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Common fixed points of *G*-nonexpansive mappings on Banach spaces with a graph

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Abstract

In this paper, we prove the weak and strong convergence of a sequence $\{x_n\}$ generated by the Ishikawa iteration to some common fixed points of two *G*-nonexpansive mappings defined on a Banach space endowed with a graph.

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Keywords: common fixed point; *G*-nonexpansive mappings; Ishikawa iteration; Banach space; directed graph

1 Introduction

In 1922, Banach proved a remarkable and powerful result called the *Banach contraction principle*. Because of its fruitful applications, the principle has been generalized in many directions. The recent version of the theorem was given in Banach spaces endowed with a graph. In 2008, Jachymski [1] gave a generalization of the Banach contraction principle to mappings on a metric space endowed with a graph. In 2012, Aleomraninejad *et al.* [2] presented some iterative scheme results for *G*-contractive and *G*-nonexpansive mappings on graphs. In 2015, Alfuraidan and Khamsi [3] defined the concept of *G*-monotone nonexpansive multivalued mappings defined on a metric space with a graph. In the same year, Alfuraidan [4] gave a new definition of the *G*-contraction and obtained sufficient conditions for the existence of fixed points for multivalued mappings on a metric space with a graph, and also in [5], he proved the existence of a fixed point of monotone nonexpansive mapping defined in a Banach space endowed with a graph. Recently, Tiammee *et al.* [6] proved Browder's convergence theorem for *G*-nonexpansive mapping in a Banach space with a directed graph. They also proved the strong convergence of the Halpern iteration for a *G*-nonexpansive mapping.

Inspired by all aforementioned references, the author proves strong and weak convergence theorems for *G*-nonexpansive mappings using the Ishikawa iteration generated from arbitrary x_0 in a closed convex subset C of a uniformly convex Banach space X endowed with a graph.

2 Preliminaries

In this section, we recall some standard graph notations and terminology and also some needed results.

Let (X,d) be a metric space, and $\triangle = \{(x,x)|x \in X\}$. Consider a directed graph G for which the set $V(G)$ of its vertices coincides with X and the set $E(G)$ of its edges contains all loops. Assume that G has no parallel edges. Then $G = (V(G),E(G))$, and by assigning to each edge the distance between its vertices, G may be treated as a *weighted* graph.

Definition 2.1 The *conversion* of a graph G is the graph obtained from G by reversing the direction of edges denoted by G^{-1}, and

$$E(G^{-1}) = \{(x,y) \in X \times X|(y,x) \in E(G)\}.$$

Definition 2.2 Let x and y be vertices of a graph G. A *path* in G from x to y of length N ($N \in \mathbb{N} \cup \{0\}$) is a sequence $\{x_i\}_{i=0}^{N}$ of $N+1$ vertices for which

$$x_0 = x, \qquad x_N = y, \quad \text{and} \quad (x_i, x_{i+1}) \in E(G) \quad \text{for } i = 0,1,\dots,N-1.$$

Definition 2.3 A graph G is said to be *connected* if there is a path between any two vertices of the graph G.

Definition 2.4 A directed graph $G = (V(G),E(G))$ is said to be *transitive* if, for any $x,y,z \in V(G)$ such that (x,y) and (y,z) are in $E(G)$, we have $(x,z) \in E(G)$.

The definition of a *G-nonexpansive mapping* is given as follows.

Definition 2.5 Let C be a nonempty convex subset of a Banach space X, and $G = (V(G),E(G))$ a directed graph such that $V(G) = C$. Then a mapping $T: C \to C$ is *G-nonexpansive* (see [3], Definition 2.3(iii)) if it satisfies the following conditions.

(i) T is edge-preserving.

(ii) $\|Tx - Ty\| \leq \|x - y\|$ whenever $(x,y) \in E(G)$ for any $x,y \in C$.

