Hindawi Publishing Corporation
Fixed Point Theory and Applications
Volume 2008, Article ID 401947, 15 pages
doi:10.1155/2008/401947

Research Article

Bounded and Periodic Solutions of Semilinear Impulsive Periodic System on Banach Spaces

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Received 20 February 2008; Revised 6 April 2008; Accepted 7 July 2008

Recommended by Jean Mawhin

A class of semilinear impulsive periodic system on Banach spaces is considered. First, we introduce the T_0-periodic PC-mild solution of semilinear impulsive periodic system. By virtue of Gronwall lemma with impulse, the estimate on the PC-mild solutions is derived. The continuity and compactness of the new constructed *Poincaré* operator determined by impulsive evolution operator corresponding to homogenous linear impulsive periodic system are shown. This allows us to apply Horn's fixed-point theorem to prove the existence of T_0-periodic PC-mild solutions when PC-mild solutions are ultimate bounded. This extends the study on periodic solutions of periodic system without impulse to periodic system with impulse on general Banach spaces. At last, an example is given for demonstration.

1. Introduction

It is well known that impulsive periodic motion is a very important and special phenomenon not only in natural science but also in social science such as climate, food supplement, insecticide population, and sustainable development. There are many results, such as existence, the relationship between bounded solutions and periodic solutions, stability, food limited, and robustness, about impulsive periodic system on finite dimensional spaces (see [1–7]).

Although, there are some papers on periodic solution of periodic systems on infinite dimensional spaces (see [8–13]) and some results about the impulsive systems on infinite dimensional spaces (see [14–18]). Particulary, Professor Jean Mawhin investigated the periodic solutions of all kinds of systems on (in)finite dimensional spaces extensively (see [2, 19–23]). However, to our knowledge, nonlinear impulsive periodic systems on infinite dimensional spaces (with unbounded operator) have not been extensively investigated. There are only few works done by us about the impulsive periodic system (with unbounded operator) on infinite dimensional spaces (see [24–27]). We have been established periodic solution theory under the existence of a bounded solution for the linear impulsive periodic system on infinite dimensional spaces. Several criteria were obtained to ensure the existence, uniqueness, global asymptotical stability, alternative theorem, Massera's theorem, and Robustness of a T_0-periodic PC-mild solution for the linear impulsive periodic system.

Herein, we go on studying the semilinear impulsive periodic system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + f(t,x), \quad t \neq \tau_k, \\ \Delta x(t) &= B_k x(t) + c_k, \quad t = \tau_k, \end{aligned} \tag{1.1}$$

on infinite dimensional Banach space X, where $0 = \tau_0 < \tau_1 < \tau_2 < \cdots < \tau_k \cdots$, $\lim_{k\to\infty}\tau_k = \infty$, $\tau_{k+\delta} = \tau_k + T_0$, $\Delta x(\tau_k) = x(\tau_k^+) - x(\tau_k^-)$, $k \in \mathbb{Z}_0^+$, T_0 is a fixed positive number and $\delta \in \mathbb{N}$ denoted the number of impulsive points between 0 and T_0. The operator A is the infinitesimal generator of a C_0-semigroup $\{T(t),\ t \geq 0\}$ on X, f is a measurable function from $[0,\infty) \times X$ to X and is T_0-periodic in t, and$B_{k+\delta} = B_k$, $c_{k+\delta} = c_k$. This paper is mainly concerned with the existence of periodic solution for semilinear impulsive periodic system on infinite dimensional Banach space X.

In this paper, we use Horn's fixed-point theorem to obtain the existence of periodic solution for semilinear impulsive periodic system (1.1). First, by virtue of impulsive evolution operator corresponding to homogeneous linear impulsive system, we construct a new *Poincaré* operator P for semilinear impulsive periodic system (1.1), then we overcome some difficulties to show the continuity and compactness of *Poincaré* operator P which are very important. By virtue of Gronwall lemma with impulse, the estimate of PC-mild solutions is given. Therefore, the existence of T_0-periodic PC-mild solutions for semilinear impulsive periodic system when PC-mild solutions are ultimate bounded is shown.

This paper is organized as follows. In Section 2, some results of linear impulsive periodic system and properties of impulsive evolution operator corresponding to homogeneous linear impulsive periodic system are recalled. In Section 3, the Gronwall's lemma with impulse is collected and the T_0-periodic PC-mild solution of semilinear impulsive periodic system (1.1) is introduced. The new *Poincaré* operator P is constructed and the relation between T_0-periodic PC-mild solution and the fixed point of *Poincaré* operator P is given. After the continuity and compactness of *Poincaré* operator P are shown, the existence of T_0-periodic PC-mild solutions for semilinear impulsive periodic system is established by virtue of Horn's fixed-point theorem when PC-mild solutions are ultimate bounded. At last, an example is given to demonstrate the applicability of our result.

2. Linear impulsive periodic system

Let X be a Banach space. $\pounds(X)$ denotes the space of linear operators in X; $\pounds_b(X)$ denotes the space of bounded linear operators in X. $\pounds_b(X)$ is the Banach space with the usual supremum norm. Define $\widetilde{D} = \{\tau_1, \ldots, \tau_\delta\} \subset [0, T_0]$. We introduce $PC([0,T_0];X) \equiv \{x : [0,T_0] \to X \mid x$ is continuous at $t \in [0,T_0] \setminus \widetilde{D}$, x is continuous from left and has right-hand limits at $t \in \widetilde{D}\}$, and $PC^1([0,T_0];X) \equiv \{x \in PC([0,T_0];X) \mid \dot{x} \in PC([0,T_0];X)\}$. Set

$$\|x\|_{PC} = \max\left\{\sup_{t\in[0,T_0]}\|x(t+0)\|,\ \sup_{t\in[0,T_0]}\|x(t-0)\|\right\}, \qquad \|x\|_{PC^1} = \|x\|_{PC} + \|\dot{x}\|_{PC}. \tag{2.1}$$

It can be seen that endowed with the norm $\|\cdot\|_{PC}(\|\cdot\|_{PC^1})$, $PC([0,T_0];X)(PC^1([0,T_0];X))$ is a Banach space.

In order to study the semilinear impulsive periodic system, we first recall linear impulse periodic system here.

Firstly, we recall homogeneous linear impulsive periodic system

$$\begin{aligned} \dot{x}(t) &= Ax(t), \quad t \neq \tau_k, \\ \Delta x(t) &= B_k x(t), \quad t = \tau_k. \end{aligned} \tag{2.2}$$

We introduce the following assumption [H1].

