paper $\mathbb{R}$ and $\mathbb{R}^+$ denote the set of real numbers and the set of nonnegative real numbers, respectively.

Definition 2.1 ([14]) A quasi-partial b-metric on a non-empty set X is a function $qp_b: X \times X \to [0,+\infty)$ such that for some real number $\rho \geq 1$ and all $\alpha,\beta,\gamma \in X$:

(QPb_1) $qp_b(\alpha,\alpha) = qp_b(\alpha,\beta) = qp_b(\beta,\beta)$ implies $\alpha = \beta$,

(QPb_2) $qp_b(\alpha,\alpha) \leq qp_b(\alpha,\beta)$,

(QPb_3) $qp_b(\alpha,\alpha) \leq qp_b(\beta,\alpha)$,

(QPb_4) $qp_b(\alpha,\beta) + qp_b(\gamma,\gamma) \leq \rho\{qp_b(\alpha,\gamma) + qp_b(\gamma,\beta)\}$.

(X,qp_b) is called a quasi-partial b-metric space where X is a non-empty set and qp_b defines a quasi-partial b-metric on X. The number ρ is called the coefficient of (X,qp_b).

Let us see a new example of quasi-partial b-metric space.

Example 2.1 Let $X = \mathbb{R}$. Define the metric

$$qp_b(\alpha,\beta) = \left|\alpha^2 - \beta^2\right| + \left|\alpha^2\right| + |\alpha-\beta|^2$$

for all $\alpha,\beta \in X$.

It can be shown that (X,qp_b) is a quasi-partial b-metric space.

If $qp_b(\alpha,\alpha) = qp_b(\alpha,\beta) = qp_b(\beta,\alpha)$ i.e. $|\alpha^2| = |\alpha^2-\beta^2| + |\alpha^2| + |\alpha-\beta|^2 = |\beta^2|$, then $\alpha = \beta$, which shows (QPb_1) is true.

Now, $qp_b(\alpha,\alpha) = |\alpha^2| \leq |\alpha^2-\beta^2| + |\alpha^2| + |\alpha-\beta|^2$ i.e. $qp_b(\alpha,\alpha) \leq qp_b(\alpha,\beta)$, which proves (QPb_2).

Since

$$\begin{aligned}\left|\alpha^2\right| - \left|\beta^2\right| &\leq \left\||\alpha^2| - |\beta^2|\right\| \\ &\leq \left|\alpha^2 - \beta^2\right| \\ &\leq \left|\beta^2 - \alpha^2\right| + |\beta-\alpha|^2,\end{aligned}$$

which proves (QPb_3).

Now we will prove (QPb_4) with $\rho = 2$, that is,

$$qp_b(\alpha,\beta) \leq 2\left[qp_b(\alpha,\gamma) + qp_b(\gamma,\beta)\right] - qp_b(\gamma,\gamma).$$

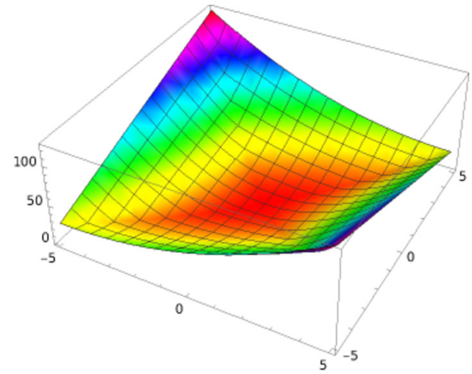

Figure 1 The 3-D representation for $qp_b(\alpha,\beta) = |\alpha^2 - \beta^2| + |\alpha^2| + |\alpha - \beta|^2$

In addition, since $|\alpha - \beta|^2 \leq (|\alpha - \gamma| + |\gamma - \beta|)^2 \leq 2(|\alpha - \gamma|^2 + |\gamma - \beta|^2)$, we have

$$\begin{aligned} qp_b(\alpha,\beta) + qp_b(\gamma,\gamma) &= \left|\alpha^2 - \beta^2\right| + \left|\alpha^2\right| + |\alpha - \beta|^2 \\ &\leq 2\left[\left|\alpha^2 - \gamma^2\right| + \left|\gamma^2 - \beta^2\right| + \left|\alpha^2\right| + \left|\gamma^2\right| + |\alpha - \gamma|^2 + |\gamma - \beta|^2\right]. \end{aligned}$$

Rearranging proves (QPb_4).

Hence, (X, qp_b) is a quasi-partial b-metric space with $\rho = 2$ whose 3-D presentation is given in Fig. 1.

Lemma 2.1 *Let (X, qp_b) be a quasi-partial b-metric space and $\{\alpha_n\}$ be a convergent sequence in X with $\lim_{n\to+\infty} \alpha_n = \alpha$. Then, for all $\beta \in X$ and $\rho \geq 1$, we have*

$$\rho^{-1} qp_b(\alpha,\beta) \leq \lim_{n\to+\infty} \inf qp_b(\alpha_n,\beta) \leq \sup qp_b(\alpha_n,\beta) \leq \rho qp_b(\alpha,\beta).$$

Proof If we apply the triangle inequality (QPB_4) twice, we get for every $n \in N$

$$\rho^{-1} qp_b(\alpha,\beta) - qp_b(\alpha_n,\alpha) \leq qp_b(\alpha_n,\beta) \leq \rho qp_b(\alpha_n,\alpha).$$

If we take lim inf on the left-hand side inequality and lim sup on the right-hand side inequality, we obtain the desired property. □

Wardowski [8] introduced a new concept of contraction that generalizes Banach contraction principle as follows.

Definition 2.2 ([8]) Let (X, d) be a metric space, and there exists a mapping $F: (0,+\infty) \to \mathbb{R}$ which satisfies the following conditions:

(F_1) F is strictly increasing.

(F_2) For any sequence $\{x_n\}_{n\in N}$, $\lim_{n\to+\infty} x_n = 0$ if and only if $\lim_{n\to+\infty} F(x_n) = -\infty$.

(F_3) $\lim_{x\to 0^+} x^k F(x) = 0$ for some $k \in (0,1)$.

Then a mapping $P: X \to X$ is said to be Wardowski F-contraction if $d(P_\alpha, P_\beta) > 0$ implies

$$\delta + F\big(d(P_\alpha, P_\beta)\big) \leq F\big(d(\alpha,\beta)\big)$$

for all $\alpha, \beta \in X$.

