Fixed Point Theory and Algorithms for Sciences and Engineering

RESEARCH

Open Access

Solving an integral equation via generalized controlled fuzzy metrics

Gunaseelan Mani[1], Arul Joseph Gnanaprakasam[2], Abdollah Dinmohammadi[3], Vahid Parvaneh[4*] and Babak Mohammadi[5]

*Correspondence:
zam.dalahoo@gmail.com
[4]Department of Mathematics, Gilan-E-Gharb Branch, Islamic Azad University, Gilan-E-Gharb, Iran
Full list of author information is available at the end of the article

Abstract

The purpose of this manuscript is to obtain some fixed point results in generalized controlled fuzzy metric spaces. Our results generalize and extend many of the previous findings in the same approach. Moreover, two examples to support our theorems are obtained. Finally, to examine and strengthen the theoretical results, the existence and uniqueness of the solution to a nonlinear integral equation is studied as a kind of application.

MSC: 54H25; 47H10; 47S40; 54A40

Keywords: Fuzzy metric space; Controlled fuzzy metric space; Generalized controlled fuzzy metric space; Contraction; Fixed point

1 Introduction

The concept of metric space was introduced by Fréchet in 1906 [1], and it plays an important role in mathematics and its applications. Many mathematicians did research on metric spaces because there are many mathematical concepts that can be discussed in this structure.

The Banach contraction principle is one of the famous and useful results in mathematics. In the last 100 years it has been extended in many different directions. The substitution of the metric space by other generalized metric spaces is one normal way to strengthen the Banach contraction principle, which was introduced by Banach [2] in 1922. Many researchers afterward applied this approach to other areas.

Later, Jleli and Samet [3] developed the concept of a generalized metric space in 2015 and proved certain fixed point theorems in it. In 1975, Kramosil and Michalek [4] for the first time introduced the concept of a fuzzy metric space, which can be regarded as a generalization of the statistical (probabilistic) metric space. Clearly, this work provides an important basis for the construction of fixed point theory in fuzzy metric spaces. Then fuzzy metric space was extended by many authors (see [5–10]). Recently, Ashraf et al. [11] established some fuzzy fixed point theorems in generalized fuzzy metric spaces in the year 2020.

In this paper, we introduce the concept of generalized controlled fuzzy metric space, which generalizes the notion of controlled fuzzy metric space, and state various fixed point theorems with an application, as suggested by Jleli and Samet [3] and Elkouch and Marhrani [12].

2 Preliminaries

Now, we begin with some basic concepts of a generalized metric space developed recently by Jleli and Samet [3].

Definition 2.1 ([3]) Let Φ be a nonempty set and $\Gamma : \Phi^2 \to [0,\infty]$ be a given mapping. For every $\sigma \in \Phi$, define the set $\mathcal{C}(\Gamma,\Phi,\sigma) = \{\{\sigma_n\} \subset \Phi : \lim_{n\to\infty} \Gamma(\sigma_n,\sigma) = 0\}$. Then Γ is a generalized metric on Φ^2 if it satisfies the following conditions:

(1) For every $(\sigma,\vartheta) \in \Phi^2$, we have $\Gamma(\sigma,\vartheta) = 0 \implies \sigma = \vartheta$;
(2) For every $(\sigma,\vartheta) \in \Phi^2$, we have $\Gamma(\sigma,\vartheta) = \Gamma(\vartheta,\sigma)$;
(3) There exists $\mathfrak{c} > 0$ such that if $(\sigma,\vartheta) \in \Phi^2$ and $\{\sigma_n\} \in \mathcal{C}(\Gamma,\Phi,\sigma)$, then $\Gamma(\sigma,\vartheta) \le \mathfrak{c} \limsup_{n\to\infty} \Gamma(\sigma_n,\vartheta)$.

Then the pair (Φ,Γ) is a generalized metric space (GMS).

Remark 2.2 ([2]) The above concept is also known as sequential metric spaces. For more details, see [13] and [14].

Remark 2.3 ([3]) If the set $\mathcal{C}(\Gamma,\Phi,\sigma)$ is empty for every $\sigma \in \Phi$, then (Φ,Γ) is a generalized metric space if and only if (1) and (2) are satisfied.

Definition 2.4 ([3]) Let Φ be a nonempty set and $\beta : \Phi^2 \to [1,\infty)$. Let $\Gamma : \Phi^2 \to [0,\infty]$ be a given function. For every $\sigma \in \Phi$, define the set $\mathcal{C}(\Gamma,\Phi,\sigma) = \{\{\sigma_n\} \subset \Phi : \lim_{n\to\infty} \Gamma(\sigma_n,\sigma) = 0\}$. Then Γ is a generalized controlled metric on Φ^2 if it satisfies the following conditions:

(1) For every $(\sigma,\vartheta) \in \Phi^2$, we have $\Gamma(\sigma,\vartheta) = 0 \implies \sigma = \vartheta$;
(2) For every $(\sigma,\vartheta) \in \Phi^2$, we have $\Gamma(\sigma,\vartheta) = \Gamma(\vartheta,\sigma)$;
(3) If $(\sigma,\vartheta) \in \Phi^2$ and $\{\sigma_n\} \in \mathcal{C}(\Gamma,\Phi,\sigma)$, then

$$\Gamma(\sigma,\vartheta) \le \limsup_{n\to\infty} \beta(\sigma_n,\vartheta)\Gamma(\sigma_n,\vartheta).$$

Then the pair (Φ,Γ) is a generalized controlled metric space (GCMS).

Schweizer and Sklar [15] introduced the continuous triangular norm in 1960 as follows.

Definition 2.5 ([15]) A binary operation $\star : [0,1] \times [0,1] \to [0,1]$ is a continuous triangular norm (γ-norm) if the following conditions are satisfied:

(1) $\star$ is commutative, i.e., $\mathfrak{a} \star \mathfrak{b} = \mathfrak{b} \star \mathfrak{a}$ for all $\mathfrak{a}, \mathfrak{b} \in [0,1]$ and $\star$ is associative, i.e., $\mathfrak{a} \star (\mathfrak{b} \star \mathfrak{c}) = (\mathfrak{a} \star \mathfrak{b}) \star \mathfrak{c}$ for all $\mathfrak{a}, \mathfrak{b}, \mathfrak{c} \in [0,1]$;
(2) $\mathfrak{a} \star 1 = \mathfrak{a}$ for all $\mathfrak{a} \in [0,1]$;
(3) $\star$ is continuous; and
(4) $\mathfrak{a} \star \mathfrak{b} \le \mathfrak{c} \star \mathfrak{d}$ whenever $\mathfrak{a} \le \mathfrak{c}$ and $\mathfrak{b} \le \mathfrak{d}$ for all $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d} \in [0,1]$.

With the help of the γ-norm, George and Veeramani [16] extended the concept of fuzzy metric space introduced by Kramosil and Michalek [17] and presented the following formulation in 1994.