[H1.1]: A is the infinitesimal generator of a C_0-semigroup $\{T(t),\ t \geq 0\}$ on X with domain $D(A)$.

[H1.2]: There exists δ such that $\tau_{k+\delta} = \tau_k + T_0$.

[H1.3]: For each $k \in \mathbb{Z}_0^+$, $B_k \in \pounds_b(X)$ and $B_{k+\delta} = B_k$.

In order to study system (2.2), we need to consider the associated Cauchy problem

$$\begin{aligned} \dot{x}(t) &= Ax(t), \quad t \in [0,T_0] \setminus \widetilde{D}, \\ \Delta x(\tau_k) &= B_k x(\tau_k), \quad k = 1,2,\ldots,\delta, \\ x(0) &= \overline{x}. \end{aligned} \tag{2.3}$$

If $\overline{x} \in D(A)$ and $D(A)$ is an invariant subspace of B_k, using [28, Theorem 5.2.2, page 144], step by step, one can verify that the Cauchy problem (2.3) has a unique classical solution $x \in PC^1([0,T_0];X)$ represented by $x(t) = S(t,0)\overline{x}$, where

$$S(\cdot,\cdot) : \Delta = \{(t,\theta) \in [0,T_0] \times [0,T_0] \mid 0 \leq \theta \leq t \leq T_0\} \longrightarrow \pounds(X), \tag{2.4}$$

given by

$$S(t,\theta) = \begin{cases} T(t-\theta), & \tau_{k-1} \leq \theta \leq t \leq \tau_k, \\ T(t-\tau_k^+)(I+B_k)T(\tau_k-\theta), & \tau_{k-1} \leq \theta < \tau_k < t \leq \tau_{k+1}, \\ T(t-\tau_k^+)\left[\prod_{\theta<\tau_j<t}(I+B_j)T(\tau_j-\tau_{j-1}^+)\right](I+B_i)T(\tau_i-\theta), & \\ & \tau_{i-1} \leq \theta < \tau_i \leq \cdots < \tau_k < t \leq \tau_{k+1}. \end{cases} \tag{2.5}$$

Definition 2.1. The operator $\{S(t,\theta),\ (t,\theta) \in \Delta\}$ given by (2.5) is called the impulsive evolution operator associated with $\{T(t),\ t \geq 0\}$ and $\{B_k;\tau_k\}_{k=1}^{\infty}$.

We introduce the PC-mild solution of Cauchy problem (2.3) and T_0-periodic PC-mild solution of system (2.2).

Definition 2.2. For every $\overline{x} \in X$, the function $x \in PC([0, T_0]; X)$ given by $x(t) = S(t, 0)\overline{x}$ is said to be the PC-mild solution of the Cauchy problem (2.3).

Definition 2.3. A function $x \in PC([0, +\infty); X)$ is said to be a T_0-periodic PC-mild solution of system (2.2) if it is a PC-mild solution of Cauchy problem (2.3) corresponding to some $\overline{x}$ and $x(t + T_0) = x(t)$ for $t \geq 0$.

The following lemma gives the properties of the impulsive evolution operator $\{S(t, \theta),\ (t, \theta) \in \Delta\}$ associated with $\{T(t),\ t \geq 0\}$ and $\{B_k; \tau_k\}_{k=1}^{\infty}$ are widely used in this paper.

Lemma 2.4 (see [24, Lemma 1]). *Impulsive evolution operator $\{S(t, \theta),\ (t, \theta) \in \Delta\}$ has the following properties.*

(1) *For $0 \leq \theta \leq t \leq T_0$, $S(t, \theta) \in \pounds_b(X)$, that is, there exists a constant $M_{T_0} > 0$ such that*

$$\sup_{0 \leq \theta \leq t \leq T_0} \|S(t, \theta)\| \leq M_{T_0}. \tag{2.6}$$

(2) *For $0 \leq \theta < r < t \leq T_0$, $r \neq \tau_k$, $S(t, \theta) = S(t, r)S(r, \theta)$.*

(3) *For $0 \leq \theta \leq t \leq T_0$ and $N \in \mathbb{Z}_0^+$, $S(t + NT_0, \theta + NT_0) = S(t, \theta)$.*

(4) *For $0 \leq t \leq T_0$ and $N \in \mathbb{Z}_0^+$, $S(NT_0 + t, 0) = S(t, 0)[S(T_0, 0)]^N$.*

(5) *If $\{T(t),\ t \geq 0\}$ is a compact semigroup in X, then $S(t, \theta)$ is a compact operator for $0 \leq \theta < t \leq T_0$.*

Secondly, we recall nonhomogeneous linear impulsive periodic system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + f(t), \quad t \neq \tau_k, \\ \Delta x(t) &= B_k x(t) + c_k, \quad t = \tau_k, \end{aligned} \tag{2.7}$$

where $f \in L^1([0, T_0]; X)$, $f(t + T_0) = f(t)$ for $t \geq 0$ and $c_{k+\delta} = c_k$.

In order to study system (2.7), we need to consider the associated Cauchy problem

$$\begin{aligned} \dot{x}(t) &= Ax(t) + f(t), \quad t \in [0, T_0] \setminus \widetilde{D}, \\ \Delta x(\tau_k) &= B_k x(\tau_k) + c_k, \quad k = 1, 2, \ldots, \delta, \\ x(0) &= \overline{x}, \end{aligned} \tag{2.8}$$

and introduce the PC-mild solution of Cauchy problem (2.8) and T_0-periodic PC-mild solution of system (2.7).

Definition 2.5. A function $x \in PC([0, T_0]; X)$, for finite interval $[0, T_0]$, is said to be a PC-mild solution of the Cauchy problem (2.8) corresponding to the initial value $\overline{x} \in X$ and input $f \in L^1([0, T_0]; X)$ if x is given by

$$x(t) = S(t, 0)\overline{x} + \int_0^t S(t, \theta) f(\theta) d\theta + \sum_{0 \leq \tau_k < t} S(t, \tau_k^+) c_k. \tag{2.9}$$

Definition 2.6. A function $x \in PC([0, +\infty); X)$ is said to be a T_0-periodic PC-mild solution of system (2.7) if it is a PC-mild solution of Cauchy problem (2.8) corresponding to some $\overline{x}$ and $x(t + T_0) = x(t)$ for $t \geq 0$.