In 2015, Casentino et al. [31] extended the concept of F-contraction in metric space to F-contraction in b-metric space by introducing the following condition:

(F_4) For any sequence $\{x_n\}_{n\in N}$,

$$\delta + F(sx_n) \le F(x_{n-1})$$

for all $s \in \mathbb{R}$ and $n \in N$ and for some $\delta > 0$,

$$\delta + F\left(s^n x_n\right) \le F\left(s^{n-1} x_{n-1}\right)$$

for all $n \in N$.

Here, we have extended the concept of F-contraction in b-metric space to F-contraction in quasi-partial b-metric space.

Definition 2.3 For a quasi-partial b-metric space (X, qp_b), a mapping $P\colon X \to X$ is said to be an F-contractive type mapping if there exists $\delta > 0$ such that if $qp_b(\alpha, P_\alpha)qp_b(\beta, P_\beta) \neq 0$, then

$$\begin{aligned}\delta + F\left(\rho qp_b(P_\alpha, P_\beta)\right) \le{}& \frac{1}{3}\left[F\left(qp_b(\alpha, \beta)\right) + F\left(qp_b(\alpha, P_\alpha)\right) + F\left(qp_b(\beta, P_\beta)\right)\right] \\ & - F\left(qp_b(\gamma, P_\gamma)\right),\end{aligned}$$

and if $qp_b(\alpha, P_\alpha)qp_b(\beta, P_\beta) = 0$, then

$$\begin{aligned}\delta + F\left(\rho qp_b(P_\alpha, P_\beta)\right) \le{}& \frac{1}{3}\left[F\left(qp_b(\alpha, \beta)\right) + F\left(qp_b(\alpha, P_\beta)\right) + F\left(qp_b(\beta, P_\alpha)\right)\right] \\ & - F\left(qp_b(\gamma, P_\gamma)\right)\end{aligned}$$

for all $\alpha, \beta, \gamma \in X$ and $\rho \ge 1$.

We now justify Definition 2.3 by illustrating it with the following example.

Example 2.2 Let $X = [0, +\infty)$. Define a quasi-partial b-metric space

$$qp_b(\alpha, \beta) = \begin{cases} |\alpha - \beta| + \alpha, & \alpha \neq \beta, \\ 0, & \alpha = \beta, \end{cases}$$

and there exists an F-contractive function $F(\alpha) = \log \alpha$ for which conditions (F_1) and (F_2) reduce to the following:

$qp_b(\alpha, P_\alpha)qp_b(\beta, P_\beta) \neq 0$ implies

$$\rho^3 qp_b(P_\alpha, P_\beta)^3 qp_b(\gamma, P_\gamma) \le e^{-3\delta} qp_b(\alpha, \beta) qp_b(\alpha, P_\alpha) qp_b(\beta, P_\beta)$$

and $qp_b(\alpha, P_\alpha)qp_b(\beta, P_\beta) = 0$ implies

$$\rho^3 qp_b(P_\alpha, P_\beta)^3 qp_b(\gamma, P_\gamma) \le e^{-3\delta} qp_b(\alpha, \beta) qp_b(\alpha, P_\beta) qp_b(\beta, P_\alpha)$$

for all $\alpha, \beta, \gamma \in X$ and $\rho \geq 1$. Also, we define a mapping $P\colon X \to X$ by

$$P(\alpha) = \begin{cases} 1, & 0 \leq \alpha \leq 2, \\ \frac{1}{\alpha}, & \alpha > 2, \end{cases}$$

which is discontinuous at $\alpha = 2$.

Case I For $\alpha, \beta \in [0,2]$ with $\alpha \neq \beta$, (F_2) holds and $qp_b(P_\alpha, P_\beta) = 0$.

Case II For $\alpha \neq \beta, \alpha, \beta > 2$,

$$qp_b(P_\alpha, P_\beta)^3 = \left(\left|\frac{1}{\alpha} - \frac{1}{\beta}\right| + \frac{1}{\alpha}\right)^3 < 1,$$

and

$$qp_b(\alpha,\beta)qp_b(\alpha,P_\alpha)qp_b(\beta,P_\beta) = \left(|\alpha-\beta| + \alpha\right)\left(\left|\alpha - \frac{1}{\alpha}\right| + \alpha\right)\left(\left|\beta - \frac{1}{\beta}\right| + \beta\right) > 8.$$

Case III For $1 \neq \alpha \in [0,2]$ and $\beta > 2$,

$$qp_b(P_\alpha, P_\beta)^3 = \left(\left|1 - \frac{1}{\beta}\right| + 1\right)^3 \leq \left(\left|1 - \frac{1}{2}\right| + 1\right)^3 = \frac{27}{8},$$

and

$$\begin{aligned} qp_b(\alpha,\beta)qp_b(\alpha,P_\alpha)qp_b(\beta,P_\beta) &= \left(|\alpha-\beta| + \alpha\right)\left(|\alpha - 1| + \alpha\right)\left(\left|\beta - \frac{1}{\beta}\right| + \beta\right) \\ &\geq 2 \times 1 \times 2 = 4. \end{aligned}$$

Thus, if $qp_b(\alpha, P\alpha)qp_b(\beta, P\beta) \neq 0$, the condition holds for $e^{-3\delta} = \frac{1}{8}$ or $\delta = \frac{3\ln 2}{3} = \ln 2$.

Case IV If $\alpha > 2$,

$$qp_b(P_\alpha, P_1)^3 = \left(\left|\frac{1}{\alpha} - 1\right| + \frac{1}{\alpha}\right)^3 \leq \frac{27}{8},$$

and

$$qp_b(\alpha,1)qp_b(\alpha,P_1)qp_b(1,P_\alpha) = \left(|\alpha - 1| + \alpha\right)^2\left(\left|1 - \frac{1}{\alpha}\right| + 1\right) > 3^2 \times 1 = 9.$$

Thus, the condition holds for $e^{-3\delta} = \frac{1}{2}$ or $\delta = \frac{\ln 2}{3}$.

On taking different intervals for α, β, we have four cases with different inequalities on solving, which gives different values but satisfies the conditions to be an F-contractive mapping.

Hence, P is an F-contractive type mapping that is not continuous.

3 Main results

Here, we have given a fixed point result for an F-contractive type mapping in quasi-partial b-metric space for which the space need not be continuous.