Definition 2.6 ([17]) A nonempty set $\varPhi$ together with a fuzzy set $\digamma : \varPhi^2 \times (0,\infty) \to [0,1]$ and a continuous γ-norm $\star$ is said to be a fuzzy metric space (FMS) if $\digamma$ satisfies the following conditions:

(1) $\digamma(\sigma,\vartheta,\gamma) > 0$ for all $(\sigma,\vartheta) \in \varPhi^2$ and for all $\gamma > 0$;
(2) $\digamma(\sigma,\vartheta,\gamma) = 1$ if and only if $\sigma = \vartheta$ for all $(\sigma,\vartheta) \in \varPhi^2$ and for all $\gamma > 0$;
(3) $\digamma(\sigma,\vartheta,\gamma) = \digamma(\vartheta,\sigma,\gamma)$ for all $(\sigma,\vartheta) \in \varPhi^2$ and $\gamma > 0$;
(4) $\digamma(\sigma,\delta,\gamma+s) \geq \digamma(\sigma,\vartheta,\gamma) \star \digamma(\vartheta,\delta,s)$ for all $(\sigma,\vartheta,\delta) \in \varPhi$ and $\gamma, s > 0$;
(5) $\digamma(\sigma,\vartheta,\cdot)$ is continuous for all $(\sigma,\vartheta) \in \varPhi^2$.

Many authors researched fuzzy metric spaces, and some useful fixed point theorems have been established in these spaces. Take, for example, [18–24]. Sezen [9] recently introduced the concept of controlled fuzzy metric spaces and demonstrated some related fixed point results.

Definition 2.7 ([9]) Let $\varPhi$ be a nonempty set and a function $\beta : \varPhi^2 \to [1,\infty)$. Suppose that $\star$ is a continuous γ-norm and $\digamma$ is a fuzzy set on $\varPhi^2 \times (0,\infty)$ satisfying the following conditions:

(1) $\digamma(\sigma,\vartheta,0) = 0$ for all $(\sigma,\vartheta) \in \varPhi^2$;
(2) $\digamma(\sigma,\vartheta,\gamma) = 1$ if and only if $\sigma = \vartheta$ for all $(\sigma,\vartheta) \in \varPhi^2$ and for all $\gamma > 0$;
(3) $\digamma(\sigma,\vartheta,\gamma) = \digamma(\vartheta,\sigma,\gamma)$ for all $(\sigma,\vartheta) \in \varPhi^2$ and for all $\gamma > 0$;
(4) $\digamma(\sigma,\delta,\gamma+s) \geq \digamma(\sigma,\vartheta,\frac{\gamma}{\beta(\sigma,\vartheta)}) \star \digamma(\vartheta,\delta,\frac{s}{\beta(\vartheta,\delta)})$ for all $\sigma,\vartheta,\delta \in \varPhi$ and for all $\gamma, s > 0$;
(5) $\digamma(\sigma,\vartheta,\cdot)$ is continuous for all $(\sigma,\vartheta) \in \varPhi^2$.

Then the triplet $(\varPhi,\digamma,\star)$ is called controlled fuzzy metrics space (CFMS), and $\digamma$ is called a controlled fuzzy metric on $\varPhi^2 \times (0,\infty)$.

3 Main results

We now propose the concept of a generalized controlled fuzzy metric space (GCFMS), as inspired by Jleli and Samet [3].

Definition 3.1 Consider a nonempty set $\varPhi$, a function $\beta : \varPhi^2 \to [1,\infty)$, and a function $\varPi : \varPhi^2 \times [0,\infty) \to [0,1]$. Define a set

$$\mathcal{C}(\varPi,\varPhi,\sigma) = \Bigl\{\{\sigma_n\} \subset \varPhi : \lim_{n\to\infty} \varPi(\sigma_n,\sigma,\gamma) = 1 \text{ for all } \gamma > 0\Bigr\}$$

for every $\sigma \in \varPhi$. Then $\varPi$ is said to be a generalized controlled fuzzy metric if it satisfies the following conditions:

(1) $\varPi(\sigma,\vartheta,\gamma) > 0$ for all $(\sigma,\vartheta) \in \varPhi^2$ and $\gamma > 0$;
(2) $\varPi(\sigma,\vartheta,\gamma) = 1 \implies \sigma = \vartheta$ for all $(\sigma,\vartheta) \in \varPhi^2$ and $\gamma > 0$;
(3) $\varPi(\sigma,\vartheta,\gamma) = \varPi(\vartheta,\sigma,\gamma)$ for all $(\sigma,\vartheta) \in \varPhi^2$ and $\gamma > 0$;
(4) If $\{\sigma_n\} \in \mathcal{C}(\varPi,\varPhi,\sigma)$, then $\varPi(\sigma,\vartheta,\gamma) \geq \lim_{n\to\infty} \sup \varPi(\sigma_n,\vartheta,\frac{\gamma}{\beta(\sigma_n,\vartheta)})$ for all $(\sigma,\vartheta) \in \varPhi^2$ and $\gamma > 0$;
(5) $\varPi(\sigma,\vartheta,\cdot)$ is continuous for all $(\sigma,\vartheta) \in \varPhi^2$, $\lim_{\gamma\to\infty} \varPi(\sigma,\vartheta,\gamma) = 1$.

Then $(\varPi,\varPhi,\star)$ is called GCFMS.

The following example exemplifies the above definition.

Example 3.2 Consider a generalized controlled metrics space (Φ, Γ). Define a mapping $\Pi : \Phi^2 \times [0,\infty) \to [0,1]$ by

$$\Pi(\sigma,\vartheta,\gamma) = e^{-\frac{\Gamma(\sigma,\vartheta)}{\gamma}}, \tag{3.1}$$

and $\mathcal{C}(\Pi,\Phi,\sigma) = \{\{\sigma_n\} \subset \Phi : \lim_{n\to\infty} \Pi(\sigma_n,\sigma,\gamma) = 1\}$ for every $\sigma \in \Phi$ and $\gamma > 0$. Then $(\Pi,\Phi,\star)$ is a GCFMS, where the γ-norm "$\star$" is taken as a product norm, i.e., $\sigma \star \vartheta = \sigma\vartheta$.

Proof We only prove that Π satisfies property (4) of Definition 3.1. Let $(\sigma,\vartheta) \in \Phi^2$ and $\{\sigma_n\} \in \mathcal{C}(\Gamma,\Phi,\sigma)$: Since Γ is a condition (3) of Definition 2.4 and from equation (3.1), it is clear that $\{\sigma_n\}$ also belongs to $\mathcal{C}(\Pi,\Phi,\sigma)$. Then

$$\begin{aligned}
\Pi(\sigma,\vartheta,\gamma) = e^{-\frac{\Gamma(\sigma,\vartheta)}{\gamma}} \\
&\geq e^{-\frac{\beta(\sigma_n,\vartheta)\limsup_{n\to\infty}\Gamma(\sigma_n,\vartheta)}{\gamma}} \\
&= e^{-\frac{\limsup_{n\to\infty}\Gamma(\sigma_n,\vartheta)}{\frac{\gamma}{\beta(\sigma_n,\vartheta)}}} \\
&= \lim_{n\to\infty}\sup e^{-\frac{\Gamma(\sigma_n,\vartheta)}{\frac{\gamma}{\beta(\sigma_n,\vartheta)}}} \\
&= \lim_{n\to\infty}\sup \Pi\left(\sigma_n,\vartheta,\frac{\gamma}{\beta(\sigma_n,\vartheta)}\right).
\end{aligned}$$

