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Research Article

Fuzzy Stability of the Pexiderized Quadratic Functional Equation: A Fixed Point Approach

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The fixed point alternative methods are implemented to give generalized Hyers-Ulam-Rassias stability for the Pexiderized quadratic functional equation in the fuzzy version. This method introduces a metrical context and shows that the stability is related to some fixed point of a suitable operator.

1. Introduction

The aim of this article is to extend the applications of the fixed point alternative method to provide a fuzzy version of Hyers-Ulam-Rassias stability for the functional equation:

$$f(x+y)+f(x-y)=2g(x)+2h(y), \tag{1.1}$$

which is said to be a Pexiderized quadratic functional equation or called a quadratic functional equation for $f = g = h$. During the last two decades, the Hyers-Ulam-Rassias stability of (1.1) has been investigated extensively by several mathematicians for the mapping f with more general domains and ranges [1–4]. In view of fuzzy space, Katsaras [5] constructed a fuzzy vector topological structure on the linear space. Later, some other type fuzzy norms and some properties of fuzzy normed linear spaces have been considered by some mathematicians [6–12]. Recently, considerable attention has been increasing to the problem of fuzzy stability of functional equations. Several various fuzzy stability results concerning Cauchy, Jensen, quadratic, and cubic functional equations have been investigated [13–16].

As we see, the powerful method for studying the stability of functional equation was first suggested by Hyers [17] while he was trying to answer the question originated from the problem of Ulam [18], and it is called a direct method because it allows us to construct the additive function directly from the given function f. In 2003, Radu [19] proposed the fixed point alternative method for obtaining the existence of exact solutions and error estimations. Subsequently, Miheţ [20] applied the fixed alternative method to study the fuzzy stability of the Jensen functional equation on the fuzzy space which is defined in [14].

Practically, the application of the two methods is successfully extended to obtain a fuzzy approximate solutions to functional equations [14, 20]. A comparison between the direct method and fixed alternative method for functional equations is given in [19]. The fixed alternative method can be considered as an advantage of this method over direct method in the fact that the range of approximate solutions is much more than the latter [14].

2. Preliminaries

Before obtaining the main result, we firstly introduce some useful concepts: a fuzzy normed linear space is a pair (X, N), where X is a real linear space and N is a fuzzy norm on X, which is defined as follow.

Definition 2.1 (cf. [6]). A function $N : X \times \mathbb{R} \to [0,1]$ (the so-called fuzzy subset) is said to be a fuzzy norm on X if for all $x, y \in X$ and all $s, t \in \mathbb{R}$, $N(x,\cdot)$ is left continuous for every x and satisfies

(N1) $N(x,c) = 0$ for $c \leq 0$;

(N2) $x = 0$ if and only if $N(x,c) = 1$ for all $c > 0$;

(N3) $N(cx,t) = N(x,t/|c|)$ if $c \neq 0$;

(N4) $N(x+y,s+t) \geq \min\{N(x,s), N(y,t)\}$;

(N5) $N(x,\cdot)$ is a nondecreasing function on $\mathbb{R}$ and $\lim_{t\to\infty} N(x,t) = 1$.

Let (X, N) be a fuzzy normed linear space. A sequence $\{x_n\}$ in X is said to be convergent if there exists $x \in X$ such that $\lim_{n\to\infty} N(x_n - x, t) = 1 (t > 0)$. In that case, x is called the limit of the sequence $\{x_n\}$ and we write $N - \lim x_n = x$.

A sequence $\{x_n\}$ in a fuzzy normed space (X, N) is called Cauchy if for each $\varepsilon > 0$ and $\delta > 0$, there exists $n_0 \in N$ such that $N(x_m - x_n, \delta) > 1 - \varepsilon(m, n \geq n_0)$. If each Cauchy sequence is convergent, then the fuzzy norm is said to be complete and the fuzzy normed space is called a fuzzy Banach space.

We recall the following result by Margolis and Diaz.