Here, we note that system (2.2) has a T_0-periodic PC-mild solution x if and only if $S(T_0, 0)$ has a fixed point. The impulsive periodic evolution operator $\{S(t,\theta),\ (t,\theta) \in \Delta\}$ can be used to reduce the existence of T_0-periodic PC-mild solutions for system (2.7) to the existence of fixed points for an operator equation. This implies that we can use the uniform framework in [8, 13] to study the existence of periodic PC-mild solutions for impulsive periodic system on Banach space.

3. Semilinear impulsive periodic system

In order to derive the estimate of PC-mild solutions, we collect the following Gronwall's lemma with impulse which is widely used in sequel.

Lemma 3.1. *Let $x \in PC([0, T_0]; X)$ and satisfy the following inequality:*

$$\|x(t)\| \le a + b\int_0^t \|x(\theta)\| d\theta + \sum_{0<\tau_k<t} \zeta_k \|x(\tau_k)\|, \tag{3.1}$$

where $a, b, \zeta_k \ge 0$, are constants. Then, the following inequality holds:

$$\|x(t)\| \le a \prod_{0<\tau_k<t} (1+\zeta_k) e^{bt}. \tag{3.2}$$

Proof. Defining

$$u(t) = a + b\int_0^t \|x(\theta)\| d\theta + \sum_{0<\tau_k<t} \zeta_k \|x(\tau_k)\|, \tag{3.3}$$

we get

$$\begin{gathered} \dot{u}(t) = b\|x(t)\| \le bu(t), \quad t \ne \tau_k, \\ u(0) = a, \\ u(\tau_k^+) = u(\tau_k) + \zeta_k \|x(\tau_k)\| \le (1+\zeta_k) u(\tau_k). \end{gathered} \tag{3.4}$$

For $t \in (\tau_k, \tau_{k+1}]$, by (3.4), we obtain

$$u(t) \le u(\tau_k^+) e^{b(t-\tau_k)} \le (1+\zeta_k) u(\tau_k) e^{b(t-\tau_k)}, \tag{3.5}$$

further,

$$u(t) \le a \prod_{0<\tau_k<t} (1+\zeta_k) e^{bt}, \tag{3.6}$$

thus,

$$\|x(t)\| \le a \prod_{0<\tau_k<t} (1+\zeta_k) e^{bt}. \tag{3.7}$$

For more details the reader can refer to [5, Lemma 1.7.1]. □

Now, we consider the following semilinear impulsive periodic system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + f(t,x), \quad t \neq \tau_k, \\ \Delta x(t) &= B_k x(t) + c_k, \quad t = \tau_k. \end{aligned} \tag{3.8}$$

and introduce a suitable *Poincaré* operator and study the T_0-periodic PC-mild solutions of system (3.8).

In order to study the system (3.8), we first consider the associated Cauchy problem

$$\begin{aligned} \dot{x}(t) &= Ax(t) + f(t,x), \quad t \in [0,T_0] \setminus \widetilde{D}, \\ \Delta x(\tau_k) &= B_k x(\tau_k) + c_k, \quad k = 1,2,\ldots,\delta, \\ x(0) &= \overline{x}. \end{aligned} \tag{3.9}$$

Now, we can introduce the PC-mild solution of the Cauchy problem (3.9).

Definition 3.2. A function $x \in PC([0,T_0];X)$ is said to be a PC-mild solution of the Cauchy problem (3.9) corresponding to the initial value $\overline{x} \in X$ if x satisfies the following integral equation:

$$x(t) = S(t,0)\overline{x} + \int_0^t S(t,\theta)f(\theta,x(\theta))d\theta + \sum_{0 \le \tau_k < t} S(t,\tau_k^+)c_k. \tag{3.10}$$

Remark 3.3. Since one of the main difference of system (3.9) and other ODEs is the middle "jumping condition," we need verify that the PC-mild solution defined by (3.10) satisfies the middle "jumping condition" in (3.9). In fact, it comes from (3.10) and $S(\tau_k^+,\theta) = (I + B_k)S(\tau_k,\theta)$, for $0 \le \theta < \tau_k$, $k = 1,2,\ldots,\delta$, that

$$\begin{aligned} x(\tau_k^+) &= S(\tau_k^+,0)\overline{x} + \int_0^{\tau_k^+} S(\tau_k^+,\theta)f(\theta,x(\theta))d\theta + \sum_{0 \le \tau_k < \tau_k^+} S(\tau_k^+,\tau_k^+)c_k \\ &= (I + B_k)\left(S(\tau_k,0)\overline{x} + \int_0^{\tau_k} S(\tau_k,\theta)f(\theta,x(\theta))d\theta + \sum_{0 \le \tau_{k-1} < \tau_k} S(\tau_k,\tau_{k-1}^+)c_k \right) + c_k \\ &= (I + B_k)x(\tau_k) + c_k. \end{aligned} \tag{3.11}$$

It shows that $\Delta x(\tau_k) = B_k x(\tau_k) + c_k$, $k = 1,2,\ldots,\delta$.

In order to show the existence of the PC-mild solution of Cauchy problem (3.9) and T_0-periodic PC-mild solutions for system (3.8), we introduce assumption [H2].

[H2.1]: $f : [0,\infty) \times X \to X$ is measurable for $t \ge 0$ and for any $x, y \in X$ satisfying $\|x\|, \|y\| \le \rho$, there exists a positive constant $L_f(\rho) > 0$ such that

$$\|f(t,x) - f(t,y)\| \le L_f(\rho)\|x - y\|. \tag{3.12}$$

[H2.2]: There exists a positive constant $M_f > 0$ such that

$$\|f(t,x)\| \le M_f(1 + \|x\|) \quad \forall x \in X. \tag{3.13}$$

[H2.3]: $f(t,x)$ is T_0-periodic in t, that is, $f(t+T_0,x) = f(t,x)$, $t \geq 0$.

[H2.4]: For each $k \in \mathbb{Z}_0^+$ and $c_k \in X$, there exists $\delta \in \mathbb{N}$ such that $c_{k+\delta} = c_k$.

Now, we state the following result which asserts the existence of PC-mild solution for Cauchy problem (3.9) and gives the estimate of PC-mild solutions for Cauchy problem (3.9) by virtue of Lemma 3.1. A similar result for a class of generalized nonlinear impulsive integral differential equations is given by Xiang and Wei in [17]. Thus, we only sketch the proof here.