Therefore, $\Pi(\sigma,\vartheta,\gamma) \geq \lim_{n\to\infty}\sup \Pi(\sigma_n,\vartheta,\frac{\gamma}{\beta(\sigma_n,\vartheta)})$. □

Proposition 3.3 *The set $\mathcal{C}(\Pi,\Phi,\sigma)$ is nonempty if and only if $\Pi(\sigma,\sigma,\gamma) = 1$.*

Proof If $\mathcal{C}(\Pi,\Phi,\sigma) \neq \emptyset$, then there is a sequence $\{\sigma_n\} \subset \Phi$ such that $\lim_{n\to\infty}\Pi(\sigma_n,\sigma,\gamma) = 1$ for all $\gamma > 0$. By property (4) of Definition 3.1, we obtain

$$\Pi(\sigma,\sigma,\gamma) \geq \lim_{n\to\infty}\sup \Pi\left(\sigma_n,\sigma,\frac{\gamma}{\beta(\sigma_n,\sigma)}\right).$$

Thus, $\Pi(\sigma,\sigma,\gamma) = 1$.

Conversely, if $\Pi(\sigma,\sigma,\gamma) = 1$, then we can take the sequence $\{\sigma_n\} \subset \Phi$ such that for all $n \in \mathbb{N}$, $\sigma_n = \sigma$ and $\lim_{n\to\infty}\Pi(\sigma_n,\sigma,\gamma) = 1$. Hence, $\mathcal{C}(\Pi,\Phi,\sigma) \neq \emptyset$. □

Proposition 3.4 *Every CFMS $(\digamma,\Phi,\star)$ is a GCFMS.*

Proof We only prove that $\digamma$ satisfies property (4) of Definition 3.1. Now, for all $(\sigma,\vartheta) \in \Phi^2$ and $\{\sigma_n\} \in \mathcal{C}(\Pi,\Phi,\sigma)$, by using the triangular inequality,

$$\begin{aligned}
\digamma(\sigma,\vartheta,\gamma) \geq \digamma\left(\sigma_n,\sigma,\frac{\gamma}{\beta(\sigma_n,\sigma_n)}\right) \star \digamma\left(\sigma_n,\vartheta,\frac{\gamma}{\beta(\sigma_n,\vartheta)}\right) \\
&= 1 \star \digamma\left(\sigma_n,\vartheta,\frac{\gamma}{\beta(\sigma_n,\vartheta)}\right) \\
&= \digamma\left(\sigma_n,\vartheta,\frac{\gamma}{\beta(\sigma_n,\vartheta)}\right).
\end{aligned}$$

□

Definition 3.5 Let $(\varPi,\varPhi,\star)$ be a GCFMS. A sequence $\{\sigma_n\}\in\varPhi$ is said to be a $\varPi$-convergent sequence if there exists $\sigma\in\varPhi$ such that $\{\sigma_n\}\in\mathcal{C}(\varPi,\varPhi,\sigma)$.

The notion of a Cauchy sequence in GCFMSs can be expanded as follows, according to Grabiec [25]:

Definition 3.6 Let $(\varPi,\varPhi,\star)$ be a GCFMS. A sequence $\{\sigma_n\}\in\varPhi$ is said to be a $\varPi$-Cauchy sequence if $\lim_{n,\mathrm{m}\to\infty}\varPi(\sigma_n,\sigma_{n+\mathrm{m}},\gamma)=1$ for all $\gamma>0$.

Definition 3.7 A $\varPi$-complete GCFMS is a GCFMS in which every $\varPi$-Cauchy sequence is $\varPi$-convergent.

A $\varPi$-convergent sequence in a GCFMS may not be a $\varPi$-Cauchy sequence, as we will prove now.

Example 3.8 Let $\varPhi=\mathbb{R}^+\cup\{0\}$. Define a set

$$\mathcal{C}(\varPi,\varPhi,\sigma)=\Big\{\{\sigma_n\}\subset\varPhi:\lim_{n\to\infty}\varPi(\sigma_n,\sigma,\gamma)=1\Big\}$$

for every $\sigma\in\varPhi$ and $\gamma>0$, where $\varPi:\varPhi^2\times(0,\infty)\to[0,1]$ is defined by

$$\varPi\left(\sigma,\vartheta,\frac{\gamma}{\beta(\sigma,\vartheta)}\right)=\begin{cases}e^{-\beta(\sigma,\vartheta)\frac{\sigma+\vartheta}{\gamma}} & \text{if either } \sigma=0 \text{ or } \vartheta=0,\\ e^{-\beta(\sigma,\vartheta)\frac{1+\sigma+\vartheta}{\gamma}} & \text{otherwise.}\end{cases}$$

Then $(\varPi,\varPhi,\star)$ is a GCFMS, where $\star$ is the product γ-norm, i.e., $\sigma\star\vartheta=\sigma\vartheta$.

Consider a sequence $\{\sigma_n\}$ as $\sigma_n=\frac{1}{n}$ for all $n\in\mathbb{N}$. So,

$$\lim_{n\to\infty}\varPi\left(\sigma_n,0,\frac{\gamma}{\beta(\sigma_n,0)}\right)=\lim_{n\to\infty}e^{-\beta(\sigma_n,0)\frac{\sigma_n}{\gamma}}=\lim_{n\to\infty}e^{-\frac{\beta(\sigma_n,0)}{n\gamma}}.$$

Therefore, $\{\sigma_n\}\varPi$-converges to 1.

Now,

$$\begin{aligned}\lim_{n,m\to\infty}\varPi\left(\sigma_n,\sigma_{n+m},\frac{\gamma}{\beta(\sigma_n,\sigma_{n+m})}\right)&=\lim_{n,m\to\infty}e^{-\beta(\sigma_n,\sigma_{n+m})\frac{1+\sigma_n+\sigma_{n+m}}{\gamma}}\\&=\lim_{n,m\to\infty}e^{-\beta(\sigma_n,\sigma_{n+m})\frac{1+\frac{1}{n}+\frac{1}{n+m}}{\gamma}}\\&=\lim_{n,m\to\infty}e^{-\beta(\sigma_n,\sigma_{n+m})\frac{1}{\gamma}}e^{-\beta(\sigma_n,\sigma_{n+m})\frac{1}{n\gamma}}e^{-\beta(\sigma_n,\sigma_{n+m})\frac{1}{(n+m)\gamma}}\\&=e^{-\beta(\sigma_n,\sigma_{n+m})\frac{1}{\gamma}}.1.1=e^{-\beta(\sigma_n,\sigma_{n+m})\frac{1}{\gamma}}\neq1.\end{aligned}$$

Hence, $\{\sigma_n\}$ is not a $\varPi$-Cauchy sequence.