Definition 3.1 In a quasi-partial b-metric space (X, qp_b), a self-map $P: X \to X$ is said to be a Picard operator if it has a unique point $\gamma \in X$ and the Picard iteration $\{\alpha\}_{n=0}^{+\infty}$ defined by $\alpha_{n+1} = P_{\alpha_n}, n = 0, 1, 2, \ldots$, converges to γ for any $\alpha_0 \in X$.

Theorem 3.1 *Let (X, qp_b) be a quasi-partial b-metric space and $P: X \to X$ be an F-contractive type mapping. Then P is a Picard operator.*

Proof Let $\alpha_0 \in X$ be an arbitrary point and consider a sequence $\{\alpha_n\}$, where $\alpha_n = P\alpha_n, n \in N$. Since P is an F-contractive type mapping, we have

$$\begin{aligned} \delta + F(\rho u_n) \leq {} & \frac{1}{3}\big[F\big(qp_b(\alpha_{n-1}, \alpha_n)\big) + F\big(qp_b(\alpha_{n-1}, \alpha_n)\big) + F\big(qp_b(\alpha_n, \alpha_{n+1})\big)\big] \\ & - F\big(qp_b(\alpha_{n-1}, \alpha_{n+1})\big). \end{aligned} \tag{3.1}$$

Denoting $qp_b(\alpha_n, \alpha_n)$ by u_n in equation (3.1) for all $\alpha_n > 0$ and $n \in N$, we get

$$F(\rho u_n) \leq F(u_{n-1}) - \frac{3}{2}\delta - \frac{3}{2}F(u_n).$$

By condition (F_4),

$$F\big(\rho^n u_n\big) \leq F\big(\rho^{n-1} u_{n-1}\big) - \frac{3}{2}\delta - \frac{3}{2}F\big(\rho^n u_n\big),$$

and by induction,

$$F\big(\rho^n u_n\big) \leq F\big(\rho^{n-1} u_{n-1}\big) - \frac{3}{2}\delta - \frac{3}{2}F\big(\rho^n u_n\big) \leq F(u_0) - \frac{3}{2}n - \frac{3}{2}F(u). \tag{3.2}$$

Letting $n \to +\infty$, we get

$$\lim_{n \to +\infty} F\big(\rho^n u_n\big) = -\infty$$

so that

$$\lim_{n \to +\infty} \rho^n u_n = 0.$$

From condition (F_2), there exists $k \in (0, 1)$ such that

$$\lim_{n \to +\infty} \big(\rho^n u_n\big)^k F\big(\rho^n u_n\big) = 0.$$

Multiplying equation (3.2) by $(\rho^n u_n)^k$ gives

$$0 \leq \big(\rho^n u_n\big)^k F\big(\rho^n u_n\big) + \frac{3}{2}\big(\rho^n u_n\big)^k \big(n + F(u)\big) \leq F(u_0)\big(\rho^n u_n\big)^k.$$

Taking limit as $n \to +\infty$, we have

$$\lim_{n \to +\infty} n\big(\rho^n u_n\big)^k = 0.$$

Also, we shall show that $\{\alpha_n\}$ is a Cauchy sequence as there exists $\tau \in X$ such that

$$\lim_{n\to+\infty} \alpha_n = \tau.$$

Applying Lemma 2.1, we get

$$\lim_{n\to+\infty} qp_b(\tau,\alpha_n) = \lim_{n\to+\infty} \sup qp_b(\tau,\alpha_n) \leq \rho\big(qp_b(\tau,\tau)\big) = 0.$$

Also, using equation (3.1), we have

$$\begin{aligned}\delta + F\big(\rho qp_b(P_\tau, P_{\alpha_n})\big) \leq \frac{1}{3}\big[&F\big(qp_b(\tau,\alpha_n)\big) + F\big(qp_b(\tau,P_\tau)\big) + F\big(qp_b(\alpha_n,\alpha_{n+1})\big)\big] \\ &- F(qp_b(\alpha_{n-1},\alpha_{n+1}).\end{aligned}$$

As $n \to +\infty$, we conclude that

$$\delta + \lim_{n\to+\infty} F(\rho qp_b(P_\tau, P_{\alpha_n}) \leq -\infty.$$

That is,

$$\lim_{n\to+\infty} qp_b(P_\tau,\alpha_{n+1}) = \lim_{n\to+\infty} qp_b(P_\tau, P_{\alpha_n}) = 0.$$

Therefore, $\{\alpha_n\}$ converges to both τ and P_τ. This implies

$$P_\tau = \tau.$$

To show uniqueness, let if possible τ' be another fixed point of $P(\tau \neq \tau')$. Then

$$\begin{aligned}\delta + F\big(\rho qp_b(P_\tau, P_\tau')\big) \leq \frac{1}{3}\big[&F\big(qp_b(\tau,\tau')\big) + F\big(qp_b(\tau,P_\tau')\big) + F\big(qp_b(P_\tau,\tau')\big)\big] \\ &- F\big(qp_b(P_\tau,P_\tau')\big)\end{aligned}$$

or

$$F\big(\rho qp_b(\tau,\tau')\big) \leq F\big(qp_b(\tau,\tau')\big),$$

which is a contradiction for equation (3.1), and hence P has a unique fixed point that is τ. □

The following example illustrates the above result.

Example 3.1 Let $X = [0,3] \cup [6,+\infty)$ with

$$qp_b(\alpha,\beta) = \begin{cases} \min\{\alpha-\beta, 6\}, & \alpha \neq \beta, \\ 0, & \alpha = \beta. \end{cases}$$

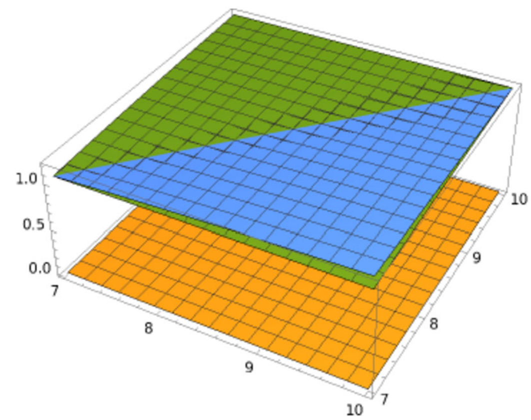

Figure 2 The plane in blue colour denotes the left-hand side of the inequality and the plane in orange colour denotes the right-hand side of the inequality. Hence right-hand side of the inequality is dominant for $\alpha, \beta \geq 6$

Also, we define a mapping $P: X \to X$ by

$$P(\alpha) = \begin{cases} \frac{3}{2}, & 0 \leq \alpha \leq 3, \\ 0, & \alpha = 3, \\ \frac{3}{2} - \frac{3}{\alpha}, & \alpha \geq 6. \end{cases}$$

Here, P is discontinuous at $\alpha = 3$.