Theorem 3.4. *Assumptions [H1.1], [H2.1], and [H2.2] hold, and for each $k \in \mathbb{Z}_0^+$, $B_k \in £_b(X)$, $c_k \in X$ be fixed. Let $\overline{x} \in X$ be fixed. Then Cauchy problem* (3.9) *has a unique PC-mild solution given by*

$$x(t,\overline{x}) = S(t,0)\overline{x} + \int_0^t S(t,\theta) f(\theta, x(\theta,\overline{x}))d\theta + \sum_{0\leq\tau_k<t} S(t,\tau_k^+)c_k. \tag{3.14}$$

Further, suppose $\overline{x} \in \Xi \subset X$, Ξ is a bounded subset of X, then there exits a constant $M^* > 0$ such that

$$\|x(t,\overline{x})\| \leq M^* \quad \forall t \in [0,T_0]. \tag{3.15}$$

Proof. Under the assumptions [H1.1], [H2.1], and [H2.2], using the similar method of [28, Theorem 5.3.3, page 169], Cauchy problem

$$\begin{gathered} \dot{x}\,(t) = Ax(t) + f(t,x), \quad t \in [s,\tau], \\ x(s) = \overline{x} \in X, \end{gathered} \tag{3.16}$$

has a unique mild solution

$$x(t) = T(t)\overline{x} + \int_s^t T(t-\theta) f(\theta, x(\theta))d\theta. \tag{3.17}$$

In general, for $t \in (\tau_k, \tau_{k+1}]$, Cauchy problem

$$\begin{gathered} \dot{x}\,(t) = Ax(t) + f(t,x), \quad t \in (\tau_k,\tau_{k+1}], \\ x(\tau_k) = x_k \equiv (I+B_k)x(\tau_k) + c_k \in X \end{gathered} \tag{3.18}$$

has a unique PC-mild solution

$$x(t) = T(t-\tau_k)x_k + \int_{\tau_k}^t T(t-\theta) f(\theta, x(\theta))d\theta. \tag{3.19}$$

Combining all solutions on$[\tau_k, \tau_{k+1}]$ $(k = 1,\ldots,\delta)$, one can obtain the PC-mild solution of the Cauchy problem (3.9) given by

$$x(t,\overline{x}) = S(t,0)\overline{x} + \int_0^t S(t,\theta) f(\theta, x(\theta,\overline{x}))d\theta + \sum_{0\leq\tau_k<t} S(t,\tau_k^+)c_k. \tag{3.20}$$

Further, by assumption [H2.2] and (1) of Lemma 2.4, we obtain

$$\|x(t,\overline{x})\| \leq \left(M_{T_0}\|\overline{x}\| + M_{T_0}M_f T_0 + M_{T_0}\sum_{0\leq\tau_k<T_0}\|c_k\| \right) + M_{T_0}\int_0^t \|x(\theta,\overline{x})\|d\theta. \tag{3.21}$$

Since $\overline{x} \in \Xi \subset X$, Ξ is a bounded subset of X, using Lemma 3.1, one can obtain

$$\|x(t,\overline{x})\| \leq \left(M_{T_0}\|\overline{x}\| + M_{T_0}M_f T_0 + M_{T_0}\sum_{0\leq\tau_k<T_0}\|c_k\| \right)e^{M_{T_0}T_0} \equiv M^*, \quad \forall\, t \in [0,T_0]. \tag{3.22}$$

□

Now, we introduce the T_0-periodic PC-mild solution of system (3.8).

Definition 3.5. A function $x \in PC([0,+\infty);X)$ is said to be a T_0-periodic PC-mild solution of system (3.8) if it is a PC-mild solution of Cauchy problem (3.9) corresponding to some $\overline{x}$ and $x(t+T_0) = x(t)$ for $t \geq 0$.

In order to study the periodic solutions of the system (3.8), we construct a new *Poincaré* operator from X to X as follows:

$$P(\overline{x}) = x(T_0,\overline{x}) = S(T_0,0)\overline{x} + \int_0^{T_0} S(T_0,\theta)f(\theta,x(\theta,\overline{x}))d\theta + \sum_{0\leq\tau_k<T_0} S(T_0,\tau_k^+)c_k, \tag{3.23}$$

where $x(\cdot,\overline{x})$ denote the PC-mild solution of the Cauchy problem (3.9) corresponding to the initial value $x(0) = \overline{x}$.

We can note that a fixed point of P gives rise to a periodic solution as follows.

Lemma 3.6. *System* (3.8) *has a T_0-periodic PC-mild solution if and only if P has a fixed point.*

Proof. Suppose $x(\cdot) = x(\cdot + T_0)$, then $x(0) = x(T_0) = P(x(0))$. This implies that $x(0)$ is a fixed point of P. On the other hand, if $Px_0 = x_0$, $x_0 \in X$, then for the PC-mild solution $x(\cdot,x_0)$ of Cauchy problem (3.9) corresponding to the initial value $x(0) = x_0$, we can define $y(\cdot) = x(\cdot + T_0,x_0)$, then $y(0) = x(T_0,x_0) = Px_0 = x_0$. Now, for $t > 0$, we can use (2), (3), and (4) of Lemma 2.4 and assumptions [H1.2], [H1.3], [H2.3], [H2.4] to obtain

$$\begin{aligned}
y(t) &= x(t+T_0,x_0) \\
&= S(t+T_0,T_0)S(T_0,0)x_0 + \int_0^{T_0} S(t+T_0,T_0)S(T_0,\theta)f(\theta,x(\theta,x_0))d\theta \\
&\quad + \sum_{0\leq\tau_k<T_0} S(t+T_0,T_0)S(T_0,\tau_k^+)c_k + \int_{T_0}^{t+T_0} S(t+T_0,\theta)f(\theta,x(\theta,x_0))d\theta \\
&\quad + \sum_{T_0\leq\tau_{k+\delta}<t+T_0} S(t+T_0,\tau_{k+\delta}^+)c_{k+\delta} \\
&= S(t,0)\left\{ S(T_0,0)x_0 + \int_0^{T_0} S(T_0,\theta)f(\theta,x(\theta,x_0))d\theta + \sum_{0\leq\tau_k<T_0} S(T_0,\tau_k^+)c_k \right\} \\
&\quad + \int_0^t S(t+T_0,s+T_0)f(s+T_0,x(s+T_0,x_0))ds + \sum_{0\leq\tau_k<t} S(t,\tau_k^+)c_k \\
&= S(t,0)y(0) + \int_0^t S(t,s)f(s,y(s,y(0)))ds + \sum_{0\leq\tau_k<t} S(t,\tau_k^+)c_k.
\end{aligned} \tag{3.24}$$

This implies that $y(\cdot, y(0))$ is a PC-mild solution of Cauchy problem (3.9) with initial value $y(0) = x_0$. Thus, the uniqueness implies that $x(\cdot, x_0) = y(\cdot, y(0)) = x(\cdot + T_0, x_0)$ so that $x(\cdot, x_0)$ is a T_0-periodic. □

Next, we show that the operator P is continuous.