Proposition 3.9 *Let $(\varPi,\varPhi,\star)$ be a GCFMS, $\{\sigma_n\}$ be a sequence in $\varPhi$, and $(\sigma,\vartheta)\in\varPhi^2$. If $\{\sigma_n\}\varPi$-converges to σ and $\{\sigma_n\}\varPi$-converges to ϑ, then $\sigma=\vartheta$.*

Proof By property (4) of Definition 3.1, we obtain

$$\Pi(\sigma,\vartheta,\gamma) \geq \lim_{n\to\infty} \sup \Pi\left(\sigma_n,\vartheta,\frac{\gamma}{\beta(\sigma_n,\vartheta)}\right) = 1,$$

so we have $\sigma = \vartheta$. □

Definition 3.10 Let $(\Pi,\Phi,\star)$ be a GCFMS and $\sigma \in \Phi$. A self mapping $\mathfrak{g}:\Phi \to \Phi$ is said to be fuzzy continuous at σ if the Π-convergence of the sequence $\{\sigma_n\} \in \Phi$ to σ implies that the sequence $\{\mathfrak{g}\sigma_n\}$ Π-converges to $\mathfrak{g}\sigma$.

Definition 3.11 Let $(\Pi,\Phi,\star)$ be a GCFMS. A self mapping $\mathfrak{g}:\Phi \to \Phi$ is a *GCFMS*-contraction of type-*I* if for all $(\sigma,\vartheta) \in \Phi^2$ and for some $\mathfrak{k} \in (0,1)$, we have

$$\Pi\big(\mathfrak{g}(\sigma),\mathfrak{g}(\vartheta),\mathfrak{k}\gamma\big) \geq \Pi\big(\sigma,\mathfrak{g}(\sigma),\gamma\big) \quad \text{for all } \gamma > 0.$$

Proposition 3.12 *Let $(\Pi,\Phi,\star)$ be a GCFMS and g be a GCFMS-contraction of type-I. If $\Pi(\mathfrak{v},\mathfrak{v},\gamma) = 1$ for any fixed point $\mathfrak{v}$ of $\mathfrak{g}$, which satisfies $\Pi(\mathfrak{v},\mathfrak{v},\gamma) > 0$.*

Proof Let $\mathfrak{v} \in \Phi$ be a fixed point of $\mathfrak{g}$. Since $\mathfrak{g}$ is a *GCFMS*-contraction of type-*I*, so

$$\begin{aligned}
\Pi(\mathfrak{v},\mathfrak{v},\gamma) &= \Pi\big(\mathfrak{g}(\mathfrak{v}),\mathfrak{g}(\mathfrak{v}),\gamma\big) \\
&\geq \Pi\left(\mathfrak{v},\mathfrak{v},\frac{\gamma}{\mathfrak{k}}\right) \\
&\geq \Pi\left(\mathfrak{v},\mathfrak{v},\frac{\gamma}{\mathfrak{k}^2}\right) \\
&\ \ \vdots \\
&\geq \Pi\left(\mathfrak{v},\mathfrak{v},\frac{\gamma}{\mathfrak{k}^n}\right) \to 1 \quad \text{as } n \to \infty \text{ for all } \mathfrak{k} \in (0,1).
\end{aligned}$$

So, $\Pi(\mathfrak{v},\mathfrak{v},\gamma) = 1$. □

Theorem 3.13 *Let $(\Pi,\Phi,\star)$ be a Π-complete GCFMS and $\mathfrak{g}$ be a GCFMS-contraction of type-I which is fuzzy continuous. If there exists $\mathfrak{a}_0 \in \Phi$ such that $\delta(\Pi,\mathfrak{g},\mathfrak{a}_0,\gamma) > 0$, where*

$$\delta(\Pi,\mathfrak{g},\mathfrak{a}_0,\gamma) = \inf\big\{\Pi\big(\mathfrak{g}^{\mathfrak{i}}(\mathfrak{a}_0),\mathfrak{g}^{\mathfrak{j}}(\mathfrak{a}_0),\gamma\big); \mathfrak{i},\mathfrak{j} \in \mathbb{N}, \gamma > 0\big\},$$

then $\{\mathfrak{g}^n(\mathfrak{a}_0)\}$ converges to a fixed point of $\mathfrak{g}$.

Proof Since $\mathfrak{g}$ is a *GCFMS*-contraction of type-*I*, so, for all $\mathfrak{i},\mathfrak{j} \in \mathbb{N}$, we obtain

$$\Pi\big(\mathfrak{g}^{n+\mathfrak{i}}(\mathfrak{a}_0),\mathfrak{g}^{n+\mathfrak{j}}(\mathfrak{a}_0),\gamma\big) \geq \Pi\left(\mathfrak{g}^{n-1+\mathfrak{i}}(\mathfrak{a}_0),\mathfrak{g}^{n+\mathfrak{i}}(\mathfrak{a}_0),\frac{\gamma}{\mathfrak{k}}\right).$$

Therefore

$$\inf\big\{\Pi\big(\mathfrak{g}^{n+\mathfrak{i}}(\mathfrak{a}_0),\mathfrak{g}^{n+\mathfrak{j}}(\mathfrak{a}_0),\gamma\big)\big\} \geq \inf\left\{\Pi\left(\mathfrak{g}^{n-1+\mathfrak{i}}(\mathfrak{a}_0),\mathfrak{g}^{n+\mathfrak{i}}(\mathfrak{a}_0),\frac{\gamma}{\mathfrak{k}}\right)\right\}.$$

So

$$\delta\big(\Pi,\mathfrak{g},\mathfrak{g}^n(\mathfrak{a}_0),\gamma\big)\ge\delta\left(\Pi,\mathfrak{g},\mathfrak{g}^{n-1}(\mathfrak{a}_0),\frac{\gamma}{\mathfrak{k}}\right).$$

For all $n>0$, we obtain

$$\delta\big(\Pi,\mathfrak{g},\mathfrak{g}^n(\mathfrak{a}_0),\gamma\big)\ge\delta\left(\Pi,\mathfrak{g},\mathfrak{a}_0,\frac{\gamma}{\mathfrak{k}^n}\right).\tag{3.2}$$

For every $n,m\in\mathbb{N}$ such that $m>n$, we use (3.2) to obtain

$$\begin{aligned}\Pi\big(\mathfrak{g}^n(\mathfrak{a}_0),\mathfrak{g}^m(\mathfrak{a}_0),\gamma\big)&\ge\delta\big(\Pi,\mathfrak{g},\mathfrak{g}^n(\mathfrak{a}_0),\gamma\big)\\&\ge\delta\left(\Pi,\mathfrak{g},\mathfrak{a}_0,\frac{\gamma}{\mathfrak{k}^n}\right).\end{aligned}$$