Lemma 2.2 (cf. [19, 21]). *Let (X, d) be a complete generalized metric space and let $J : X \to X$ be a strictly contractive mapping, that is,*

$$d(Jx, Jy) \leq Ld(x,y), \quad \forall x, y \in X, \tag{2.1}$$

for some $L \leq 1$. Then, for each fixed element $x \in X$, either

$$d\left(J^n x, J^{n+1} x\right) = +\infty, \quad \forall n \geq 0, \tag{2.2}$$

or

$$d\left(J^n x, J^{n+1} x\right) < +\infty, \quad \forall n \geq n_0, \tag{2.3}$$

for some natural number n_0*. Moreover, if the second alternative holds, then:*

(i) *the sequence* $\{J^n x\}$ *is convergent to a fixed point* y^* *of* J*;*

(ii) y^* *is the unique fixed point of* J *in the set* $Y := \{y \in X \mid d(J^{n_0} x, y) < +\infty\}$ *and* $d(y, y^*) \leq (1/(1-L))d(y, Jy)$*, for all* $x, y \in Y$*.*

3. Main Results

We start our works with a fuzzy generalized Hyers-Ulam-Rassias stability theorem for the Pexiderized quadratic functional equation (1.1). Due to some technical reasons, we first examine the stability for odd and even functions and then we apply our results to a general function.

The aim of this section is to give an alternative proof for that result in [15, Section 3], based on the fixed point method. Also, our method even provides a better estimation.

Theorem 3.1. *Let* X *be a linear space and let* (Z, N') *be a fuzzy normed space. Let* $\varphi : X \times X \to Z$ *be a function such that*

$$\varphi(2x, 2y) = \alpha\varphi(x, y), \quad \forall x, y \in X, \ t > 0, \tag{3.1}$$

for some real number α *with* $0 < |\alpha| < 2$*. Let* (Y, N) *be a fuzzy Banach space and let* f, g*, and* h *be odd functions from* X *to* Y *such that*

$$N(f(x+y) + f(x-y) - 2g(x) - 2h(y), t) \geq N'(\varphi(x, y), t), \quad \forall x, y \in X, \ t > 0. \tag{3.2}$$

Then there exists a unique additive mapping $T : X \to Y$ *such that*

$$N(T(x) - f(x), t) \geq M_1\left(x, \frac{2-|\alpha|}{2}t\right), \tag{3.3}$$

$$N(g(x) + h(x) - T(x), t) \geq M_1\left(x, \frac{6-3|\alpha|}{10-2|\alpha|}t\right), \tag{3.4}$$

where $M_1(x, t) = \min\{N'(\varphi(x, x), (2/3)t), N'(\varphi(x, 0), (2/3)t), N'(\varphi(0, x), (2/3)t)\}$*.*

The next Lemma 3.2 has been proved in [15, Proposition 3.1].

Lemma 3.2. *If* $\alpha > 0$*, then* $N(f(x) - 2^{-1}f(2x), t) \geq M_1(x, t)$ *and* $M_1(2x, t) = M_1(x, t/\alpha)$*, for all* $x \in X$*,* $t > 0$*.*

Proof of Theorem 3.1. Without loss of generality we may assume that $\alpha > 0$. By changing the roles of x and y in (3.2), we obtain

$$N(f(x+y) - f(x-y) - 2g(y) - 2h(x), t) \geq N'(\varphi(y,x), t). \tag{3.5}$$

It follows from (3.2), (3.5), and (N4) that

$$N(f(x+y) - g(x) - h(y) - g(y) - h(x), t) \geq \min\{N'(\varphi(x,y), t), N'(\varphi(y,x), t)\}. \tag{3.6}$$

Putting $y = 0$ in (3.6), we get

$$N(f(x) - g(x) - h(x), t) \geq \min\{N'(\varphi(x,0), t), N'(\varphi(0,x), t)\}. \tag{3.7}$$