Lemma 3.7. *Assumptions [H1.1], [H2.1], and [H2.2] hold. Then, operator P is a continuous operator of $\overline{x}$ on X.*

Proof. Let $\overline{x}, \overline{y} \in \Xi \subset X$, where Ξ is a bounded subset of X. Suppose $x(\cdot, \overline{x})$ and $x(\cdot, \overline{y})$ are the PC-mild solutions of Cauchy problem (3.9) corresponding to the initial value $\overline{x}$ and $\overline{y} \in X$, respectively, given by

$$\begin{aligned} x(t,\overline{x}) &= S(t,0)\overline{x} + \int_0^t S(t,\theta) f(\theta, x(\theta,\overline{x}))d\theta + \sum_{0\le\tau_k<t} S(T_0,\tau_k^+)c_k; \\ x(t,\overline{y}) &= S(t,0)\overline{y} + \int_0^t S(t,\theta) f(\theta, x(\theta,\overline{y}))d\theta + \sum_{0\le\tau_k<t} S(T_0,\tau_k^+)c_k. \end{aligned} \tag{3.25}$$

Thus, by assumption [H2.2] and (1) of Lemma 2.4, we obtain

$$\begin{aligned} \|x(t,\overline{x})\| &\le \left(M_{T_0}\|\overline{x}\| + M_{T_0}M_f T_0 + M_{T_0}\sum_{0\le\tau_k<T_0}\|c_k\| \right) + M_{T_0}\int_0^t \|x(\theta,\overline{x})\|d\theta; \\ \|x(t,\overline{y})\| &\le \left(M_{T_0}\|\overline{y}\| + M_{T_0}M_f T_0 + M_{T_0}\sum_{0\le\tau_k<T_0}\|c_k\| \right) + M_{T_0}\int_0^t \|x(\theta,\overline{y})\|d\theta. \end{aligned} \tag{3.26}$$

By Lemma 3.1, one can verify that there exist constants M_1^* and $M_2^* > 0$ such that

$$\|x(t,\overline{x})\| \le M_1^*, \qquad \|x(t,\overline{y})\| \le M_2^*. \tag{3.27}$$

Let $\rho = \max\{M_1^*, M_2^*\} > 0$, then $\|x(\cdot,\overline{x})\|, \|x(\cdot,\overline{y})\| \le \rho$. By assumption [H2.1] and (1) of Lemma 2.4, we obtain

$$\begin{aligned} \|x(t,\overline{x}) - x(t,\overline{y})\| &\le \|S(t,0)\|\|\overline{x}-\overline{y}\| + \int_0^t \|S(t,\theta)\|\|f(\theta,x(\theta,\overline{x})) - f(\theta,x(\theta,\overline{y}))\|d\theta \\ &\le M_{T_0}\|\overline{x}-\overline{y}\| + M_{T_0}L_f(\rho)\int_0^t \|x(\theta,\overline{x}) - x(\theta,\overline{y})\|d\theta. \end{aligned} \tag{3.28}$$

By Lemma 3.1 again, one can verify that there exists a constant $M > 0$ such that

$$\|x(t,\overline{x}) - x(t,\overline{y})\| \le MM_{T_0}\|\overline{x}-\overline{y}\| \equiv L\|\overline{x}-\overline{y}\|, \quad \forall\, t \in [0,T_0], \tag{3.29}$$

which implies that

$$\|P(\overline{x}) - P(\overline{y})\| = \|x(T_0,\overline{x}) - x(T_0,\overline{y})\| \le L\|\overline{x}-\overline{y}\|. \tag{3.30}$$

Hence, P is a continuous operator of $\overline{x}$ on X. □

In the sequel, we need to prove the compactness of operator P, so we assume the following.

Assumption [H3]: The semigroup $\{T(t),\ t \geq 0\}$ is compact on X.

Now, we are ready to prove the compactness of operator P defined by (3.23).

Lemma 3.8. *Assumptions [H1.1], [H2.1], [H2.2], and [H3] hold. Then, the operator P is a compact operator.*

Proof. We only need to verify that P takes a bounded set into a precompact set on X. Let Γ is a bounded subset of X. Define $K = P\Gamma = \{P(\overline{x}) \in X \mid \overline{x} \in \Gamma\}$. For $0 < \varepsilon < t \leq T_0$, define $K_\varepsilon = P_\varepsilon \Gamma = S(T_0, T_0 - \varepsilon)\{x(T_0 - \varepsilon, \overline{x}) \mid \overline{x} \in \Gamma\}$.

Next, we show that K_ε is precompact on X. In fact, for $\overline{x} \in \Gamma$ fixed, we have

$$\begin{aligned} \|x(T_0-\varepsilon,\overline{x})\| &= \left\| S(T_0-\varepsilon,0)\overline{x} + \int_0^{T_0-\varepsilon} S(T_0-\varepsilon,\theta) f(\theta, x(\theta,\overline{x}))d\theta + \sum_{0\leq\tau_k<T_0-\varepsilon} S(T_0-\varepsilon,\tau_k^+)c_k \right\| \\ &\leq M_{T_0}\|\overline{x}\| + M_{T_0}M_f T_0 + \int_0^{T_0} \|x(\theta,\overline{x})\|d\theta + M_{T_0}\sum_{0\leq\tau_k<T_0}\|c_k\| \\ &\leq M_{T_0}\|\overline{x}\| + M_{T_0}M_f T_0 + T_0\rho + M_{T_0}\sum_{k=1}^{\delta}\|c_k\|. \end{aligned} \tag{3.31}$$

This implies that the set $\{x(T_0 - \varepsilon, \overline{x}) \mid \overline{x} \in \Gamma\}$ is bounded.

By assumption [H3] and (5) of Lemma 2.4, $S(T_0, T_0 - \varepsilon)$ is a compact operator. Thus, K_ε is precompact on X.