Definition 2.6 ([7]) Let C be a nonempty closed convex subset of a real uniformly convex Banach space X. The mappings T_i ($i = 1,2$) on C are said to satisfy *Condition* B if there exists a nondecreasing function $f: [0,\infty) \to [0,\infty)$ with $f(0) = 0$ and $f(r) > 0$ for all $r > 0$ such that, for all $x \in C$,

$$\max\{\|x - T_1x\|, \|x - T_2x\|\} \geq f(d(x,F)),$$

where $F = F(T_1) \cap F(T_2)$ and $F(T_i)$ ($i = 1,2$) are the sets of fixed points of T_i.

Definition 2.7 ([7]) Let C be a subset of a metric space (X,d). A mapping T is *semicompact* if for a sequence $\{x_n\}$ in C with $\lim_{n\to\infty} d(x_n, Tx_n) = 0$, there exists a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ such that $x_{n_i} \to p \in C$.

Definition 2.8 A Banach space X is said to satisfy *Opial's property* if the following inequality holds for any distinct elements x and y in X and for each sequence $\{x_n\}$ weakly convergent to x:

$$\liminf_{n\to\infty} \|x_n - x\| < \liminf_{n\to\infty} \|x_n - y\|.$$

Definition 2.9 Let X be a Banach space. A mapping T with domain D and range R in X is *demiclosed at* 0 if, for any sequence $\{x_n\}$ in D such that $\{x_n\}$ converges weakly to $x \in D$ and $\{Tx_n\}$ converges strongly to 0, we have $Tx = 0$.

Lemma 2.10 ([8]) *Let X be a uniformly convex Banach space, and $\{\alpha_n\}$ a sequence in $[\delta, 1-\delta]$ for some $\delta \in (0,1)$. Suppose that sequences $\{x_n\}$ and $\{y_n\}$ in X are such that $\limsup_{n\to\infty} \|x_n\| \leq c$, $\limsup_{n\to\infty} \|y_n\| \leq c$ and $\limsup_{n\to\infty} \|\alpha x_n + (1-\alpha_n)y_n\| = c$ for some $c \geq 0$. Then $\lim_{n\to\infty} \|x_n - y_n\| = 0$.*

Lemma 2.11 ([9]) *Let X be a Banach space, and $R > 1$ be a fixed number. Then X is uniformly convex if and only if there exists a continuous, strictly increasing, and convex function $g : [0,\infty) \to [0,\infty)$ with $g(0) = 0$ such that*

$$\big\|\lambda x + (1-\lambda)y\big\|^2 \leq \lambda\|x\|^2 + (1-\lambda)\|y\|^2 - \lambda(1-\lambda)g\big(\|x-y\|\big)$$

for all $x, y \in B_R(0) = \{x \in X \mid \|x\| \leq R\}$ and $\lambda \in [0,1]$.

Lemma 2.12 ([10]) *Let X be a Banach space that satisfies Opial's property, and let $\{x_n\}$ be a sequence in X. Let x, y in X be such that $\lim_{n\to\infty} \|x_n - x\|$ and $\lim_{n\to\infty} \|x_n - y\|$ exist. If $\{x_{n_j}\}$ and $\{x_{n_k}\}$ are subsequences of $\{x_n\}$ that converge weakly to x and y, respectively, then $x = y$.*

3 Main results

Throughout the section, we let C be a nonempty closed convex subset of a Banach space X endowed with a directed graph G such that $V(G) = C$ and $E(G)$ is convex. We also suppose that the graph G is transitive. The mappings T_i $(i = 1,2)$ are G-nonexpansive from C to C with $F = F(T_1) \cap F(T_2)$ nonempty. Let $\{x_n\}$ be a sequence generated from arbitrary $x_0 \in C$,

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T_1 y_n, \\ y_n &= (1-\beta_n)x_n + \beta_n T_2 x_n, \end{aligned}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are real sequences in $[0,1]$.

We first begin by proving the following useful results.