Since $\delta(\Pi,\mathfrak{g},\mathfrak{a}_0,\frac{\gamma}{\mathfrak{k}^n})>0$ and $\mathfrak{k}\in(0,1)$,

$$\lim_{n,m\to\infty}\Pi\big(\mathfrak{g}^n(\mathfrak{a}_0),\mathfrak{g}^m(\mathfrak{a}_0),\gamma\big)=1,$$

which shows that $\{\mathfrak{g}^n(\mathfrak{a}_0)\}$ is a Π-Cauchy sequence. Since $(\Pi,\Phi,\star)$ is a Π-complete GCFMS, there exists a point $\mathfrak{d}\in\Phi$ such that $\{\mathfrak{g}^n(\mathfrak{a}_0)\}$ Π-converges to $\mathfrak{d}$. Since $\mathfrak{g}$ is fuzzy continuous, $\{\mathfrak{g}^{n+1}(\mathfrak{a}_0)\}$ Π-converges to $\mathfrak{g}\mathfrak{d}$. Using Proposition 3.12, we have $\mathfrak{g}(\mathfrak{d})=\mathfrak{d}$. Hence, $\mathfrak{d}$ is a fixed point of $\mathfrak{g}$. □

Example 3.14 Let $\Phi=[0,1]$ and define $\Pi:\Phi^2\times(0,\infty)\to[0,1]$ by

$$\begin{cases}\Pi(\sigma,\vartheta,\gamma)=e^{-\frac{\sigma+\vartheta}{\gamma}} & \text{if } \sigma\ne0 \text{ and } \vartheta\ne0,\\ \Pi(0,\sigma,\gamma)=\Pi(\sigma,0,\gamma)=e^{-\frac{\sigma}{2\gamma}} & \text{for all } \sigma\in\Phi,\end{cases}$$

where $\vartheta\in\Phi$ and $\gamma>0$. It is easy to verify that $(\Pi,\Phi,\star)$ is a Π-complete GCFMS. For a constant $\mathfrak{k}\in(0,1)$, we define $\mathfrak{g}:\Phi\to\Phi$ by

$$\begin{cases}\mathfrak{g}(\sigma)=\frac{\mathfrak{k}\sigma}{n} & \text{if } \sigma\in[0,1),\\ \mathfrak{g}(1)=1,\end{cases}$$

where $n\ge1$. For any $\sigma\in\Phi$, we obtain

$$\begin{aligned}\Pi\big(\mathfrak{g}(\sigma),\mathfrak{g}(0),\mathfrak{k}\gamma\big)&=\Pi\left(\frac{\mathfrak{k}\sigma}{n},0,\mathfrak{k}\gamma\right)\\&=e^{-\frac{\frac{\mathfrak{k}\sigma}{n}}{2\mathfrak{k}\gamma}}\\&=e^{-\frac{\sigma}{2n\gamma}}\\&\ \ \vdots\\&\ge e^{\frac{-(\sigma+\frac{\mathfrak{k}\sigma}{n})}{2\gamma}}\\&=\Pi\big(\sigma,\mathfrak{g}(\sigma),\gamma\big).\end{aligned}$$

For $(\sigma,\vartheta) \in \Phi - \{0\}$ and $\mathfrak{k} \in (0,1)$, we obtain

$$\begin{aligned}
\Pi\big(\mathfrak{g}(\sigma),\mathfrak{g}(\vartheta),\mathfrak{k}\gamma\big) &= \Pi\left(\frac{\mathfrak{k}\sigma}{n},\frac{\mathfrak{k}\vartheta}{n},\mathfrak{k}\gamma\right) \\
&= e^{-\frac{\frac{\mathfrak{k}\sigma}{n}+\frac{\mathfrak{k}\vartheta}{n}}{\mathfrak{k}\gamma}} \\
&= e^{-\frac{\sigma+\vartheta}{n\gamma}} \\
&\ \ \vdots \\
&\geq e^{-\frac{\sigma+\frac{\mathfrak{k}\sigma}{n}}{\gamma}} \\
&= \Pi\big(\sigma,\mathfrak{g}(\sigma),\gamma\big).
\end{aligned}$$

Then

$$\Pi\big(\mathfrak{g}(\sigma),\mathfrak{g}(\vartheta),\mathfrak{k}\gamma\big) \geq \Pi\big(\sigma,\mathfrak{g}(\sigma),\gamma\big) \quad \text{for all } \gamma > 0.$$

Thus, $\mathfrak{g}$ is a *GCFMS*-contraction of type-*I*. Hence, all the conditions of Theorem 3.13 are satisfied. Therefore, $\mathfrak{g}$ has a fixed point.

Definition 3.15 Let $(\Pi,\Phi,\star)$ be a GCFMS. A self mapping $\mathfrak{g}:\Phi\to\Phi$ is a *GCFMS*-contraction of type-*II* if for all $\sigma,\vartheta\in\Phi$ and for some $\mathfrak{k}\in(0,1)$, we have

$$\Pi\big(\mathfrak{g}(\sigma),\mathfrak{g}(\vartheta),\mathfrak{k}\gamma\big) \geq \Pi\big(\sigma,\mathfrak{g}(\vartheta),\gamma\big) \quad \text{for all } \gamma > 0.$$

Theorem 3.16 *Let $(\Pi,\Phi,\star)$ be a Π-complete GCFMS and $\mathfrak{g}$ be a GCFMS-contraction of type-II which is fuzzy continuous. If there exists $\mathfrak{a}_0\in\Phi$ such that $\delta(\Pi,\mathfrak{g},\mathfrak{a}_0,\gamma)>0$, where*

$$\delta(\Pi,\mathfrak{g},\mathfrak{a}_0,\gamma) = \inf\big\{\Pi\big(\mathfrak{g}^{\mathfrak{i}}(\mathfrak{a}_0),\mathfrak{g}^{\mathfrak{j}}(\mathfrak{a}_0),\gamma\big); \mathfrak{i},\mathfrak{j}\in\mathbb{N},\gamma>0\big\},$$

then $\{\mathfrak{g}^n(\mathfrak{a}_0)\}$ converges to a unique fixed point of $\mathfrak{g}$.

Proof Since $\mathfrak{g}$ is a *GCFMS*-contraction of type-*II*, for all $\mathfrak{i},\mathfrak{j}\in\mathbb{N}$, we obtain

$$\Pi\big(\mathfrak{g}^{n+\mathfrak{i}}(\mathfrak{a}_0),\mathfrak{g}^{n+\mathfrak{j}}(\mathfrak{a}_0),\gamma\big) \geq \Pi\left(\mathfrak{g}^{n+\mathfrak{i}-1}(\mathfrak{a}_0),\mathfrak{g}^{n+\mathfrak{j}}(\mathfrak{a}_0),\frac{\gamma}{\mathfrak{k}}\right).$$

Therefore

$$\inf\big\{\Pi\big(\mathfrak{g}^{n+\mathfrak{i}}(\mathfrak{a}_0),\mathfrak{g}^{n+\mathfrak{j}}(\mathfrak{a}_0),\gamma\big)\big\} \geq \inf\left\{\Pi\left(\mathfrak{g}^{n-1+\mathfrak{i}}(\mathfrak{a}_0),\mathfrak{g}^{n+\mathfrak{j}}(\mathfrak{a}_0),\frac{\gamma}{\mathfrak{k}}\right)\right\}.$$

So

$$\delta\big(\Pi,\mathfrak{g},\mathfrak{g}^n(\mathfrak{a}_0),\gamma\big) \geq \delta\left(\Pi,\mathfrak{g},\mathfrak{g}^{n-1}(\mathfrak{a}_0),\frac{\gamma}{\mathfrak{k}}\right).$$