On the other hand, for arbitrary $\overline{x} \in \Gamma$,

$$P_\varepsilon(\overline{x}) = S(T_0,0)\overline{x} + \int_0^{T_0-\varepsilon} S(T_0,\theta) f(\theta, x(\theta,\overline{x}))d\theta + \sum_{0\leq\tau_k<T_0-\varepsilon} S(T_0,\tau_k^+)c_k, \tag{3.32}$$

thus, combined with (3.23), we have

$$\begin{aligned} \|P_\varepsilon(\overline{x}) - P(\overline{x})\| &\leq \left\| \int_0^{T_0-\varepsilon} S(T_0,\theta) f(\theta, x(\theta))d\theta - \int_0^{T_0} S(T_0,\theta) f(\theta, x(\theta))d\theta \right\| \\ &\quad + \left\| \sum_{0\leq\tau_k<T_0-\varepsilon} S(T_0,\tau_k^+)c_k - \sum_{0\leq\tau_k<T_0} S(T_0,\tau_k^+)c_k \right\| \\ &\leq \int_{T_0-\varepsilon}^{T_0} \|S(T_0,\theta)\|\|f(\theta,x(\theta))\|d\theta + M_{T_0}\sum_{T_0-\varepsilon\leq\tau_k<T_0}\|c_k\| \\ &\leq 2M_{T_0}M_f(1+\rho)\varepsilon + M_{T_0}\sum_{T_0-\varepsilon\leq\tau_k<T_0}\|c_k\|. \end{aligned} \tag{3.33}$$

It is showing that the set K can be approximated to an arbitrary degree of accuracy by a precompact set K_ε. Hence, K itself is precompact set on X. That is, P takes a bounded set into a precompact set on X. As a result, P is a compact operator. □

After showing the continuity and compactness of operator P, we can follow and derive periodic PC-mild solutions for system (3.8). In the sequel, we define the following definitions. The following definitions are standard, we state them here for convenient references. Note that the uniform boundedness and uniform ultimate boundedness are not required to obtain the periodic PC-mild solutions here, so we only define the (local) boundedness and ultimate boundedness.

Definition 3.9. PC-mild solutions of Cauchy problem (3.9) are said to be bounded if for each $B_1 > 0$, there is a $B_2 > 0$ such that $\|\overline{x}\| \le B_1$ implies $\|x(t,\overline{x})\| \le B_2$ for $t \ge 0$.

Definition 3.10. PC-mild solutions of Cauchy problem (3.9) are said to be locally bounded if for each $B_1 > 0$ and $k_0 > 0$, there is a $B_2 > 0$ such that $\|\overline{x}\| \le B_1$ implies $\|x(t,\overline{x})\| \le B_2$ for $0 \le t \le k_0$.

Definition 3.11. PC-mild solutions of Cauchy problem (3.9) are said to be ultimate bounded if there is a bound $B > 0$, such for each $B_3 > 0$, there is a $k > 0$ such that $\|\overline{x}\| \le B_3$ and $t \ge k$ imply $\|x(t,\overline{x})\| \le B$.

We also need the following results as a reference.

Lemma 3.12 (see [11, Theorem 3.1]). *Local boundedness and ultimate boundedness implies boundedness and ultimate boundedness.*

Lemma 3.13 (see [10, Lemma 3.1], Horn's fixed point theorem). *Let $E_0 \subset E_1 \subset E_2$ be convex subsets of Banach space X, with E_0 and E_2 compact subsets and E_1 open relative to E_2. Let $P : E_2 \to X$ be a continuous map such that for some integer m, one has*

$$\begin{aligned} P^j(E_1) &\subset E_2, \quad 1 \le j \le m-1, \\ P^j(E_1) &\subset E_0, \quad m \le j \le 2m-1, \end{aligned} \tag{3.34}$$

then P has a fixed point in E_0.

With these preparations, we can prove our main result in this paper.

Theorem 3.14. *Let assumptions [H1], [H2], and [H3] hold. If the PC-mild solutions of Cauchy problem* (3.9) *are ultimate bounded, then system* (3.8) *has a T_0-periodic PC-mild solution.*

Proof. By Theorem 3.4 and Definition 3.10, Cauchy problem (3.9) corresponding to the initial value $x(0) = \overline{x}$ has a PC-mild solution $x(\cdot,\overline{x})$ which is locally bound. From ultimate boundedness and Lemma 3.12, $x(\cdot,\overline{x})$ is bound. Next, let $B > 0$ be the bound in the definition of ultimate boundedness. Then, by boundedness, there is a $B_1 > B$ such that $\|\overline{x}\| \le B$ implies $\|x(t,\overline{x})\| \le B_1$ for $t \ge 0$. Furthermore, there is a $B_2 > B_1$ such that $\|\overline{x}\| \le B_1$ implies $\|x(t,\overline{x})\| \le B_2$ for $t \ge 0$. Now, using ultimate boundedness again, there is a positive integer m such that $\|\overline{x}\| \le B_1$ implies $\|x(t,\overline{x})\| \le B$ for $t \ge (m-2)T_0$.

Define $y(\cdot, y(0)) = x(\cdot + T_0, \overline{x})$, then $y(0) = x(T_0,\overline{x}) = P(\overline{x})$. From (3.24) in Lemma 3.6, we obtain $P(y(0)) = y(T_0, y(0)) = x(2T_0,\overline{x})$. Thus, $P^2(\overline{x}) = P(P(\overline{x})) = P(y(0)) = x(2T_0,\overline{x})$. Suppose there exists integer $m-1$ such that $P^{m-1}(\overline{x}) = x((m-1)T_0,\overline{x})$. By induction, we get the following:

$$P^m(\overline{x}) = P^{m-1}(P(\overline{x})) = P^{m-1}(y(0)) = y((m-1)T_0, y(0)) = x(mT_0,\overline{x}). \tag{3.35}$$

Thus, we obtain

$$\|P^{j-1}(\overline{x})\| = \|x((j-1)T_0, \overline{x})\| < B_2, \quad j = 1, 2, \ldots, m-1, \ \|\overline{x}\| < B_1;$$
$$\|P^{j-1}(\overline{x})\| = \|x((j-1)T_0, \overline{x})\| < B, \quad j \geq m, \ \|\overline{x}\| < B_1. \tag{3.36}$$