Let $E := \{\phi \mid \phi : X \to Y,\ \phi(0) = 0\}$ and introduce the generalized metric d_{M_1}, define it on E by

$$d_{M_1}(\phi_1, \phi_2) = \inf\{\varepsilon \in (0,\infty) \mid N(\phi_1(x) - \phi_2(x), \varepsilon t) \geq M_1(x,t), \forall x \in X, t > 0\}. \tag{3.8}$$

Then, it is easy to verify that d_{M_1} is a complete generalized metric on E (see the proof of [22] or [23]). We now define a function $J_1 : E \to E$ by

$$J_1\phi(x) = \frac{1}{2}\phi(2x), \quad \forall x \in X. \tag{3.9}$$

We assert that J_1 is a strictly contractive mapping with the Lipschitz constant $\alpha/2$. Given $\phi_1, \phi_2 \in E$, let $\varepsilon \in (0,\infty)$ be an arbitrary constant with $d_{M_1}(\phi_1, \phi_2) \leq \varepsilon$. From the definition of d_{M_1}, it follows that

$$N(\phi_1(x) - \phi_1(x), \varepsilon t) \geq M_1(x,t), \quad \forall x \in X, t > 0. \tag{3.10}$$

Therefore,

$$\begin{aligned} N\left(J_1\phi_1(x) - J_1\phi_2(x), \frac{\alpha}{2}\varepsilon t\right) &= N\left(\frac{1}{2}\phi_1(2x) - \frac{1}{2}\phi_2(2x), \frac{\alpha}{2}\varepsilon t\right) \\ &= N(\phi_1(2x) - \phi_2(2x), \alpha\varepsilon t) \\ &\geq M_1(2x, \alpha t) = M_1(x,t), \quad \forall x \in X,\ t > 0. \end{aligned} \tag{3.11}$$

Hence, it holds that $d_{M_1}(J_1\phi_1, J_1\phi_2) \leq (\alpha/2)\varepsilon$, that is, $d_{M_1}(J_1\phi_1, J_1\phi_2) \leq (\alpha/2) d_{M_1}(\phi_1, \phi_2)$, for all $\phi_1, \phi_2 \in E$.

Next, from $N(f(x) - 2^{-1}f(2x), t) \geq M_1(x,t)$ (see Lemma 3.2), it follows that $d_{M_1}(f, J_1 f) \leq 1$. From the fixed point alternative, we deduce the existence of a fixed point of J_1, that is, the existence of a mapping $T : X \to Y$ such that $T(2x) = 2T(x)$ for each $x \in X$. Moreover, we have $d_{M_1}(J_1^n f, T) \to 0$, which implies

$$N - \lim_{n\to\infty} \frac{f(2^n x)}{2^n} = T(x), \quad \forall x \in X. \tag{3.12}$$

Also, $d_{M_1}(f,T) \leq (1/(1-L)) d_{M_1}(f, J_1 f)$ implies the inequality

$$d_{M_1}(f,T) \leq \frac{1}{1-(\alpha/2)} = \frac{2}{2-\alpha}. \tag{3.13}$$

If ε_n is a decreasing sequence converging to $2/(2-\alpha)$, then

$$N\big(T(x) - f(x), \varepsilon_n t\big) \geq M_1(x,t), \quad \forall x \in X,\ t > 0,\ n \in \mathbb{N}. \tag{3.14}$$

Then implies that

$$N\big(T(x) - f(x), t\big) \geq M_1\left(x, \frac{1}{\varepsilon_n} t\right), \quad \forall x \in X,\ t > 0,\ n \in \mathbb{N}, \tag{3.15}$$

that is, (as M_1 is left continuous)

$$N\big(T(x) - f(x), t\big) \geq M_1\left(x, \frac{2-\alpha}{2} t\right), \quad \forall x \in X,\ t > 0. \tag{3.16}$$