For all $n > 0$, we obtain

$$\delta\big(\Pi, \mathfrak{g}, \mathfrak{g}^n(\mathfrak{a}_0), \gamma\big) \geq \delta\left(\Pi, \mathfrak{g}, \mathfrak{a}_0, \frac{\gamma}{\mathfrak{k}^n}\right). \tag{3.3}$$

For every $n, m \in \mathbb{N}$ such that $m > n$, we use (3.3) to obtain

$$\begin{aligned} \Pi\big(\mathfrak{g}^n(\mathfrak{a}_0), \mathfrak{g}^m(\mathfrak{a}_0), \gamma\big) &\geq \delta\big(\Pi, \mathfrak{g}, \mathfrak{g}^n(\mathfrak{a}_0), \gamma\big) \\ &\geq \delta\left(\Pi, \mathfrak{g}, \mathfrak{a}_0, \frac{\gamma}{\mathfrak{k}^n}\right). \end{aligned}$$

Since $\delta(\Pi, \mathfrak{g}, \mathfrak{a}_0, \frac{\gamma}{\mathfrak{k}^n}) > 0$ and $\mathfrak{k} \in (0,1)$,

$$\lim_{n,m\to\infty} \Pi\big(\mathfrak{g}^n(\mathfrak{a}_0), \mathfrak{g}^m(\mathfrak{a}_0), \gamma\big) = 1,$$

which implies that $\{\mathfrak{g}^n(\mathfrak{a}_0)\}$ is a Π-Cauchy sequence. Since $(\Pi, \Phi, \star)$ is a Π-complete GCFMS, there exists a point $\mathfrak{d} \in \Phi$ such that $\{\mathfrak{g}^n(\mathfrak{a}_0)\}$ Π-converges to $\mathfrak{d}$. Since $\mathfrak{g}$ is fuzzy continuous, $\{\mathfrak{g}^{n+1}(\mathfrak{a}_0)\}$ Π-converges to $\mathfrak{g}\mathfrak{d}$. Using Proposition 3.12, we obtain that $\mathfrak{g}(\mathfrak{d}) = \mathfrak{d}$. Therefore, $\mathfrak{d}$ is a fixed point of $\mathfrak{g}$.

Let $\mathfrak{e} \in \Phi$ be another fixed point of $\mathfrak{g}$ such that $\Pi(\mathfrak{d}, \mathfrak{e}, \gamma) > 0$. Since $\mathfrak{g}$ is a *GCFMS*-contraction of type-*II*, we obtain

$$\begin{aligned} \Pi(\mathfrak{d}, \mathfrak{e}, \gamma) &= \Pi\big(\mathfrak{g}(\mathfrak{d}), \mathfrak{g}(\mathfrak{e}), \gamma\big) \\ &\geq \Pi\left(\mathfrak{d}, \mathfrak{g}(\mathfrak{e}), \frac{\gamma}{\mathfrak{k}}\right) \\ &= \Pi\left(\mathfrak{d}, \mathfrak{e}, \frac{\gamma}{\mathfrak{k}}\right) \\ &\ \ \vdots \\ &\geq \Pi\left(\mathfrak{d}, \mathfrak{e}, \frac{\gamma}{\mathfrak{k}^n}\right) \to 1 \quad \text{as } n \to \infty \text{ for all } \mathfrak{k} \in (0,1). \end{aligned}$$

Thus, $\mathfrak{d} = \mathfrak{e}$. □

Example 3.17 Let $\Phi = [0,1]$ and define $\Pi : \Phi^2 \times (0,\infty) \to [0,1]$ by

$$\begin{cases} \Pi(\sigma, \vartheta, \gamma) = e^{-\frac{\sigma+\vartheta}{\gamma}} & \text{if } \sigma \neq 0 \text{ and } \vartheta \neq 0, \\ \Pi(0, \sigma, \gamma) = \Pi(\sigma, 0, \gamma) = e^{-\frac{\sigma}{2\gamma}} & \text{for all } \sigma \in \Phi, \end{cases}$$

for all $(\sigma, \vartheta) \in \Phi^2$ and $\gamma > 0$. It is easy to verify that $(\Pi, \Phi, \star)$ is a Π-complete GCFMS. For $\mathfrak{k} \in (0,1)$, we define $\mathfrak{g} : \Phi \to \Phi$ by $\mathfrak{g}(\sigma) = \frac{\mathfrak{k}\sigma}{n}$ for all $\sigma \in \Phi$ and for some $n \geq 1$. For any $\sigma \in \Phi$, we have

$$\begin{aligned} \Pi\big(\mathfrak{g}(\sigma), \mathfrak{g}(0), \mathfrak{k}\gamma\big) &= \Pi\left(\frac{\mathfrak{k}\sigma}{n}, 0, \mathfrak{k}\gamma\right) \\ &= e^{-\frac{\frac{\mathfrak{k}\sigma}{n}}{2\mathfrak{k}\gamma}} \end{aligned}$$

$$\begin{aligned}&= e^{-\frac{\sigma}{2n\gamma}}\\&\geq e^{-\frac{\sigma}{2\gamma}}\\&= \Pi\big(\sigma, \mathfrak{g}(0), \gamma\big).\end{aligned}$$

For $(\sigma, \vartheta) \in \Phi - \{0\}$ and for constant $\mathfrak{k} \in (0,1)$, we obtain

$$\begin{aligned}\Pi\big(\mathfrak{g}(\sigma), \mathfrak{g}(\vartheta), \mathfrak{k}\gamma\big) &= \Pi\left(\frac{\mathfrak{k}\sigma}{n}, \frac{\mathfrak{k}\vartheta}{n}, \mathfrak{k}\gamma\right)\\&= e^{-\frac{\frac{\mathfrak{k}\sigma}{n}+\frac{\mathfrak{k}\vartheta}{n}}{\mathfrak{k}\gamma}}\\&= e^{-\frac{\sigma+\vartheta}{n\gamma}}\\&\geq e^{-\frac{\sigma+\frac{\mathfrak{k}\vartheta}{n}}{\gamma}}\\&= \Pi\big(\sigma, \mathfrak{g}(\vartheta), \gamma\big).\end{aligned}$$

Then

$$\Pi\big(\mathfrak{g}(\sigma), \mathfrak{g}(\vartheta), \mathfrak{k}\gamma\big) \geq \Pi\big(\sigma, \mathfrak{g}(\vartheta), \gamma\big) \quad \text{for all } \gamma > 0.$$

Thus, $\mathfrak{g}$ is a *GCFMS*-contraction of type-*II*. Hence, all the conditions of Theorem 3.16 are satisfied and $\sigma = 0$ is a unique fixed point of $\mathfrak{g}$.