It comes from Lemma 3.8 that $P(\overline{x}) = x(T_0, \overline{x})$ on X is compact. Now let

$$H = \{\overline{x} \in X : \|\overline{x}\| < B_2\}, \qquad E_2 = \text{cl.}(\text{cov.}(P(H))),$$
$$W = \{\overline{x} \in X : \|\overline{x}\| < B_1\}, \qquad E_1 = W \cap E_2, \tag{3.37}$$
$$G = \{\overline{x} \in X : \|\overline{x}\| < B\}, \qquad E_0 = \text{cl.}(\text{cov.}(P(G))),$$

where cov.(Y) is the convex hull of the set Y defined by cov.$(Y) = \{\sum_{i=1}^{n} \lambda_i y_i \mid n \geq 1, \ y_i \in Y, \ \lambda_i \geq 0, \ \sum_{i=1}^{n} \lambda_i = 1\}$, and cl. denotes the closure. Then, we see that $E_0 \subset E_1 \subset E_2$ are convex subset of X with E_0, E_2 compact subsets, and E_1 open relative to E_2, and from (3.36), one has

$$P^j(E_1) \subset P^j(W) = PP^{j-1}(W) \subset P(H) \subset E_2, \quad j = 1, 2, \ldots, m-1;$$
$$P^j(E_1) \subset P^j(W) = PP^{j-1}(W) \subset P(G) \subset E_0, \quad j = m, m+1, \ldots, 2m-1. \tag{3.38}$$

We see that $P : E_2 \to X$ is a continuous map continuous from Lemma 3.7. Consequently, from Horn's fixed-point theorem, we know that the operator P has a fixed point $x_0 \in E_0 \subset X$. By Lemma 3.6, we know that the PC-mild solution $x(\cdot, x_0)$ of Cauchy problem (3.9), corresponding to the initial value $x(0) = x_0$, is just T_0-periodic. Therefore, $x(\cdot, x_0)$ is a T_0-periodic PC-mild solution of system (3.8). This proves the theorem. □

4. Application

In this section, an example is given to illustrate our theory. Consider the following boundary value problem

$$\frac{\partial}{\partial t} x(t, y) = \Delta x(t, y) + \sqrt{x^2(t, y) + 1} + \sin(t, y), \quad y \in \Omega, \ t \neq \tau_i, \ i = 1, 2, 3, 5, 6, 7, \ldots,$$
$$\Delta x(\tau_i, y) = \begin{cases} 0.05 I x(\tau_i, y), & i = 1, \\ -0.05 I x(\tau_i, y), & i = 2, \\ 0.05 I x(\tau_i, y), & i = 3, \end{cases} \quad y \in \Omega, \ \tau_i = \frac{i}{2}\pi, \ i = 1, 2, 3, 5, 6, 7, \ldots, \tag{4.1}$$
$$x(t, y) = 0, \quad y \in \partial\Omega, \ t > 0,$$

and the associated initial-boundary value problem

$$
\begin{aligned}
&\frac{\partial}{\partial t}x(t,y)=\Delta x(t,y)+\sqrt{x^2(t,y)+1}+\sin(t,y),\quad y\in\Omega,\ t\in(0,2\pi]\setminus\left\{\frac{1}{2}\pi,\pi,\frac{3}{2}\pi\right\},\\
&\Delta x(\tau_i,y)=\begin{cases}0.05Ix(\tau_i,y), & i=1,\\ -0.05Ix(\tau_i,y), & i=2,\\ 0.05Ix(\tau_i,y), & i=3,\end{cases}\quad y\in\Omega,\ \tau_i=\frac{i}{2}\pi,\ i=1,2,3,\\
&x(t,y)=0,\quad y\in\partial\Omega,\ t>0,\quad x(0,y)=x(2\pi,y),
\end{aligned}
\tag{4.2}
$$

where $\Omega\subset\mathbb{R}^3$ is bounded domain and $\partial\Omega\in C^3$.

Define $X=L_2(\Omega)$, $D(A)=H^2(\Omega)\cap H_0^1(\Omega)$, and $Ax=-(\partial^2x/\partial y_1^2+\partial^2x/\partial y_2^2+\partial^2x/\partial y_3^2)$ for $x\in D(A)$. Then, A generates a compact semigroup $\{T(t),\ t\geq 0\}$. Define $x(\cdot)(y)=x(\cdot,y)$, $\sin(\cdot)(y)=\sin(\cdot,y)$, $f(\cdot,x(\cdot))(y)=\sqrt{x^2(\cdot,y)+1}+\sin(\cdot,y)$, and

$$
B_i=\begin{cases}0.05I, & i=3m-2,\\ -0.05I, & i=3m-1,\\ 0.05I, & i=3m,\end{cases}\quad i,m\in\mathbb{N},
\tag{4.3}
$$

and $\tau_i=((i+m-1)/2)\pi$, $i,m\in\mathbb{N}$.

Thus, problem (4.1) can be rewritten as

$$
\begin{aligned}
\dot{x}(t)&=Ax(t)+f(t,x),\quad t\neq\tau_i,\ i=1,2,3,5,6,7,\ldots,\\
\Delta x(t)&=B_ix(t),\quad t=\tau_i,\ i=1,2,3,5,6,7,\ldots,
\end{aligned}
\tag{4.4}
$$

and problem (4.2) can be rewritten as

$$
\begin{aligned}
\dot{x}(t)&=Ax(t)+f(t,x),\quad t\in(0,2\pi]\setminus\left\{\frac{1}{2}\pi,\pi,\frac{3}{2}\pi\right\},\\
\Delta x\left(\frac{i}{2}\pi\right)&=B_ix\left(\frac{i}{2}\pi\right),\quad i=1,2,3,\\
x(0)&=x(2\pi).
\end{aligned}
\tag{4.5}
$$

If the PC-mild solutions of Cauchy problem (4.5) are ultimate bounded, then all the assumptions in Theorem 3.14 are met, our results can be used to system (4.4). That is, problem (4.1) has a 2π-periodic PC-mild solution $x_{2\pi}(\cdot,y)\in PC_{2\pi}([0+\infty);L_2(\Omega))$, where

$$
PC_{2\pi}([0,+\infty);L_2(\Omega))\equiv\{x\in PC([0,+\infty);L_2(\Omega))\mid x(t)=x(t+2\pi),\ t\geq 0\}.
\tag{4.6}
$$

Acknowledgments

This work is supported by National Natural Science foundation of China (no. 10661044) and Guizhou Province Found (no. 2008008). This work is partially supported by undergraduate carve out project of department of Guiyang City Science and Technology.