Case I For $\alpha, \beta \geq 6$ and $\alpha \neq \beta$, we have

$$qp_b(P_\alpha, P_\beta) = \min\left\{\frac{3}{\beta} - \frac{3}{\alpha}, 6\right\}^3 = \left(\frac{3}{\beta} - \frac{3}{\alpha}\right)^3 < 1$$

and

$$qp_b(\alpha, \beta) qp_b(\alpha, P_\alpha) qp_b(\beta, P_\beta) = \min\left\{\alpha + \frac{3}{2} - \frac{3}{\alpha}, 6\right\} \min\left\{\beta + \frac{3}{2} - \frac{3}{\beta}, 6\right\} = 36.$$

The right-hand side of the inequality is dominant, as shown in Fig. 2.

Case II If $\alpha \in [0, 3)$ and $\beta \geq 6$,

$$qp_b(P_\alpha, P_\beta) = qp_b\left(\frac{3}{2}, \frac{3}{2} - \frac{3}{\alpha}\right)^3 = \min\left\{3 - \frac{3}{\beta}, 6\right\}^3 = \left(3 - \frac{3}{\beta}\right)^3 < 3$$

and

$$qp_b(\alpha, \beta) qp_b(\alpha, P_\alpha) qp_b(\beta, P_\beta) = \min\{\alpha - \beta, 6\} \min\left\{\alpha + \frac{3}{2}, 6\right\} \min\left\{\beta + \frac{3}{2} - \frac{3}{\beta}, 6\right\}$$
$$= 1 \times \left(\alpha + \frac{3}{2}\right) \times 6 \geq 6.$$

Clearly, the right-hand side of the inequality is dominant, as shown in Fig. 3.

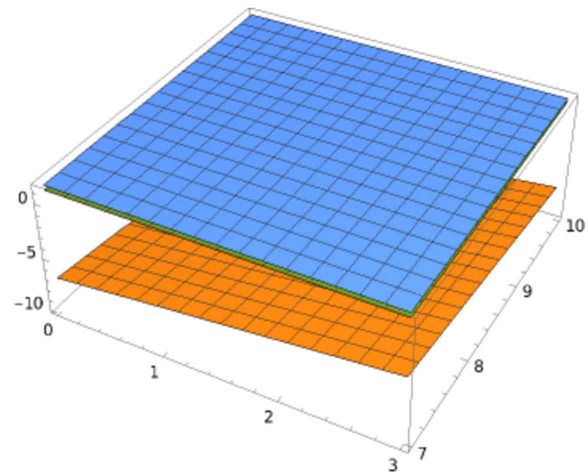

Figure 3 The plane in blue colour denotes the left-hand side of the inequality and the plane in orange colour denotes the right-hand side of the inequality. Hence the right-hand side of the inequality is dominant for $\alpha \in [0,3), \beta \geq 6$

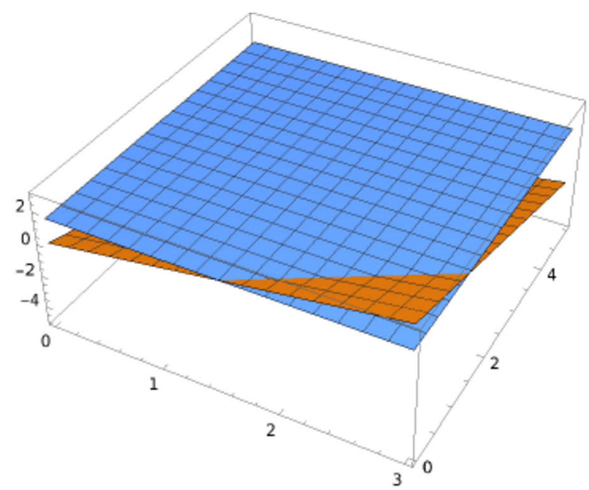

Figure 4 The plane in blue colour denotes the left-hand side of the inequality and the plane in orange colour denotes the right-hand side of the inequality. Hence the left-hand side of the inequality is dominant for $\alpha \in (0,3)$

Case III For $\alpha \in (0,3)$,

$$qp_b(P_\alpha, P_3)^3 = qp_b\left(\frac{3}{2}, 0\right)^3 = \min\left\{\frac{3}{2}, 6\right\}^3 = \frac{27}{8}$$

and

$$qp_b(\alpha,3)qp_b(\alpha,P_\alpha)qp_b(3,P_3) = \min\{\alpha-3,6\}\min\left\{\alpha-\frac{3}{2},6\right\}\min\{3,6\} \geq \frac{3}{2}.$$

Clearly, the left-hand side of the inequality is dominant, as shown in Fig. 4.

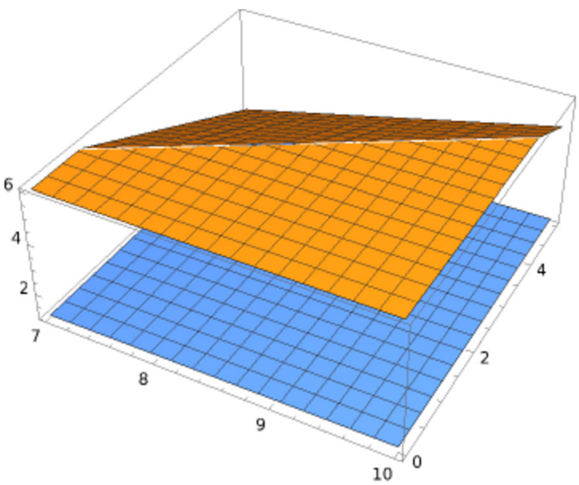

Figure 5 The plane in blue colour denotes the left-hand side of the inequality and the plane in orange colour denotes the right-hand side of the inequality. Hence the left-hand side of the inequality is dominant for $\alpha \geq 6$