4 Application

Consider $\Phi = C[0,1]$, the class of all real-valued continuous functions defined on $[0,1]$. Define a Π-complete generalized controlled fuzzy metric $\Pi : \Phi^2 \times (0,\infty) \to [0,1]$ by

$$\Pi(\sigma, \vartheta, \gamma) = e^{-\frac{\sup_{\kappa\in[0,1]} |\sigma(\kappa)-\vartheta(\kappa)|}{\gamma}}$$

for all $(\sigma, \vartheta) \in \Phi^2$ and $\gamma > 0$. Consider the integral equation

$$\sigma(\kappa) = \mathfrak{f}(\kappa) + \int_0^1 \mathfrak{z}(\kappa, s)\mathcal{F}\big(\kappa, s, \sigma(s)\big)\, ds, \tag{4.1}$$

where $\mathfrak{f} : [0,1] \to \mathbb{R}$, $\mathfrak{z} : [0,1] \times [0,1] \to \mathbb{R}$ and $\mathcal{F} : [0,1] \times [0,1] \times \mathbb{R} \to \mathbb{R}$ are continuous functions.

Theorem 4.1 *Let $(\Pi, \Phi, \star)$ be a Π-complete GCFMS defined in the above. Let $\mathfrak{g} : \Phi \to \Phi$ be the integral operator defined by*

$$\mathfrak{g}\big(\sigma(\kappa)\big) = \mathfrak{f}(\kappa) + \int_0^1 \mathfrak{z}(\kappa, s)\mathcal{F}\big(\kappa, s, \sigma(s)\big)\, ds$$

for all $\sigma \in \Phi$ and $\kappa, s \in [0,1]$. Suppose that the following conditions are satisfied:

(1) *For all $\kappa, s \in [0,1]$ and $(\sigma, \vartheta) \in \Phi^2$, we have*

$$\big|\mathcal{F}\big(\kappa, s, \sigma(s)\big) - \mathcal{F}\big(\kappa, s, \vartheta(s)\big)\big| < \big|\sigma(s) - \mathfrak{g}\big(\vartheta(s)\big)\big|.$$

(2) *For all* $\kappa, s \in [0,1]$ *and for some* $\mathfrak{k} < 1$,

$$\sup_{\kappa\in[0,1]}\left|\int_0^1 \mathfrak{z}(\kappa,s)\,ds\right| \le \mathfrak{k} < 1.$$

Then integral equation (4.1) *has a unique solution* $\sigma^\star \in \Phi$.

Proof Note that, for all $\sigma^\star \in \Phi$, we have

$$\begin{aligned}\delta\big(\Pi,\mathfrak{g},\sigma^\star,\gamma\big) &= \inf\big\{\Pi\big(\mathfrak{g}^{\mathrm{i}}\big(\sigma^\star\big),\mathfrak{g}^{\mathrm{j}}\big(\sigma^\star\big),\gamma\big) : \mathrm{i},\mathrm{j}\in\mathbb{N},\gamma>0\big\}\\ &= \inf\big\{e^{-\frac{\sup_{\kappa\in[0,1]}|\mathfrak{g}^{\mathrm{i}}(\sigma^\star(\kappa))-\mathfrak{g}^{\mathrm{j}}(\sigma^\star(\kappa))|}{\gamma}}\big\} > 0.\end{aligned}$$

We claim that $\mathfrak{g}$ is a *GCFMS*-contraction of type-*II*. For all $(\sigma,\vartheta)\in\Phi^2$, we obtain

$$\begin{aligned}\Pi(\mathfrak{g}\sigma,\mathfrak{g}\vartheta,\mathfrak{k}\gamma) &= e^{-\frac{\sup_{\kappa\in[0,1]}|\mathfrak{g}\sigma(\kappa)-\mathfrak{g}\vartheta(\kappa)|}{\mathfrak{k}\gamma}}\\
&= e^{-\frac{\sup_{\kappa\in[0,1]}|\int_0^1\mathfrak{z}(\kappa,\mathfrak{s})\mathcal{F}(\kappa,\mathfrak{s},\sigma(\mathfrak{s}))d\mathfrak{s}-\int_0^1\mathfrak{z}(\kappa,\mathfrak{s})\mathcal{F}(\kappa,\mathfrak{s},\vartheta(\mathfrak{s}))d\mathfrak{s}|}{\mathfrak{k}\gamma}}\\
&= e^{-\frac{\sup_{\kappa\in[0,1]}|\int_0^1\mathfrak{z}(\kappa,\mathfrak{s})[(\mathcal{F}(\kappa,\mathfrak{s},\sigma(\mathfrak{s}))-\mathcal{F}(\kappa,\mathfrak{s},\vartheta(\mathfrak{s})))]d\mathfrak{s}|}{\mathfrak{k}\gamma}}\\
&\ge e^{-\frac{\sup_{\kappa\in[0,1]}\int_0^1|\mathfrak{z}(\kappa,\mathfrak{s})||(\mathcal{F}(\kappa,\mathfrak{s},\sigma(\mathfrak{s}))-\mathcal{F}(\kappa,\mathfrak{s},\vartheta(\mathfrak{s})))|d\mathfrak{s}}{\mathfrak{k}\gamma}}\\
&= e^{-\frac{\sup_{\kappa\in[0,1]}\int_0^1|\mathfrak{z}(\kappa,\mathfrak{s})||\sigma(\mathfrak{s})-\mathfrak{g}(\vartheta(\mathfrak{s}))|d\mathfrak{s}}{\mathfrak{k}\gamma}}\\
&= e^{-\frac{\sup_{\kappa\in[0,1]}\int_0^1|\mathfrak{z}(\kappa,\mathfrak{s})|\{\sup_{\mathfrak{s}\in[0,1]}|\sigma(\mathfrak{s})-\mathfrak{g}(\vartheta(\mathfrak{s}))|\}d\mathfrak{s}}{\mathfrak{k}\gamma}}\\
&= e^{-\{\sup_{\mathfrak{s}\in[0,1]}|\sigma(\mathfrak{s})-\mathfrak{g}(\vartheta(\mathfrak{s}))|\}\frac{\sup_{\kappa\in[0,1]}\int_0^1|\mathfrak{z}(\kappa,\mathfrak{s})|d\mathfrak{s}}{\mathfrak{k}\gamma}}\\
&\ge e^{-\{\sup_{\mathfrak{s}\in[0,1]}|\sigma(\mathfrak{s})-\mathfrak{g}(\vartheta(\mathfrak{s}))|\}\frac{\mathfrak{k}}{\mathfrak{k}\gamma}}\\
&= e^{-\frac{\sup_{\mathfrak{s}\in[0,1]}|\sigma(\mathfrak{s})-\mathfrak{g}(\vartheta(\mathfrak{s}))|}{\gamma}}\\
&= \Pi(\sigma,\mathfrak{g}\vartheta,\gamma).\end{aligned}$$

Because all of Theorem 3.16's criteria are met, $\mathfrak{g}$ has a fixed point. As a result, integral equation (4.1) has only one solution. □

5 Conclusion and future work

In this article, we have obtained some fixed point results in GCFMS. An example and an application are also presented to strengthen our main results. Khalehoghli, Rahimi, and Eshaghi Gordji [9, 26] presented a real generalization of the mentioned Banach contraction principle by introducing R-metric spaces, where R is an arbitrary relation on L. We note that, in a special case, R can be considered as R := $\preceq$[partially ordered relation], R := $\perp$[orthogonal relation], etc. If one can find a suitable replacement for a Banach theorem that may determine the value of fixed point, then many problems can be solved in this R-relation. This will provide a structural method for finding a value of a fixed point. It is an interesting open problem to study the fixed point results on R-complete R-*b*-metric-like spaces.