References

[1] D. Baĭnov and P. Simeonov, *Impulsive Differential Equations: Periodic Solutions and Applications*, vol. 66 of *Pitman Monographs and Surveys in Pure and Applied Mathematics*, Longman Scientific & Technical, Harlow, UK, 1993.

[2] C. Fabry, J. Mawhin, and M. N. Nkashama, "A multiplicity result for periodic solutions of forced nonlinear second order ordinary differential equations," *Bulletin of the London Mathematical Society*, vol. 18, no. 2, pp. 173–180, 1986.

[3] V. Lakshmikantham, D. D. Baĭnov, and P. S. Simeonov, *Theory of Impulsive Differential Equations*, vol. 6 of *Series in Modern Applied Mathematics*, World Scientific, Teaneck, NJ, USA, 1989.

[4] X. Liu, "Impulsive stabilization and applications to population growth models," *The Rocky Mountain Journal of Mathematics*, vol. 25, no. 1, pp. 381–395, 1995.

[5] T. Yang, *Impulsive Control Theory*, vol. 272 of *Lecture Notes in Control and Information Sciences*, Springer, Berlin, Germany, 2001.

[6] W. Wang, J. Shen, and J. J. Nieto, "Permanence and periodic solution of predator-prey system with Holling type functional response and impulses," *Discrete Dynamics in Nature and Society*, vol. 2007, Article ID 81756, 15 pages, 2007.

[7] J. Song, "Global attractivity of positive periodic solutions for an impulsive delay periodic "food limited" population model," *Discrete Dynamics in Nature and Society*, vol. 2006, Article ID 31614, 10 pages, 2006.

[8] J. H. Liu, "Bounded and periodic solutions of differential equations in Banach space," *Applied Mathematics and Computation*, vol. 65, no. 1–3, pp. 141–150, 1994.

[9] J. H. Liu, "Bounded and periodic solutions of semilinear evolution equations," *Dynamic Systems and Applications*, vol. 4, no. 3, pp. 341–350, 1995.

[10] J. H. Liu, "Bounded and periodic solutions of finite delay evolution equations," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 34, no. 1, pp. 101–111, 1998.

[11] J. Liu, T. Naito, and N. Van Minh, "Bounded and periodic solutions of infinite delay evolution equations," *Journal of Mathematical Analysis and Applications*, vol. 286, no. 2, pp. 705–712, 2003.

[12] P. Sattayatham, S. Tangmanee, and W. Wei, "On periodic solutions of nonlinear evolution equations in Banach spaces," *Journal of Mathematical Analysis and Applications*, vol. 276, no. 1, pp. 98–108, 2002.

[13] X. Xiang and N. U. Ahmed, "Existence of periodic solutions of semilinear evolution equations with time lags," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 18, no. 11, pp. 1063–1070, 1992.

[14] N. U. Ahmed, "Some remarks on the dynamics of impulsive systems in Banach spaces," *Dynamics of Continuous, Discrete & Impulsive Systems. Series A*, vol. 8, no. 2, pp. 261–274, 2001.

[15] N. U. Ahmed, K. L. Teo, and S. H. Hou, "Nonlinear impulsive systems on infinite dimensional spaces," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 54, no. 5, pp. 907–925, 2003.

[16] W. Wei, X. Xiang, and Y. Peng, "Nonlinear impulsive integro-differential equations of mixed type and optimal controls," *Optimization*, vol. 55, no. 1-2, pp. 141–156, 2006.

[17] X. Xiang and W. Wei, "Mild solution for a class of nonlinear impulsive evolution inclusions on Banach space," *Southeast Asian Bulletin of Mathematics*, vol. 30, no. 2, pp. 367–376, 2006.

[18] X. Xiang, W. Wei, and Y. Jiang, "Strongly nonlinear impulsive system and necessary conditions of optimality," *Dynamics of Continuous, Discrete & Impulsive Systems. Series A*, vol. 12, no. 6, pp. 811–824, 2005.

[19] J. Mawhin, "Periodic solutions of nonlinear functional differential equations," *Journal of Differential Equations*, vol. 10, pp. 240–261, 1971.

[20] J. Mawhin and J. R. Ward Jr., "Periodic solutions of some forced Liénard differential equations at resonance," *Archiv der Mathematik*, vol. 41, no. 4, pp. 337–351, 1983.

[21] J. Mawhin, "Periodic solutions of some semilinear wave equations and systems: a survey," *Chaos, Solitons & Fractals*, vol. 5, no. 9, pp. 1651–1669, 1995.

[22] J. Mawhin and H. B. Thompson, "Periodic or bounded solutions of Carathéodory systems of ordinary differential equations," *Journal of Dynamics and Differential Equations*, vol. 15, no. 2-3, pp. 327–334, 2003.

[23] J. Campos and J. Mawhin, "Periodic solutions of quaternionic-valued ordinary differential equations," *Annali di Matematica Pura ed Applicata*, vol. 185, supplement 5, pp. S109–S127, 2006.

[24] J. R. Wang, "Linear impulsive periodic system on Banach space," in *Proceedings of the 4th International Conference on Impulsive and Hybrid Dynamical Systems (ICIDSA '07)*, vol. 5, pp. 20–25, Nanning, China, July 2007.

[25] J. R. Wang, X. Xiang, and W. Wei, “Linear impulsive periodic system with time-varying generating operators on Banach space,” *Advances in Difference Equations*, vol. 2007, Article ID 26196, 16 pages, 2007.

[26] J. R. Wang, X. Xiang, and W. Wei, “Existence and global asymptotical stability of periodic solution for the T-periodic logistic system with time-varying generating operators and T_0-periodic impulsive perturbations on Banach spaces,” *Discrete Dynamics in Nature and Society*, vol. 2008, Article ID 524945, 16 pages, 2008.

[27] J. R. Wang, X. Xiang, and W. Wei, “Existence of periodic solution for semilinear periodic logistic systems with periodic impulsive perturbations on Banach space,” in *Proceedings of the 6th conference of Biomathematics, Advance in BioMathematics*, vol. 1, pp. 288–291, Tai’An, China, July 2008.

[28] N. U. Ahmed, *Semigroup Theory with Applications to Systems and Control*, vol. 246 of *Pitman Research Notes in Mathematics Series*, Longman Scientific & Technical, Harlow, UK, 1991.