The additivity of T can be proved in a similar fashion as in the proof of Proposition 3.1 [15]. It follows from (3.3) and (3.7) that

$$\begin{aligned} &N\left(g(x) + h(x) - T(x), \frac{5-\alpha}{3} t\right) \\ &\quad \geq \min\left\{ N(f(x) - T(x), t), N\left(g(x) + h(x) - f(x), \frac{2-\alpha}{3} t\right)\right\} \\ &\quad \geq \min\left\{ M_1\left(x, \frac{2-\alpha}{2} t\right), N'\left(\varphi(x,0), \frac{2-\alpha}{3} t\right), N'\left(\varphi(0,x), \frac{2-\alpha}{3} t\right)\right\} \\ &\quad \geq M_1\left(x, \frac{2-\alpha}{2} t\right), \end{aligned} \tag{3.17}$$

whence we obtained (3.4).

The uniqueness of T follows from the fact that T is the unique fixed point of J_1 with the property that there exists $k \in (0, \infty)$ such that

$$N(T(x) - f(x), kt) \geq M_1(x, t), \quad \forall x \in X,\ t > 0. \tag{3.18}$$

This completes the proof of the theorem. □

Theorem 3.3. *Let X be a linear space and let (Z, N') be a fuzzy normed space. Let $\varphi : X \times X \to Z$ be a function such that*

$$\varphi(2x, 2y) = \alpha\varphi(x, y), \quad \forall x, y \in X,\ t > 0, \tag{3.19}$$

for some real number α with $0 < |\alpha| < 4$. Let (Y, N) be a fuzzy Banach space and let f, g, and h be even functions from X to Y such that $f(0) = g(0) = h(0) = 0$ and

$$N(f(x + y) + f(x - y) - 2g(x) - 2h(y), t) \geq N'(\varphi(x, y), t), \quad \forall x, y \in X,\ t > 0. \tag{3.20}$$

Then there exists a unique quadratic mapping $Q : X \to Y$ such that

$$\begin{aligned} N(Q(x) - f(x), t) &\geq M_1\left(x, \frac{4 - |\alpha|}{2} t\right), \\ N(Q(x) - g(x), t) &\geq M_1\left(x, \frac{12 - 3|\alpha|}{10 - |\alpha|} t\right), \\ N(Q(x) - h(x), t) &\geq M_1\left(x, \frac{12 - 3|\alpha|}{10 - |\alpha|} t\right), \end{aligned} \tag{3.21}$$

where $M_1(x, t) = \min\{N'(\varphi(x, x), (2/3)t), N'(\varphi(x, 0), (2/3)t), N'(\varphi(0, x), (2/3)t)\}$.

The following Lemma 3.4 has been proved in [15, Proposition 3.2].

Lemma 3.4. *If $\alpha > 0$, then $N(f(x) - 4^{-1}f(2x), t) \geq M_2(x, t)$ and $M_2(2x, t) = M_2(x, t/\alpha), \forall x \in X, t > 0$, where $M_2(x, t) = \min\{N'(\varphi(x, x), (4/3)t), N'(\varphi(x, 0), (4/3)t), N'(\varphi(0, x), (4/3)t)\}$.*

Proof of Theorem 3.3. Without loss of generality we may assume that $\alpha > 0$. By changing the roles of x and y in (3.20), we obtain

$$N(f(x + y) + f(x - y) - 2g(y) - 2h(x), t) \geq N'(\varphi(y, x), t). \tag{3.22}$$

Putting $y = x$ in (3.20), we get

$$N(f(2x) - 2g(x) - 2h(x), t) \geq N'(\varphi(x, x), t). \tag{3.23}$$

Putting $x = 0$ in (3.20), we get

$$N(2f(y) - 2h(y), t) \geq N'(\varphi(0, y), t). \tag{3.24}$$

Similarly, put $y = 0$ in (3.20) to obtain

$$N\left(2f(x) - 2g(x), t\right) \geq N'\left(\varphi(x, 0), t\right). \tag{3.25}$$