Acknowledgements
Not applicable.

Funding
This research received no external funding.

Abbreviations
GMS, Generalized metric space; GCMS, Generalized controlled metric space; FMS, Fuzzy metric space; GCFMS, Generalized controlled fuzzy metric space.

Availability of data and materials
Not applicable.

Declarations

Competing interests
The authors declare no competing interests.

Author contribution
Conceptualization, GM and AJG; formal analysis, VP and BM; funding acquisition, GM and AJG; investigation, GM, AJG and VP; methodology, GM and AJG; project administration, AD and BM; software, GM and AJG; supervision, AD and BM; writing–original draft, GM and AJG; writing–review and editing, AD, VP and BM All authors have read and agreed to the published version of the manuscript.

Author details
[1]Department of Mathematics, Saveetha School of Engineering, Saveetha Institute of Medical and Technical Sciences, Chennai 602 105, Tamil Nadu, India. [2]Department of Mathematics, College of Engineering and Technology, Faculty of Engineering and Technology, SRM Institute of Science and Technology, SRM Nagar, Kattankulathur, Kanchipuram, Chennai 603203, Tamil Nadu, India. [3]Department of Mathematics, Buein Zahra Technical University, Buein Zahra, Qazvin, Iran. [4]Department of Mathematics, Gilan-E-Gharb Branch, Islamic Azad University, Gilan-E-Gharb, Iran. [5]Department of Mathematics, Marand Branch, Islamic Azad University, Marand, Iran.

Publisher's Note

Received: 4 January 2022 Accepted: 28 September 2022 Published online: 01 November 2022

References

1. Frechet, M.: Sur quelques points du calcul fonctionnel. Rend. Circ. Mat. Palermo **22**(1), 1–74 (1906)
2. Banach, S.: Sur les operations dans les ensembles abstraits et leur application aux equations intégrales. Fundam. Math. **3**(1), 133–181 (1922)
3. Jleli, M., Samet, B.: A generalized metric space and related fixed point theorems. Fixed Point Theory Appl. **2015**(1), 61 (2015)
4. Kramosil, I., Michalek, J.: Fuzzy metric and statistical metric spaces. Kybernetika **11**, 336–344 (1975)
5. Shen, Y., Qiu, D., Chen, W.: Fixed point theorems in fuzzy metric spaces. Appl. Math. Lett. **25**(2), 138–141 (2012)
6. Beg, I., Sedghi, S., Shobe, N.: Fixed point theorems in fuzzy metric spaces. Int. J. Anal. **2013**, 1–8 (2013)
7. Gupta, V., Kanwar, A.: V-fuzzy metric space and related fixed point theorems. Fixed Point Theory Algorithms Sci. Eng. **2016**, 51 (2016)
8. Rakić, D., Došenović, T., Mitrović, Z.D., de la Sen, M., Radenović, S.: Some fixed point theorems of Ćirić type in fuzzy metric spaces. Mathematics **8**(2), 297 (2020)
9. Sezen, M.S.: Controlled fuzzy metric spaces and some related fixed point results. Numer. Methods Partial Differ. Equ. **37**(1), 583–593 (2021). https://doi.org/10.1002/num.22541
10. Gunaseelan, M., Gnanaprakasam, A.J., Mitrović, Z.D., Bota, M.-F.: Solving an integral equation via fuzzy triple controlled bipolar metric spaces. Mathematics **9**, 3181 (2021)
11. Ashraf, M.S., Ali, R., Hussain, N.: New fuzzy fixed point results in generalized fuzzy metric spaces with application to integral equations. IEEE Access **8**, 91653–91660 (2020). https://doi.org/10.1109/ACCESS.2020.2994130
12. Elkouch, Y., Marhrani, E.L.: On some fixed point theorems in generalized metric space. Fixed Point Theory Appl. **2017**(1), 23 (2017)
13. Karami, A., Sedghi, S., Parvaneh, V.: Sequential extended S-metric spaces and relevant fixed point results with application to nonlinear integral equations. Adv. Math. Phys. **2021**, Article ID 9910861 (2021)
14. Roy, K., Panja, S., Saha, M., Parvaneh, V.: An extended b-metric-type space and related fixed point theorems with an application to nonlinear integral equations. Adv. Math. Phys. **2020**, Article ID 8868043 (2020). https://doi.org/10.1155/2020/8868043
15. Schweizer, B., Sklar, A.: Statistical metric spaces. Pac. J. Math. **10**, 313–334 (1960)
16. George, A., Veeramani, P.: On some results in fuzzy metric spaces. Fuzzy Sets Syst. **64**(3), 395–399 (1994)
17. Kramosil, I., Michalek, J.: Fuzzy metrics and statistical metric spaces. Kybernetika **11**(5), 326–334 (1975)
18. Sadeghi, Z., Vaezpour, S., Park, C., Saadati, R., Vetro, C.: Set-valued mappings in partially ordered fuzzy metric spaces. J. Inequal. Appl. **2014**(1), 157 (2014)
19. Turkoglu, D., Sangurlu, M.: Fixed point theorems for fuzzy contractive mappings in fuzzy metric spaces. J. Intell. Fuzzy Syst. **26**(1), 137–142 (2014)

20. Gopal, D., Vetro, C.: Some new fixed point theorems in fuzzy metric spaces. Iran. J. Fuzzy Syst. **11**(3), 95–107 (2014)
21. Vetro, C.: Fixed points in weak non-Archimedean fuzzy metric spaces. Fuzzy Sets Syst. **162**(1), 84–90 (2011)
22. Phiangsungnoen, S.: Some new fuzzy fixed point theorems for fuzzy contractive mappings in Hausdorff fuzzy metric spaces. In: Proc. Int. Conf. Control, Artif. Intell., Robot. Optim. (ICCAIRO), pp. 159–164 (2018)
23. Mohammadi, B., Hussain, A., Parvaneh, V., Saleem, N., Shahkoohi, R.J.: Fixed point results for generalized fuzzy contractive mappings in fuzzy metric spaces with application in integral equations. J. Math. **2021**, Article ID 9931066 (2021). https://doi.org/10.1155/2021/9931066
24. Rezaee, M.M., Sedghi, S., Parvaneh, V.: JS-Prešić contractive mappings in extended modular S-metric spaces and extended fuzzy S-metric spaces with an application. Azerb. J. Math. **12**(1), 162–182 (2022)
25. Grabiec, M.: Fixed points in fuzzy metric spaces. Fuzzy Sets Syst. **27**(3), 385–389 (1988)
26. Khalehoghli, S., Rahimi, H., Eshaghi Gordji, M.: Fixed point theorems in R-metric spaces with applications. AIMS Math. **5**, 3125–3137 (2020)
27. Khalehoghli, S., Rahimi, H., Eshaghi Gordji, M.: R-topological spaces and SR-topological spaces with their applications. Math. Sci. **14**, 249–255 (2020)