Proposition 3.1 *Let $z_0 \in F$ be such that (x_0, z_0), (y_0, z_0), (z_0, x_0), and (z_0, y_0) are in $E(G)$. Then (x_n, z_0), (y_n, z_0), (z_0, x_n), (z_0, y_n), and (x_n, y_n) are in $E(G)$.*

Proof We divide the proof into three parts. In the first part, with the assumption (x_0, z_0), $(y_0, z_0) \in E(G)$, we will show by induction that $(x_n, z_0), (y_n, z_0) \in E(G)$. Then, with the assumption $(z_0, x_0), (z_0, y_0) \in E(G)$, we will again prove by induction that $(z_0, x_n), (z_0, y_n) \in E(G)$. In the third part, we combine these two results using transitivity of G to get the statement in the proposition. Let (x_0, z_0) and $(y_0, z_0) \in E(G)$. Then $(T_1 y_0, z_0), (T_2 x_0, z_0) \in E(G)$ since T_i $(i = 1,2)$ are edge-preserving. By the convexity of $E(G)$ and $(T_1 y_0, z_0), (x_0, z_0) \in E(G)$, we have $(x_1, z_0) \in E(G)$. Then, by edge-preserving of T_2, $(T_2 x_1, z_0) \in E(G)$. Again, by the convexity of $E(G)$ and $(T_2 x_1, z_0), (x_1, z_0) \in E(G)$, we get $(y_1, z_0) \in E(G)$ and then $(T_1 y_1, z_0) \in E(G)$. Next, we assume that $(x_k, z_0), (y_k, z_0) \in E(G)$. Then $(T_2 x_k, z_0), (T_1 y_k, z_0) \in$

$E(G)$ since T_i $(i = 1, 2)$ are edge-preserving. Since $E(G)$ is convex, $(x_{k+1}, z_0) \in E(G)$. Indeed,

$$\alpha(T_1 y_k, z_0) + (1 - \alpha)(x_k, z_0) = \bigl(\alpha T_1 y_k + (1 - \alpha)x_k, z_0\bigr) = (x_{k+1}, z_0) \in E(G).$$

Since T_2 is edge-preserving, $(T_2 x_{k+1}, z_0) \in E(G)$. Using the convexity of $E(G)$, we get $(y_{k+1}, z_0) \in E(G)$. To be explicit,

$$\beta(T_2 x_{k+1}, z_0) + (1 - \beta)(x_{k+1}, z_0) = \bigl(\beta T_2 x_{k+1} + (1 - \beta)x_{k+1}, z_0\bigr) = (y_{k+1}, z_0) \in E(G).$$

Hence, by induction, $(x_n, z_0), (y_n, z_0) \in E(G)$ for all $n \geq 1$. Using a similar argument, we can show that $(z_0, x_n), (z_0, y_n) \in E(G)$ under the assumption that $(z_0, x_0), (z_0, y_0) \in E(G)$. Therefore, $(x_n, y_n) \in E(G)$ by the transitivity of G. □

Lemma 3.2 *Let $z_0 \in F$. Suppose that $(x_0, z_0), (y_0, z_0), (z_0, x_0), (z_0, y_0) \in E(G)$ for arbitrary x_0 in C. Then $\lim_{n\to\infty} \|x_n - z_0\|$ exists.*

Proof Notice that

$$\begin{aligned}
\|x_{n+1} - z_0\| &= \bigl\|(1 - \alpha_n)x_n + \alpha_n T_1 y_n - z_0\bigr\| \\
&\leq (1 - \alpha_n)\|x_n - z_0\| + \alpha_n \|T_1 y_n - z_0\| \\
&\leq (1 - \alpha_n)\|x_n - z_0\| + \alpha_n \|y_n - z_0\| \\
&= (1 - \alpha_n)\|x_n - z_0\| + \alpha_n \bigl\|(1 - \beta_n)x_n - (1 - \beta_n)z_0 + \beta_n(T_2 x_n - z_0)\bigr\| \\
&\leq (1 - \alpha_n)\|x_n - z_0\| + \alpha_n(1 - \beta_n)\|x_n - z_0\| + \alpha_n\beta_n\|x_n - z_0\| \\
&= (1 - \alpha_n)\|x_n - z_0\| + \alpha_n\|x_n - z_0\| \\
&= \|x_n - z_0\|.
\end{aligned}$$