Case IV For $\alpha \geq 6$,

$$qp_b(P_\alpha, P_3)^3 = qp_b\left(\frac{3}{2} - \frac{3}{\alpha}, 0\right)^3 = \min\left\{\frac{3}{2} - \frac{3}{\alpha}, 6\right\}^3 \leq \frac{27}{8},$$

and

$$qp_b(\alpha, 3)qp_b(\alpha, P_\alpha)qp_b(3, P_3) = \min\{\alpha - 3, 6\} \min\left\{\alpha + \frac{3}{2} - \frac{3}{\alpha}, 6\right\} \min\{3, 6\} \geq 6.$$

Hence, the left-hand side of the inequality is dominant, as shown in Fig. 5.
Also,

$$qp_b(P_3, P_{\frac{3}{2}})^3 = \min\left\{0 - \frac{3}{2}, 2\right\}^3 = \frac{27}{8},$$

and

$$qp_b\left(3, \frac{3}{2}\right)qp_b(3, P_{\frac{3}{2}})qp_b\left(P_3, \frac{3}{2}\right) \geq \frac{27}{8}.$$

For $\alpha \geq 6$,

$$qp_b(P_\alpha, P_{\frac{3}{2}})^3 = \min\left\{3 + \frac{3}{\alpha}, 2\right\}^3 = 3,$$

and

$$qp_b\left(\alpha, \frac{3}{2}\right)qp_b(\alpha, P_{\frac{3}{2}})qp_b\left(P_\alpha, \frac{3}{2}\right) = \frac{5}{6} \times \frac{5}{6} \times \left(3 - \frac{3}{\alpha}\right) \geq \frac{25}{18}.$$

On taking the different intervals for α, β, we can observe the existence of a fixed point for the mapping P, whose 3-D representation is also given in Figs. 2, 3, 4, 5. When both sides of the inequality have the same value, it gives us the fixed point, which is shown in Case III where the graphs for both sides intersect each other at $\alpha = \frac{3}{2}$.

Thus, P is an F-contractive mapping, and it has a unique fixed point $\alpha = \frac{3}{2}$.

If we take a self-mapping P^n instead of P in Theorem 3.1, we get a corollary.

Corollary 3.1 *Let (X, qp_b) be a quasi-partial b-metric space and $P\colon X \to X$ be a self-mapping such that for some $\delta > 0, qp_b(\alpha, P_\alpha)qp_b(\beta, P_\beta) \neq 0$ implies*

$$\begin{aligned}\delta + F\big(\rho qp_b\big(P^n_\alpha, P^n_\beta\big)\big) \leq{}& \frac{1}{3}\big[F\big(qp_b(\alpha,\beta)\big) + F\big(qp_b\big(\alpha, P^n_\alpha\big)\big) + F\big(qp_b\big(\beta, P^n_\beta\big)\big)\big] \\ &- F\big(qp_b\big(\gamma, P^n_\gamma\big)\big)\end{aligned}$$

and $qp_b(\alpha, P_\alpha)qp_b(\beta, P_\beta) = 0$ implies

$$\begin{aligned}\delta + F\big(\rho qp_b\big(P^n_\alpha, P^n_\beta\big)\big) \leq{}& \frac{1}{3}\big[F\big(qp_b(\alpha,\beta)\big) + F\big(qp_b\big(\alpha, P^n_\beta\big)\big) + F\big(qp_b\big(\beta, P^n_\alpha\big)\big)\big] \\ &- F\big(qp_b\big(\gamma, P^n_\gamma\big)\big),\end{aligned}$$

where n is a positive integer. Then P has a fixed point.

Proof Applying Theorem 3.1 to the self-mapping $S = P^n$, we can say that S has a unique fixed point say τ such that

$$P^n_\tau = S\tau = \tau.$$

Since $P^{n+1}_\tau = P_\tau$, we have

$$SP_\tau = P^n(P_\tau) = P^{n+1}_\tau = P_\tau.$$

This implies that P_τ is a fixed point of S, and by the uniqueness of fixed point we get

$$P_\tau = \tau.$$ □

3.1 Kannan F-contractive type mapping

The concept of F-contraction was extended by many researchers [32–35]. Following this direction, Kannan F-contractive type mapping is taken into consideration to prove the fixed point results.

Definition 3.2 Let (X, qp_b) be a quasi-partial b-metric space. A mapping $T\colon X \to X$ is said to be Kannan F-contractive type mapping if there exists $\delta > 0$ such that $qp_b(\alpha, T_\alpha)qp_b(\beta, T_\beta) \neq 0$ implies

$$\delta + F\big(\rho qp_b(T_\alpha, T_\beta)\big) \leq \frac{1}{2}\big[F\big(qp_b(\alpha, T_\alpha)\big) + F\big(qp_b(\beta, T_\beta)\big)\big] - F\big(qp_b(\gamma, T_\gamma)\big)$$

and $qp_b(\alpha, T_\alpha)qp_b(\beta, T_\beta) = 0$ implies

$$\delta + F\big(\rho qp_b(T_\alpha, T_\beta)\big) \leq \frac{1}{2}\big[F\big(qp_b(\alpha, T_\beta)\big) + F\big(qp_b(\beta, T_\alpha)\big)\big] - F\big(qp_b(\gamma, T_\gamma)\big)$$

for all $\alpha, \beta, \gamma \in X$ and $\rho \geq 1$.

Example 3.2 The functions defined in Example 3.1 are also Kannan F-contractive type mappings.

Theorem 3.2 *Let $T\colon X\to X$ be a Kannan F-contractive type mapping in (X,qp_b). Then T is a Picard operator.*

Proof This can also be proved as in Theorem 3.1. □

In 2017, Garai et al. [36] introduced boundedly compact metric spaces as follows.

Definition 3.3 A metric space (X,d) is said to be boundedly compact if every bounded sequence in X has a convergent subsequence.

Definition 3.3 may be extended to quasi-partial b-metric spaces as well.