Let $E := \{\psi \mid \psi : X \to Y,\ \psi(0) = 0\}$ and introduce the generalized metric d_{M_2}, define it on E by

$$d_{M_2}(\psi_1, \psi_2) = \inf\left\{\varepsilon \in (0, \infty) \mid N\left(\psi_1(x) - \psi_2(x), \varepsilon t\right) \geq M_2(x, t), \forall x \in X, t > 0\right\}. \tag{3.26}$$

Then, it is easy to verify that d_{M_2} is a complete generalized metric on E (see the proof of [22] or [23]). We now define a function $J_2 : E \to E$ by

$$J_2\psi(x) = \frac{1}{4}\psi(2x), \quad \forall x \in X. \tag{3.27}$$

We assert that J_2 is a strictly contractive mapping with the Lipschitz constant $\alpha/4$. Given $\psi_1, \psi_2 \in E$, let $\varepsilon \in (0, \infty)$ be an arbitrary constant with $d_{M_2}(\psi_1, \psi_2) \leq \varepsilon$. From the definition of d_{M_2}, it follows that

$$N\left(\psi_1(x) - \psi_2(x), \varepsilon t\right) \geq M_2(x, t), \quad \forall x \in X,\ t > 0. \tag{3.28}$$

Therefore,

$$\begin{aligned} N\left(J_2\psi_1(x) - J_2\psi_2(x), \frac{\alpha}{4}\varepsilon t\right) &= N\left(\frac{1}{4}\psi_1(2x) - \frac{1}{4}\psi_2(2x), \frac{\alpha}{4}\varepsilon t\right) \\ &= N\left(\psi_1(2x) - \psi_2(2x), \alpha\varepsilon t\right) \\ &\geq M_2(2x, \alpha t) = M_2(x, t), \quad \forall x \in X,\ t > 0. \end{aligned} \tag{3.29}$$

Hence, it holds that $d_{M_2}(J_2\psi_1, J_2\psi_2) \leq (\alpha/4)\varepsilon$, that is, $d_{M_2}(J_2\psi_1, J_2\psi_2) \leq (\alpha/4)d_{M_2}(\psi_1, \psi_2)$, $\forall \psi_2, \psi_2 \in E$.

Next, from $N(f(x) - 4^{-1}f(2x), t) \geq M_2(x, t)$ (see Lemma 3.4), it follows that $d_{M_2}(f, J_2 f) \leq 1$. From the fixed alternative, we deduce the existence of a fixed point of J_2, that is, the existence of a mapping $Q : X \to Y$ such that $Q(2x) = 4Q(x)$ for each $x \in X$. Moreover, we have $d_{M_2}(J_2^n f, Q) \to 0$, which implies that

$$N - \lim_{n \to \infty} \frac{f(2^n x)}{4^n} = Q(x), \quad \forall x \in X. \tag{3.30}$$

Also, $d_{M_2}(f, Q) \leq (1/(1 - L))d_{M_2}(f, J_2 f)$ implies the inequality

$$d_{M_2}(f, Q) \leq \frac{1}{1 - \alpha/4} = \frac{4}{4 - \alpha}. \tag{3.31}$$

If ε_n is a decreasing sequence converging to $4/(4-\alpha)$, then

$$N(Q(x) - f(x), \varepsilon_n t) \geq M_2(x,t), \quad \forall x \in X,\ t > 0,\ n \in \mathbb{N}. \tag{3.32}$$

Then implies that

$$N(Q(x) - f(x), t) \geq M_2\left(x, \frac{1}{\varepsilon_n} t\right), \quad \forall x \in X,\ t > 0,\ n \in \mathbb{N}, \tag{3.33}$$

that is, (as M_2 is left continuous)

$$\begin{aligned} N(Q(x) - f(x), t) &\geq M_2\left(x, \frac{4-\alpha}{4} t\right) \\ &= M_1\left(x, \frac{4-\alpha}{2} t\right), \quad \forall x \in X,\ t > 0. \end{aligned} \tag{3.34}$$

The quadratic of Q can be proved in a similar fashion as in the proof of Proposition 3.2 [15].