Thus, $\lim_{n\to\infty} \|x_n - z_0\|$ exists. In particular, the sequence $\{x_n\}$ is bounded. □

Lemma 3.3 *If X is uniformly convex, $\{\alpha_n\}, \{\beta_n\} \subset [\delta, 1 - \delta]$ for some $\delta \in (0, \frac{1}{2})$, and (x_0, z_0), $(y_0, z_0), (z_0, x_0), (z_0, y_0) \in E(G)$ for arbitrary x_0 in C and $z_0 \in F$, then*

$$\lim_{n\to\infty} \|x_n - T_1 x_n\| = 0 = \lim_{n\to\infty} \|x_n - T_2 x_n\|.$$

Proof Let $z_0 \in F$. Then, by the boundedness of $\{x_n\}$ and $\{T_2 x_n\}$ there exists $r > 0$ such that $x_n - z_0, y_n - z_0 \in B_r(0)$ for all $n \geq 1$. Put $c = \lim_{n\to\infty} \|x_n - z_0\|$. If $c = 0$, then by the G-nonexpansiveness of T_i $(i = 1, 2)$ we have

$$\|x_n - T_i x_n\| \leq \|x_n - z_0\| + \|z_0 - T_i x_n\| \leq \|x_n - z_0\| + \|z_0 - x_n\|.$$

Therefore, the result follows. Suppose that $c > 0$. Hence, by Lemma 2.11 together with the G-nonexpansiveness of T_2, we have

$$\begin{aligned}
\|y_n - z_0\|^2 &= \bigl\|(1 - \beta_n)x_n + \beta_n T_2 x_n - z_0\bigr\|^2 \\
&= \bigl\|\beta_n(T_2 x_n - z_0) + (1 - \beta_n)(x_n - z_0)\bigr\|^2
\end{aligned}$$

$$\begin{aligned}
&\leq \beta_n \|T_2 x_n - z_0\|^2 + (1-\beta_n)\|x_n - z_0\|^2 - \beta_n(1-\beta_n) g\big(\|T_2 x_n - x_n\|\big) \\
&\leq \beta_n \|x_n - z_0\|^2 + (1-\beta_n)\|x_n - z_0\|^2 \\
&= \|x_n - z_0\|^2.
\end{aligned}$$

Thus,

$$\limsup_{n\to\infty} \|y_n - z_0\| \leq \limsup_{n\to\infty} \|x_n - z_0\| \leq c.$$

Notice also that

$$\begin{aligned}
\|x_{n+1} - z_0\|^2 &= \big\|(1-\alpha_n)x_n + \alpha_n T_1 y_n - z_0\big\|^2 \\
&\leq \alpha_n \|y_n - z_0\|^2 + (1-\alpha_n)\|x_n - z_0\|^2 - \alpha_n(1-\alpha_n) g\big(\|T_1 y_n - x_n\|\big) \\
&\leq \|x_n - z_0\|^2 - \alpha_n(1-\alpha_n) g\big(\|T_1 y_n - x_n\|\big) \\
&\leq \|x_n - z_0\|^2 - \delta^2 g\big(\|T_1 y_n - x_n\|\big).
\end{aligned}$$

Thus,

$$\delta^2 g\big(\|T_1 y_n - x_n\|\big) \leq \|x_n - z_0\|^2 - \|x_{n+1} - z_0\|^2.$$

This implies that $\lim_{n\to\infty} g(\|T_1 y_n - x_n\|) = 0$, and since g is strictly increasing and continuous at 0,

$$\lim_{n\to\infty} \|T_1 y_n - x_n\| = 0. \tag{1}$$

Since T_1 is G-nonexpansive, we have

$$\|x_n - z_0\| \leq \|x_n - T_1 y_n\| + \|T_1 y_n - T_1 z_0\| \leq \|x_n - T_1 y_n\| + \|y_n - z_0\|.$$