Theorem 3.3 *Let (X,qp_b) be a boundedly compact quasi-partial b-metric space and $T\colon X\to X$ be a Kannan F-contractive type mapping. Then T is a Picard operator.*

Proof Let α_0 be arbitrary in X and $\{\alpha_n\}$ be a sequence such that $\alpha_n = T^n_{\alpha_0}$ for every $n\in N$. Denoting $qp_b(\alpha_n,\alpha_{n+1})$ by x_n, we have

$$\begin{aligned}\delta + F(\rho x_n) &= F\bigl(\rho qp_b\bigl(T^n_{\alpha_0},T^{n+1}_{\alpha_0}\bigr)\bigr) - F\bigl(\rho qp_b\bigl(T^{n-1}_{\alpha_0},T^{n+1}_{\alpha_0}\bigr)\bigr)\\ &= F\bigl(\rho qp_b\bigl(T\bigl(T^{n-1}_{\alpha_0}\bigr),T\bigl(T^n_{\alpha_0}\bigr)\bigr)\bigr) - F\bigl(\rho qp_b\bigl(T\bigl(T^{n-2}_{\alpha_0}\bigr),T\bigl(T^n_{\alpha_0}\bigr)\bigr)\bigr)\\ &\le \frac{1}{2}\bigl[F\bigl(qp_b\bigl(T^{n-1}_{\alpha_0},T^n_{\alpha_0}\bigr)\bigr) + F\bigl(qp_b\bigl(T^n_{\alpha_0},T^{n+1}_{\alpha_0}\bigr)\bigr)\bigr] - F\bigl(qp_b\bigl(T^{n-2}_{\alpha_0},T^n_{\alpha_0}\bigr)\bigr)\\ &= \frac{1}{2}\bigl[F(x_{n-1}) + F(x_n)\bigr] - F(x_{n-2})\\ &\le \frac{1}{2}F(\rho x_{n-1}) + \frac{1}{2}F(\rho x_n) - F(x_{n-2}).\end{aligned}$$

This implies

$$\delta' + F(\rho x_n) \le F(x_{n-1}).$$

The remaining proof can be done following the same steps as in Theorem 3.1. □

Definition 3.4 For a quasi-partial b-metric space (X,qp_b), a mapping $T\colon X\to X$ is called asymptotically regular if

$$\lim_{n\to+\infty} qp_b\bigl(T^n_\alpha,T^{n+1}_\alpha\bigr) = 0$$

for all $\alpha\in X$.

Theorem 3.4 *Let (X,qp_b) be a quasi-partial b-metric space and $T\colon X\to X$ be an asymptotically regular mapping such that, for some $\delta>0$, $qp_b(\alpha,T_\alpha)qp_b(\beta,T_\beta)\neq 0$ implies*

$$\delta + F\bigl(\rho qp_b(T_\alpha,T_\beta)\bigr) \le F\bigl(qp_b(\alpha,T_\alpha)\bigr) + F\bigl(qp_b(\beta,T_\beta)\bigr) - F\bigl(qp_b(\gamma,T_\gamma)\bigr) \tag{3.3}$$

and $qp_b(\alpha, T_\alpha)qp_b(\beta, T_\beta) = 0$ *implies*

$$\delta + F\big(\rho qp_b(T_\alpha, T_\beta)\big) \leq F\big(qp_b(\alpha, T_\beta)\big) + F\big(qp_b(\beta, T_\alpha)\big) - F\big(qp_b(\gamma, T_\gamma)\big) \tag{3.4}$$

for all $\alpha, \beta, \gamma \in X$. *Then* T *has a fixed point* $\tau \in X$.

Proof Let α_0 be an arbitrary point in X and $\{\alpha_n\}$ be a sequence such that $\alpha_n = T^n_{\alpha_0}$ for every $n \in N$. Since T is asymptotically regular, we have

$$\lim_{n\to+\infty} x_n = 0,$$

where $x_n = qp_b(\alpha_n, \alpha_{n+1})$. Since $T_{\alpha_n} \neq \alpha_n$ for all $n < m \in N$, we have

$$\begin{aligned}\delta + F\big(\rho qp_b(\alpha_{n+1}, \alpha_{m+1})\big) &\leq F\big(qp_b\big(T^n_{\alpha_0}, T^{n+1}_{\alpha_0}\big)\big) + F\big(qp_b\big(T^m_{\alpha_0}, T^{m+1}_{\alpha_0}\big)\big) - F\big(qp_b\big(T^n_{\alpha_0}, T^m_{\alpha_0}\big)\big)\\ &= F(x_n) + F(x_m) - F\big(qp_b\big(T^n_{\alpha_0}, T^m_{\alpha_0}\big)\big).\end{aligned}$$

Letting $n \to +\infty$, we get

$$\lim_{n\to+\infty} F\big(\rho qp_b(\alpha_{n+1}, \alpha_{m+1})\big) = \pm\infty$$

or

$$\lim_{n\to+\infty} \rho qp_b(\alpha_{n+1}, \alpha_{m+1}) = 0.$$

Since X is complete, there exists $\tau \in X$ such that

$$\lim_{n\to+\infty} \alpha_n = \tau.$$

By Lemma 2.1, we get

$$\lim_{n\to+\infty} qp_b(\alpha_n, \tau) = 0.$$

From equation (3.3) and equation (3.4),

$$\delta + F\big(\rho qp_b(T_\tau, T_{\alpha_n})\big) \leq F\big(qp_b(\tau, T_\tau)\big) + F\big(qp_b(x_n, T_{x_n})\big) - F\big(qp_b(T_\tau, T_{\alpha_n})\big).$$

As $n \to +\infty$, we get

$$\delta + \lim_{n\to+\infty} F\big(\rho qp_b(T_\tau, T_{\alpha_n})\big) \leq -\infty,$$

or

$$\lim_{n\to+\infty} qp_b(T_\tau, \alpha_{n+1}) = 0.$$

Since $\{\alpha_n\}$ converges to both τ and T_τ, we have

$$T_\tau = \tau.$$

Hence, τ is the fixed point of T. □

3.2 F-expanding type mapping

In 2017, Górnicki [18] defined F-expanding mappings and proved the fixed point theorem. Following this direction, we have defined a new type of mapping, which is F-expanding type mapping, and proved the fixed point result in a quasi-partial b-metric space.