It follows from (3.25) and (3.34) that

$$\begin{aligned} N\left(Q(x) - g(x), \frac{10-\alpha}{6} t\right) &\geq \min\left\{N(Q(x) - f(x), t), N\left(f(x) - g(x), \frac{4-\alpha}{6} t\right)\right\} \\ &\geq \min\left\{M_2\left(x, \frac{4-\alpha}{4} t\right), N'\left(\varphi(x,0), \frac{4-\alpha}{3} t\right)\right\} \\ &\geq M_2\left(x, \frac{4-\alpha}{4} t\right) \\ &= M_1\left(x, \frac{4-\alpha}{2} t\right), \end{aligned} \tag{3.35}$$

whence

$$N(Q(x) - g(x), t) \geq M_1\left(x, \frac{12-3\alpha}{10-\alpha} t\right). \tag{3.36}$$

A similar inequality holds for h. The rest of the proof is similar to the proof of Theorem 3.1. □

Theorem 3.5. *Let X be a linear space and let (Z, N') be a fuzzy normed space. Let $\varphi : X \times X \to Z$ be a function such that*

$$\varphi(2x, 2y) = \alpha\varphi(x,y), \quad \forall x, y \in X,\ t > 0, \tag{3.37}$$

for some real number α with $0 < |\alpha| < 2$. Let (Y, N) be a fuzzy Banach space and let f be a mapping from X to Y such that $f(0) = 0$ and

$$N(f(x+y) + f(x-y) - 2f(x) - 2f(y), t) \geq N'(\varphi(x,y), t), \quad \forall x, y \in X, \ t > 0. \tag{3.38}$$

Then there exist unique mapping T and Q from X to Y such that T is additive, Q is quadratic, and

$$N(f(x) - T(x) - Q(x), t) \geq M\left(x, \frac{2-|\alpha|}{8}t\right), \tag{3.39}$$

where $M(x,t)$=min$\{N'(\varphi(x,x), (2/3)t), N'(\varphi(-x,-x), (2/3)t), N'(\varphi(x,0), (2/3)t), N'(\varphi(0,x), (2/3)t), N'(\varphi(-x,0), (2/3)t), N'(\varphi(0,-x), (2/3)t)\}$.

Proof. Let $f_0(x) = (1/2)(f(x) - f(-x))$ for all $x \in X$, then $f_0(0) = 0$, $f_0(-x) = -f_0(x)$ and

$$N(f_0(x+y) + f_0(x-y) - 2f_0(x) - 2f_0(y), t) \geq \min\{N'(\varphi(x,y), t), N'(\varphi(-x,-y), t)\}. \tag{3.40}$$

Let $f_e(x) = (1/2)(f(x) + f(-x))$ for all $x \in X$, then $f_e(0) = 0$, $f_e(-x) = f_e(x)$ and

$$N(f_e(x+y) + f_e(x-y) - 2f_e(x) - 2f_e(y), t) \geq \min\{N'(\varphi(x,y), t), N'(\varphi(-x,-y), t)\}. \tag{3.41}$$

Using the proofs of Theorems 3.1 and 3.3, we get unique an additive mapping T and unique quadratic mapping Q satisfying

$$\begin{aligned} N(f_0(x) - T(x), t) &\geq M\left(x, \frac{2-|\alpha|}{4}t\right), \\ N(f_e(x) - Q(x), t) &\geq M\left(x, \frac{4-|\alpha|}{4}t\right). \end{aligned} \tag{3.42}$$

Therefore,

$$\begin{aligned} N(f(x) - T(x) - Q(x), t) &\geq \min\left\{N\left(f_0(x) - T(x), \frac{t}{2}\right), N\left(f_e(x) - Q(x), \frac{t}{2}\right)\right\} \\ &\geq \min\left\{M\left(x, \frac{2-|\alpha|}{8}t\right), M\left(x, \frac{4-|\alpha|}{8}t\right)\right\} \\ &= M\left(x, \frac{2-|\alpha|}{8}t\right). \end{aligned} \tag{3.43}$$

This completes the proof of the theorem. □

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