Taking $\liminf$ yields

$$c \leq \liminf_{n\to\infty} \|y_n - z_0\|.$$

Hence, we have

$$\lim_{n\to\infty} \|y_n - z_0\| = c.$$

Since

$$\lim_{n\to\infty} \big\|\beta_n(T_2 x_n - z_0) + (1-\beta_n)(x_n - z_0)\big\| = \lim_{n\to\infty} \|y_n - z_0\| = c$$

and

$$\limsup_{n\to\infty} \|T_2 x_n - z_0\| \leq c,$$

by Lemma 2.10 we have

$$\lim_{n\to\infty} \|T_2x_n - x_n\| = 0. \tag{2}$$

By the G-nonexpansiveness of T_1 together with $\|x_n - y_n\| \leq \|T_2x_n - x_n\|$ we have

$$\begin{aligned}\|T_1x_n - x_n\| &\leq \|T_1x_n - T_1y_n\| + \|T_1y_n - x_n\| \\ &\leq \|x_n - y_n\| + \|T_1y_n - x_n\| \\ &\leq \|T_2x_n - x_n\| + \|T_1y_n - x_n\|.\end{aligned}$$

Using (1) and (2), $\lim_{n\to\infty} \|T_1x_n - x_n\| = 0$. Hence, the lemma is proved. □

Lemma 3.4 *Suppose that X satisfies the Opial's property and that (x_0, z_0), (y_0, z_0) are in $E(G)$ for $z_0 \in F$ and arbitrary $x_0 \in C$. Then $I - T_i$ $(i = 1, 2)$ are demiclosed.*

Proof Suppose that $\{x_n\}$ is a sequence in C that converges weakly to q. From Lemma 3.3 we have $\lim_{n\to\infty} \|x_n - T_ix_n\| = 0$. Suppose for contradiction that $q \neq T_iq$. Then, by Opial's property we have

$$\begin{aligned}\limsup_{n\to\infty} \|x_n - q\| &< \limsup_{n\to\infty} \|x_n - T_iq\| \\ &\leq \limsup_{n\to\infty} \bigl(\|x_n - T_ix_n\| + \|T_ix_n - T_iq\|\bigr) \\ &\leq \limsup_{n\to\infty} \|x_n - q\|,\end{aligned}$$

a contradiction. Hence, $T_iq = q$, so the conclusion holds. □

Theorem 3.5 *Suppose X is uniformly convex, $\{\alpha_n\}, \{\beta_n\} \subset [\delta, 1 - \delta]$ for some $\delta \in (0, \frac{1}{2})$, T_i $(i = 1, 2)$ satisfy Condition* B, *F is dominated by x_0, F dominates x_0, and (x_0, z), (y_0, z), (z, x_0), $(z, y_0) \in E(G)$ for each $z \in F$ and arbitrary $x_0 \in C$. Then $\{x_n\}$ converges strongly to a common fixed point of T_i.*

Proof Let $z \in F$. Recall the following facts from Lemma 3.2:

(i) $\{x_n\}$ is bounded;
(ii) $\lim_{n\to\infty} \|x_n - z\|$ exists;
(iii) $\|x_{n+1} - z\| \leq \|x_n - z\|$ for all $n \geq 1$.

They imply that

$$d(x_{n+1}, F) \leq d(x_n, F).$$

Thus $\lim_{n\to\infty} d(x_n, F)$ exists. Since each T_i $(i = 1, 2)$ satisfies Condition B and $\lim_{n\to\infty} \|x_n - T_ix_n\| = 0$, we have

$$\lim_{n\to\infty} f\bigl(d(x_n, F)\bigr) = 0$$

and then

$$\lim_{n\to\infty} d(x_n, F) = 0.$$

Hence, there are a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ and a sequence $\{z_j\} \subset F$ satisfying

$$\|x_{n_j} - z_j\| \leq \frac{1}{2^j}.$$

Put $n_{j+1} = n_j + k$ for some $k \geq 1$. Then

$$\|x_{n_{j+1}} - z_j\| \leq \|x_{n_j+k-1} - z_j\| \leq \|x_{n_j} - z_j\| \leq \frac{1}{2^j}.$$