Definition 3.5 Let us consider a mapping $Q\colon X \to X$, it is said to be an F-expanding type mapping if there exists $\Delta > 0$ such that $qp_b(\alpha, Q_\alpha)qp_b(\beta, Q_\beta) \neq 0$ implies

$$\begin{aligned}\Delta + F\bigl(\rho qp_b(\alpha,\beta)\bigr) \leq{}& \frac{1}{3}\bigl[F\bigl(qp_b(Q_\alpha,Q_\beta)\bigr) + F\bigl(qp_b(\alpha,Q_\alpha)\bigr) + F\bigl(qp_b(Q_\beta,Q_\beta)\bigr)\bigr] \\ &- F\bigl(qp_b(Q_\gamma,Q_\gamma)\bigr)\end{aligned} \tag{3.5}$$

and $qp_b(\alpha, Q_\alpha)qp_b(\beta, Q_\beta) = 0$ implies

$$\begin{aligned}\Delta + F\bigl(\rho qp_b(\alpha,\beta)\bigr) \leq{}& \frac{1}{3}\bigl[F\bigl(qp_b(Q_\alpha,Q_\beta)\bigr) + F\bigl(qp_b(\alpha,Q_\beta)\bigr) + F\bigl(qp_b(Q_\beta,Q_\alpha)\bigr)\bigr] \\ &- F\bigl(qp_b(Q_\gamma,Q_\gamma)\bigr)\end{aligned} \tag{3.6}$$

for all $\alpha, \beta, \gamma \in X$.

Lemma 3.1 *Let (X, qp_b) be a quasi-partial b-metric space and $Q\colon X \to X$ be surjective. Then there exists a mapping $Q^*\colon X \to X$ such that $Q \circ Q^*$ is the identity map on X.*

Proof For any point $\alpha \in X$, let $\beta_\alpha \in X$ be any point such that $P_{\beta_\alpha} = \alpha$. Let $P^*\alpha = \beta_\alpha$ for all $\alpha \in X$. Then $(P \circ P^*)(\alpha) = P(P^*\alpha) = P_{\beta_\alpha} = \alpha$ for all $\alpha \in X$. □

Theorem 3.5 *Let (X, qp_b) be a quasi-partial b-metric space and $Q\colon X \to X$ be a surjective and F-expanding type mapping. Then Q has a unique fixed point $\gamma \in X$.*

Proof By Lemma 3.1, there exists $Q^*\colon X \to X$ such that $Q \circ Q^*$ is an identity map on X. Let α and β be arbitrary points on X such that $\alpha \neq \beta$ and $\phi = Q^*_\alpha$ and $\psi = Q^*_\beta$. Clearly, $\phi = \psi$.

Using equation (3.5), $qp_b(\phi, Q_\phi)qp_b(\psi, Q_\psi) \neq 0$ implies

$$\begin{aligned}\delta + F\bigl(\rho qp_b(\phi,\psi)\bigr) \leq{}& \frac{1}{3}\bigl[F\bigl(qp_b(Q_\phi,Q_\psi)\bigr) + F\bigl(qp_b(\phi,Q_\phi)\bigr) + F\bigl(qp_b(\psi,Q_\psi)\bigr)\bigr] \\ &- F\bigl(qp_b(\gamma,Q_\gamma)\bigr)\end{aligned}$$

and $qp_b(\phi, Q_\phi)qp_b(\psi, Q_\psi) = 0$ implies

$$\begin{aligned}\delta + F\bigl(\rho qp_b(\phi,\psi)\bigr) \leq{}& \frac{1}{3}\bigl[F\bigl(qp_b(Q_\phi,Q_\psi)\bigr) + F\bigl(qp_b(\phi,Q_\psi)\bigr) + F\bigl(qp_b(\psi,Q_\phi)\bigr)\bigr] \\ &- F\bigl(qp_b(\gamma,Q_\gamma)\bigr).\end{aligned}$$

Since $Q_\phi = Q(Q^*(\alpha)) = \alpha$ and $Q_\psi = Q(Q^*(\beta)) = \beta$, we get

$$\begin{aligned}\delta + F\bigl(\rho qp_b\bigl(Q^*_\alpha,Q^*_\beta\bigr)\bigr) \leq{}& \frac{1}{3}\bigl[F\bigl(qp_b(\alpha,\beta)\bigr) + F\bigl(qp_b\bigl(\alpha,Q^*_\alpha\bigr)\bigr) + F\bigl(qp_b\bigl(\beta,Q^*_\beta\bigr)\bigr)\bigr] \\ &- F\bigl(qp_b\bigl(\gamma,Q^*_\gamma\bigr)\bigr)\end{aligned}$$

for $qp_b(\alpha, Q_\alpha)qp_b(\beta, Q_\beta) \neq 0$ and

$$\delta + F(\rho qp_b(Q^*_\alpha, Q^*_\beta)) \leq \frac{1}{3}\left[F(qp_b(\alpha,\beta)) + F(qp_b(\alpha, Q^*_\beta)) + F(qp_b(\beta, Q^*_\alpha))\right] - F(qp_b(\gamma, Q^*_\gamma)),$$

for $qp_b(\alpha, Q_\alpha)qp_b(\beta, Q_\beta) = 0$, showing that Q^* is an F-contractive type mapping. By Theorem 3.1, Q^* has a unique fixed point $\tau \in X$ for every $\alpha_o \in X$, the sequence $\{Q^n_{\alpha_o}\}$ converges to τ. Since τ is also a fixed point of Q and $Q^*_\tau = \tau$, we can also say that

$$Q_\tau = Q(Q^*_\tau) = \tau.$$

Finally, if $\mu = Q_\mu$ is another fixed point, then from equation (3.6)

$$\delta + F(\rho qp_b(\tau,\mu)) \leq \frac{1}{3}\left[F(qp_b(Q_\tau, Q_\mu)) + F(qp_b(\tau, Q_\mu)) + F(qp_b(\mu, Q_\tau))\right] - F(qp_b(\tau,\mu)),$$

which is not possible and hence the fixed point of Q is unique.

In a similar manner, we can define Kannan F-expanding type mapping i.e. $qp_b(\alpha, Q_\alpha) \times qp_b(\beta, Q_\beta) \neq 0$ implies

$$\Delta + F(\rho qp_b(\alpha,\beta)) \leq \frac{1}{2}\left[F(qp_b(\alpha, Q_\beta)) + F(qp_b(\beta, Q_\alpha))\right] - F(qp_b(\gamma, Q_\gamma)),$$

and $qp_b(\alpha, Q_\alpha)qp_b(\beta, Q_\beta) = 0$ implies

$$\Delta + F(\rho qp_b(\alpha,\beta)) \leq \frac{1}{2}\left[F(qp_b(\alpha, Q_\alpha)) + F(qp_b(\beta, Q_\beta))\right] - F(qp_b(\gamma, Q_\gamma)),$$

and prove the fixed point result in Kannan F-expanding mapping. □

4 Application to functional equations

The widest field in which the technique of fixed point theory is used is mathematical optimization. It is well known that dynamic programming provides useful tools for mathematical optimization and computer programming. In this section, we have proved the existence and uniqueness of a solution for a class of functional equations similar to equation (4.1) in a quasi-partial b-metric space.

Assume that U and V are Banach spaces, $W \subset U$ is a state space and $D \subset V$ is a decision space. Also, R is the field of real numbers where $X = B(W)$ denotes the set of all closed and bounded real-valued functions on W, let us consider the following functional equation:

$$\theta(\alpha) = \mathrm{Sup}_{\beta\in D}\{f(\alpha,\beta) + g(\alpha,\beta,\theta(\tau(\alpha,\beta)))\}, \quad \alpha \in W. \tag{4.1}$$

Let $f\colon W \times D \to \mathbb{R}$ and $g\colon W \times D \times \mathbb{R} \to \mathbb{R}$ be bounded functions. $\tau\colon W \times D \to W$ represents the transformation of process, $\theta(\alpha)$ represents the optimal return function with initial state α, and for an arbitrary $h \in B(W)$, we have $\|h\| = \mathrm{Sup}\,|h(\alpha)|$. Also, $(B(W), \|\cdot\|)$

is a Banach space wherein convergence is uniform considering a quasi-partial b-metric space $qp_b: X \times X \to \mathbb{R}^+$ defined by

$$qp_b(\alpha, \beta) = |\beta - \alpha| + |\beta|.$$

Theorem 4.1 *Let $Q, P: B(W) \to B(W)$ be the self-mappings of qp_b, and there exists $\sigma \in [0, 1)$ such that for every $(\alpha, \beta) \in W \times D, h_1, h_2 \in B(W)$ and $\tau \in W$, it satisfies*

1. $|g(\alpha, \beta, h_1\tau(\alpha, \beta)) - g(\alpha, \beta, h_2\tau(\alpha, \beta))| \leq \rho e^{-\tau} M(h_1, h_2)$ *holds, where* $M(h_1, h_2) = \max(qp_b(h_1, h_2), qp_b(\alpha, P(\alpha)), qp_b(\beta, P(\beta)),$

$$\frac{1}{3}\{qp_b(\alpha, P(\beta)) + qp_b(\beta, P(\alpha)) - qp_b(\gamma, P(\gamma))\};$$

2. *f and g are bounded continuous functions,*

then the functional equation

$$Ph(\alpha) = \text{Sup}_{\beta \in D}\{f(\alpha, \beta) + g(\alpha, \beta, Ph(\tau(\alpha, \beta)))\}, \quad \alpha, \beta \in W \tag{4.2}$$

has a unique solution.

Proof Let σ be any arbitrary positive real number and $h_1, h_2 \in B(W)$. For $\alpha \in W$ and $\beta_1, \beta_2 \in D$, we have

$$Q(h_1(\alpha)) < f(\alpha, \beta_1) + g(\alpha, \beta_1, h_1(\tau_1)) + \sigma \tag{4.3}$$

and

$$Q(h_2(\alpha)) < f(\alpha, \beta_2) + g(\alpha, \beta_2, h_2(\tau_2)) + \sigma, \tag{4.4}$$

where $\tau_1 = \tau(\alpha, \beta_1), \tau_2 = \tau(\alpha, \beta_2)$.

From the definition of mapping Q, we have

$$Q(h_1(\alpha)) \geq f(\alpha, \beta_2) + g(\alpha, \beta_2, h_1(\tau_2)), \tag{4.5}$$

and

$$Q(h_2(\alpha)) < f(\alpha, \beta_1) + g(\alpha, \beta_1, h_2(\tau_1)). \tag{4.6}$$

From equations (4.3) and (4.6),

$$\begin{aligned} Q(h_1(\alpha)) - Q(h_2(\alpha)) &< g(\alpha, \beta_1, h_1(\tau_1)) - g(\alpha, \beta_2, h_2(\tau_2)) + \sigma, \\ &\leq |g(\alpha, \beta_1, h_1(\tau_1)) - g(\alpha, \beta_2, h_2(\tau_1))| + \sigma, \\ &\leq \rho e^{-\tau} M(h_1, h_2) + \sigma. \end{aligned}$$

This implies

$$Q(h_1(\alpha)) - Q(h_2(\alpha)) \leq \rho e^{-\tau} M(h_1, h_2) + \sigma. \tag{4.7}$$

Similarly, from equations (4.4) and (4.5),

$$Q\big(h_2(\alpha)\big) - Q\big(h_1(\alpha)\big) \le \rho e^{-\tau} M(h_1, h_2) + \sigma. \tag{4.8}$$

From equations (4.7) and (4.8),

$$\begin{aligned}
&\big|Q\big(h_1(\alpha)\big) - Q\big(h_2(\alpha)\big)\big| < \rho e^{\tau} M(h_1, h_2) + \sigma, \\
&\quad \Rightarrow \quad qp_b\big(Q(h_1), Q(h_2)\big) \le \rho e^{\tau} M(h_1, h_2) + \sigma, \\
&\quad \Rightarrow \quad qp_b\big(Q(h_1), Q(h_2)\big) \le \rho e^{\tau} M(h_1, h_2).
\end{aligned}$$

Taking logarithm on both sides,

$$\log\big(qp_b\big(Q(h_1), Q(h_2)\big)\big) \le \log\big(\rho e^{\tau} M(h_1, h_2)\big).$$

On solving, we get

$$\tau + \log\big(qp_b\big(Q(h_1), Q(h_2)\big)\big) \le \log\big(\rho M(h_1, h_2)\big).$$

We have observed that the function $F \colon \mathbb{R}^+ \to \mathbf{R}$ defined by $F(\alpha) = \log(\alpha)$ for all $\alpha \in W$ deduces that Q is an F-contraction. Since Q is continuous, we have a fixed point $h^* \in B(W)$, which is a bounded solution of functional equation (4.2). □

5 Conclusion

The significant study of this paper established the existence of a common fixed point for a new type of mapping which is a combination of F-contraction as well as Kannan contraction on quasi-partial b-metric space. The fixed point theory has found frequent applications in mathematical optimization and computer programming along with dynamic programming. In this setting, we have discussed the application of fixed point theory in functional equations to find the bounded solution of the equation.

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