Hence,

$$\|z_{j+1} - z_j\| \leq \frac{3}{2^{j+1}},$$

so that $\{z_j\}$ is a Cauchy sequence. We assume that $z_j \to q \in C$ as $n \to \infty$. Since F is closed, $q \in F$. Hence, we have $x_{n_j} \to q$ as $j \to \infty$, and since $\lim_{n\to\infty} \|x_n - q\|$ exists, the conclusion follows. □

Theorem 3.6 *Suppose that X is uniformly convex, $\{\alpha_n\}, \{\beta_n\} \subset [\delta, 1-\delta]$ for some $\delta \in (0, \frac{1}{2})$, one of T_i $(i = 1, 2)$ is semicompact, F is dominated by x_0, F dominates x_0, and $(x_0, z_0), (y_0, z_0), (z_0, x_0), (z_0, y_0) \in E(G)$ for $z_0 \in F$ and arbitrary $x_0 \in C$. Then $\{x_n\}$ converges strongly to a common fixed point of T_i.*

Proof Suppose that T_2 is semicompact; by Lemma 3.2 and Lemma 3.3 we have a bounded sequence $\{x_n\}$, and $\lim_{n\to\infty} \|x_n - T_i x_n\| = 0$. Hence, by the semicompactness of T_2 there exist $q \in C$ and a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ such that $x_{n_j} \to q$ as $j \to \infty$ and $\lim_{n\to\infty} \|x_{n_j} - T_i x_{n_j}\| = 0$. Notice that

$$\begin{aligned} \|q - T_i q\| &\leq \|q - x_{n_j}\| + \|x_{n_j} - T_i x_{n_j}\| + \|T_i x_{n_j} - T_i q\| \\ &\leq \|q - x_{n_j}\| + \|x_{n_j} - T_i x_{n_j}\| + \|x_{n_j} - q\| \\ &\to 0 \quad \text{as } n \to \infty. \end{aligned}$$

Hence, $q \in F$. Since $\lim_{n\to\infty} d(x_n, F) = 0$, it follows, by repeating the same argument as in the proof of Theorem 3.5, that $\{x_n\}$ converges strongly to a common fixed point of T_i $(i = 1, 2)$, and the proof is complete. □

Theorem 3.7 *Suppose that X is uniformly convex, $\{\alpha_n\}, \{\beta_n\} \subset [\delta, 1-\delta]$ for some $\delta \in (0, \frac{1}{2})$. If X satisfies Opial's property, $I - T_i$ is demiclosed at zero for each i, F is dominated by x_0, F dominates x_0, and $(x_0, z_0), (y_0, z_0), (z_0, x_0), (z_0, y_0) \in E(G)$ for $z_0 \in F$ and arbitrary $x_0 \in C$, then $\{x_n\}$ converges weakly to a common fixed point of T_i.*

Proof Note that by Lemma 3.2, for each $q \in F$,

$$\lim_{n\to\infty} \|x_n - q\| \quad \text{exists.} \tag{3}$$

Let $\{x_{n_k}\}$ and $\{x_{n_j}\}$ be subsequences of the sequence $\{x_n\}$ with two weak limits q_1 and q_2, respectively. Notice that, by Lemma 3.3,

$$\|x_{n_j} - T_i x_{n_j}\| \to 0 \quad \text{as } n \to \infty \quad \text{and}$$
$$\|x_{n_k} - T_i x_{n_k}\| \to 0 \quad \text{as } n \to \infty.$$

Hence, $T_i q_1 = q_1$ and $T_i q_2 = q_2$. By Lemma 3.4 we have $q_1, q_2 \in F$. In particular, $q_1 = q_2$ by Lemma 2.12. Therefore, $\{x_n\}$ converges weakly to a common fixed point in F. □

Competing interests
The author declares that she has no competing interests.

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